1.1 Understanding Simple Interest

When you **invest** money, you can earn more money in the form of INTEREST.
(Also, when you **borrow** money, you must pay a fee in the form of *interest*.)

So...there is 'good' interest, and 'bad' interest.

Terms about Interest you should know.

Invest: When you use SAVINGS (money) to earn extra income.

Interest:

i) the money earned by investing money, or:

ii) the money paid for the use of borrowed money.

Principal: the original amount of money invested or borrowed.

Interest Rate: the interest **(annual)** used to calculate the interest paid on the money invested or borrowed.

Time: the length of time for an investment (usually in years).

Simple Interest: Interest calculated on the PRINCIPAL AMOUNT invested or borrowed only.

Amount: Sum of the PRINCIPAL and INTEREST.

(ie. A = P + I)

To calculate Interest you will need to convert percents to decimals and vice versa. Also, you will need to convert time units.

Percent Refresher!

To calculate anything with percent values you must convert the percentages to decimals by dividing by 100 (or, by moving the decimal point TWICE to the left)
Try:

a) 2.5%

$\frac{2.5}{100} = 0.025$

b) 10%

$\frac{10}{100} = 0.1$

c) 33%

$\frac{33}{100} = 0.33$

To calculate the percentage of a number, you must first convert that percentage to a decimal (see above), and then MULTIPLY that decimal by the number itself.
Try:

a) 5% of 200

$0.05 \times 200 = 10$

b) 12% of 1000

$0.12 \times 1000 = 120$

c) 7% of 5000

$0.07 \times 5000 = 350$

Time and Calculating Interest

All Interest calculations use time measured in years.

1 year = 12 months = 52 weeks = 365 days

Note: to make things simple we can say that a month has 30 days.

Refer to the worksheet and try a few with Mr. Quast, then try some on your own.

Calculating Simple Interest

Simple Interest Formula: $\underset{\text{(earned or owed)}}{Interest} = \underset{\text{(original amount)}}{Principal} \times \underset{\text{(as a decimal)}}{Interest\ rate} \times \underset{\text{(in years)}}{time}$

More simply: $I = Prt$

Examples:

1. Sean invested $3500 in a Guaranteed Investment Certificate (GIC) for 2 years. The interest rate is 2.5% per year. How much interest did he earn on his investment?

$$I = Prt$$
$$I = (3500)(0.025)(2)$$
$$I = \$175$$

2. Robyn is saving to buy a new motorcycle helmet. She has saved $600 and invests it for 6 months in an aggressive mosaic of mutual funds paying interest at 5.5%/yr.

   a) How much does she make on her investment?

$$I = Prt$$
$$= (600)(0.055)(0.5)$$
$$= \$16.50$$

$$\frac{6\text{ m}}{12\text{ m}} \left| \frac{1\text{ y}}{} \right. = 0.5\text{ y}$$

   b) How much will she have to buy the helmet?

$$A = P + I$$
$$= 600 + 16.50$$
$$= \$616.50$$

3. Sue is planning a trip to the U.S.
   - She invested $5000 in a US Foreign Currency Term Deposit.
   - The annual interest rate is 1.5%.
   - The deposit matures in 120 days.

   How much will Sue have for her trip?

$$\frac{120\,d}{365\,d} \left| \frac{1\,y}{} \right. = 0.329\,y$$

$$I = Prt$$

$$I = 5000(0.015)(0.329\,y)$$

$$I = 24.68$$

$$A = P + I$$

$$= \$5000 + \$24.68$$

$$= \$5024.68$$

Simple Interest Practice:

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1.2 More Simple Interest Problems

Examples

1. Jen is completing her tax form.
   - She earned $30.24 in simple interest from her bank.
   - She remembers investing $1200 with her bank for 1 year but cannot remember the interest rate. *Calculate the interest rate.*

$$I = Prt$$

$$r = \frac{I}{Pt}$$

$$r = \frac{30.24}{(1200)(1)}$$

$$r = 0.0252 \times 100\% = \boxed{2.52\%}$$

2. Stephan is saving to buy a used boom-lift for his tree-trimming business.
   - He needs $9800 and has saved $9475.
   - He plans to invest his savings at an annual interest rate of 1.59%.
   - How long must he invest his money to earn the additional $325 he needs?

$9800 - 9475 = 325$

$$I = Prt$$

$$t = \frac{I}{Pr}$$

$$t = \frac{325}{(9475)(0.0159)} = \boxed{2.2 \text{ years}}$$

Rearranging the Simple Interest Formula

To solve for t

$$t = \frac{I}{Pr}$$

To solve for r

$$r = \frac{I}{Pt}$$

To solve for P

$$P = \frac{I}{rt}$$

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+ Simple Interest Worksheet

1.3 Understanding Compound Interest

Compound Interest

Compound Interest is interest calculated on the *principal* AND the INTEREST.

To see how Compound Interest works we can use Simple Interest and a table such as the following. In this example: r = 2.1% per year

Year	Principal Amount (at the start of the year)	Simple Interest I = Prt	Amount (at the end of the year)
1	$10 000	$I = 10000(0.021)(1)$ $= 210$	$A = P + I$ $= 10210$
2	10210	$I = 10210(0.021)(1)$ $= 214.41$	$10210 + 214.41$ $= 10424.41$
3	10424.41	$I = 10424.41(0.021)(1)$ $= 218.91$	$10424.41 + 218.91$ $= 10643.32$
4	10643.32	$I = 10643.32(0.021)(1)$ $= 223.51$	$10643.32 + 223.51$ $= \boxed{10866.83}$

The Compound Interest Formula

$$A = P(1 + i)^n$$

Where

- A = AMOUNT (P + I)
- P = PRINCIPAL
- i = interest rate per compounding period ($i = \frac{r}{\#\text{ of comp. periods per yr.}}$)
- n = number of compounding periods total

Examples

1. Madelyn invested $10,000 four years ago. The investment earned compound interest at 2.1% /yr, compounded annually. Use the Compound Interest Formula to calculate the interest she earned.

$i = \frac{0.021}{1} = 0.021$

$A = P(1 + i)^n \qquad n = 4$

$A = 10000(1 + 0.021)^4$

$A = \boxed{\$10866.83}$

2. Owen invests $80,000 in a savings account that earns 2.75% /yr compounded annually. How much will he have in 3 years?

$i = \frac{0.0275}{1} = 0.0275$

$A = P(1 + i)^n$

$A = 80000(1 + 0.0275)^3$

$A = \boxed{\$86783.16}$

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1.5 Compounding Periods

Although interest rates are usually advertised as a RATE PER YEAR, they can by compounded in different ways.

Compounding Period Name	Number of times compounded during the year.
ANNUALLY	1
SEMI-ANNUALLY	2
QUARTERLY	4
MONTHLY	12
WEEKLY	52
DAILY	365

Changing the Compound Interest Formula

When calculating interest that is compounded more than once a year, we must change the Compound Interest Formula in two ways:

1. Change the interest rate. (i)

   *remember, i is the rate per ***compounding period***. So,

   $$i = \frac{r}{\#of\ compounding\ periods\ per\ year}$$

2. Change the number of times the interest is compounded (n)

   *to find n, multiply the number of years the investment exists by the number of compounding periods per year.

Complete the worksheet.

Ex. Kevin opens an account that has an interest rate of 1.8% /yr with the interest compounded monthly. His initial deposit is $2000. Calculate how much he will have at the end of 5 years.

$A = P(1 + i)^n$

$i = \frac{0.018}{12} = 0.0015$

$n = 5 \times 12 = 60$

$A = 2000(1 + 0.0015)^{60}$

$A = 2000(1.0015)^{60}$

$A = 2000(1.0941)$

$A = 2188.20$

Ex. Catrina has $40 000 to invest.

1. She decides to invest $20 000 in an account that pays 1.8% /yr compounded semi-annually.
2. The other $20 000 she invests in an account that pays 1.8% /yr compounded daily.

How much interest will she earn on each investment after 3 years?

Account 1

$A = P(1 + i)^n$

$i = \frac{0.018}{2} = 0.009$

$n = 3 \times 2 = 6$

$A = 20000(1 + 0.009)^6$

$A = 20000(1.0552)$

$A = 21,104.59$

$I = A - P$

$= 1104.59$

Account 2

$A = P(1 + i)^n$

$i = \frac{0.018}{365} = 0.00004932$

$n = 3 \times 365 = 1095$

$A = 20000(1 + 0.00004932)^{1095}$

$A = 20000(1.0555)$

$A = 21,109.66$

$I = A - P$

$= 1109.66$

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1.6 Compound Interest Problems

Review the Compound Interest Formula:

$$A = P(1+i)^n$$

A: Amount; P: Principal; i: interest rate per compounding period; n: # of times compounded

$$i = \frac{r}{\#\text{ of comp. periods per yr.}}$$

We can solve for any of the variables in the compound interest formula using our algebra skills (solving equations). Note: you will not be asked to ever solve for n! Requires knowledge of logarithms (part of Pre-Calculus Math 12).

Example. Daisy wants to start saving for a renovation of her business space in 5 years. The interest rate for Daisy's investment is 4.2% /yr compounded semi-annually. How much does Daisy need to invest now to have $6000 in 5 years?

$$A = P(1+i)^n$$

$$6000 = P(1+0.021)^{10}$$

$$P = \frac{6000}{(1+0.021)^{10}}$$

$$P = \$4874.09$$

$$i = \frac{0.042}{2}$$

$$i = 0.021$$

$$n = (5)(2)$$

$$n = 10$$

$$A = 6000$$

The Rule of 72

- The Rule of 72 is used to estimate how long it will take for an investment (Principal amount) to DOUBLE.
- It uses the following simple formula:

$$\text{Number of years for investment to double} = \frac{72}{\text{annual int. rate as a \%}}$$

Example. James runs his own business and as such is responsible for saving for his retirement. He invests in an RRSP (Registered Retirement Savings Plan) every year.

One year he invested $3000 at 3.6% /yr compounded annually. How long will it take for his money to double?

Estimate using the Rule of 72.

$$\text{\# of years to double} = \frac{72}{3.6} = \boxed{20 \text{ years}}$$

For fun: Check your estimate using the Compound Interest Formula.

$P = \$3000$

$A = \$6000$ (doubled!)

$i = 3.6\% = 0.036$

$n = ?$ try 20!

$$A = P(1+i)^n$$

$$6000 = 3000(1+0.036)^{20}$$

$$6000 = 6085.80$$

very close!

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