6.1 Slope

Slope

- The measure of the STEEPNESS or PITCH of a line.
- The ratio of the vertical (y) change to the horizontal (x) change between two points in a two-dimensional plane.
- Represented by the variable m.
- To ***calculate*** slope, TWO points on a line are required.
- To keep things simple and consistent, we will represent the two points with the general coordinates (x_1, y_1) and (x_2, y_2).

$$\text{Slope } (m) = \frac{\text{RISE}}{\text{RUN}} = \frac{\text{change in vertical } (y)}{\text{change in horizontal } (x)} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

when given two points: (x_1, y_1) (x_2, y_2)

Examples

Ex. 1: For each of the following pairs of points, calculate the slope of the line that connects them :

a. (1, 3) and (2, 7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{2-1} = \frac{4}{1} = \boxed{4}$$

b. (-5, 4) and (3, -1)

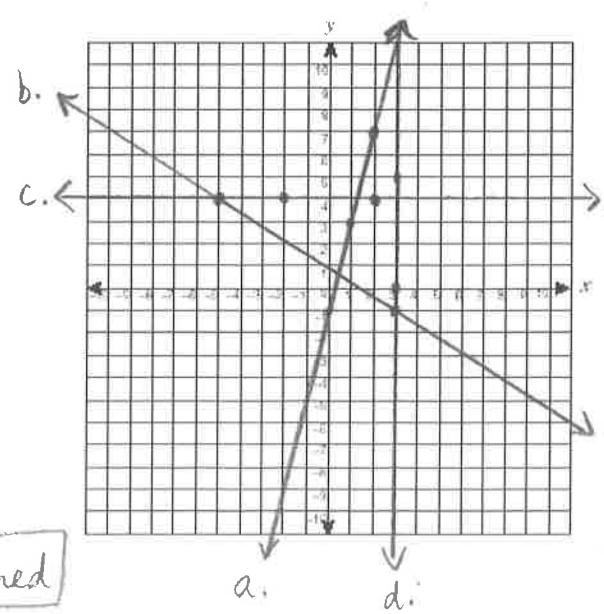
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{3-(-5)} = \boxed{\frac{-5}{8}}$$

c. (2, 4) and (-2, 4)

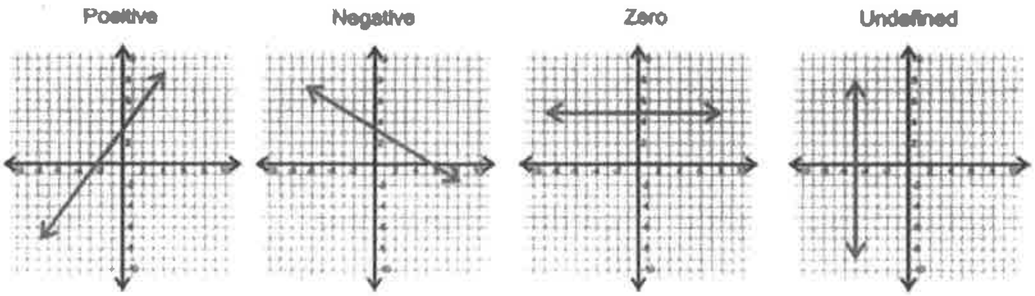
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{-2-2} = \frac{0}{-4} = \boxed{0}$$

d. (3, 5) and (3, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{3-3} = \frac{-5}{0} = \boxed{\text{undefined}}$$

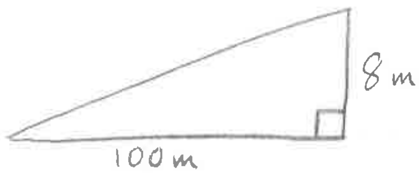


Conclusion: There are four different TYPES of slopes:



Ex. 2. Suppose you were approaching a hill that rose 8m for every 100m that you drove horizontally.

a) Draw a diagram to represent the situation.



NOT to scale!

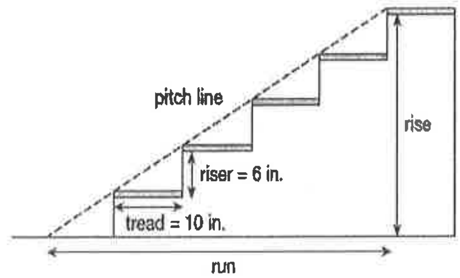
b) Calculate the slope of the road.

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{8}{100} = \boxed{\frac{2}{25}}$$

c) Often, slopes of hills on roads are called *grades* and are represented as a percentage. What would the grade of this hill be on a roadsign?

$$\frac{8}{100} = \boxed{8\% \text{ grade}}$$

Ex. 3. You are building a set of stairs that must have a 6 in. rise and a 10 in. tread depth (see diagram to the right):



a) Referring to the diagram; specifically the dotted pitch line, what is the pitch of each step?

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{6}{10} = \boxed{\frac{3}{5}}$$

b) What is the pitch of the entire staircase?

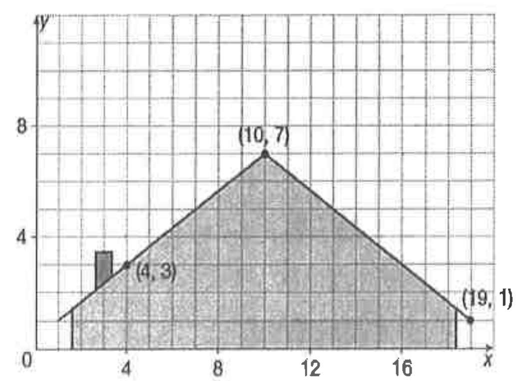
Total rise : 5 steps × 6 in. = 30 in.
Total run : 5 steps × 10 in. = 50 in.

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{30}{50} = \boxed{\frac{3}{5}}$$

c) What do the answers from a and b tell you?

Regardless of the two points used, the slope of a line is CONSISTENT.

Ex. 4. The following diagram represents a drawing of a roof design for a house:



a) What is the slope of the left side of the house?

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{10-4} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

b) What is the slope of the right side of the house?

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{10-19} = \frac{6}{-9} = \boxed{-\frac{2}{3}}$$

Opposite slopes, same PITCH / STEEPNESS.

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6.2 Comparing Slopes

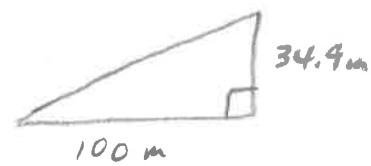
A new ski run has coordinates (50,780) and (1140, 1160) on a map. What is the grade of the new ski run?

a) Find the slope.

$$m = \frac{\text{RISE}}{\text{RUN}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1160 - 780}{1140 - 50} = \frac{380}{1090} = \boxed{\frac{38}{109}}$$

b) Find the percent (grade).

$$\frac{38}{109} = 0.349 = \boxed{34.9\%}$$ ie.



Comparing Slopes

Slopes can be represented and compared in three ways.

1. FRACTION Example: $\frac{3}{10}$ $\left(\frac{\text{rise}}{\text{run}}\right)$

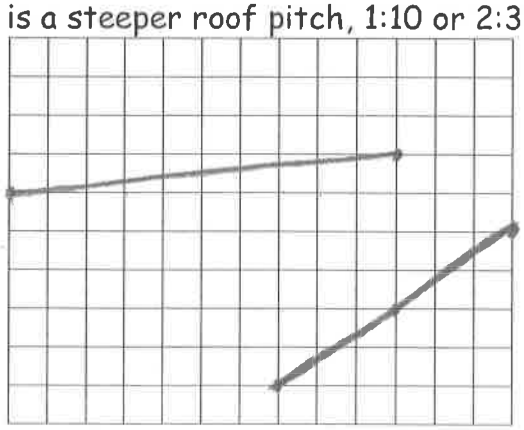
2. RATIO Example: 3 : 10 (rise : run)

3. GRADE Example: 30%

In order to most efficiently compare slopes, put them in GRADE / % form.

Examples

Ex. 1. Which is a steeper roof pitch, 1:10 or 2:3? Draw and calculate.



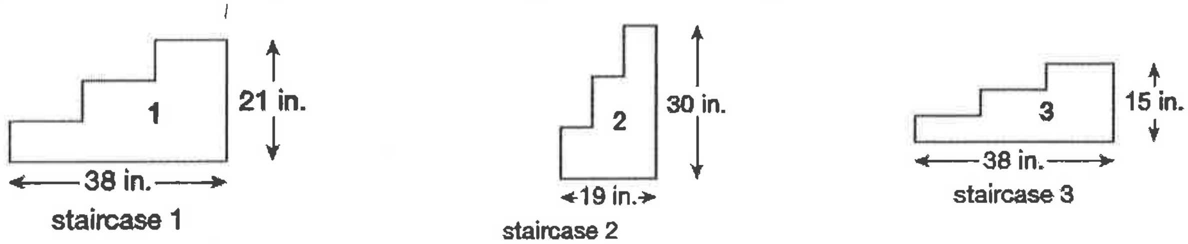
1 : 10 → 10%

2 : 3 → 67%

Why does roof pitch matter?

Water run-off / snow melt-off

Ex. 2. Milo is a carpenter. For a project he suggests staircases with a maximum grade of 80%. Compare the following staircase designs.



	Rise	Run	Grade
Staircase 1	21 in.	38 in.	$\frac{21}{38} = 0.553 = 55.3\%$
Staircase 2	30 in.	19 in.	$\frac{30}{19} = 1.579 = 157.9\%$
Staircase 3	15 in.	38 in.	$\frac{15}{38} = 0.395 = 39.5\%$

a) Order the staircases from least to greatest pitch/steepness.

3, 1, 2

b) Which staircase(s) could Milo use for the project?

1 and 3

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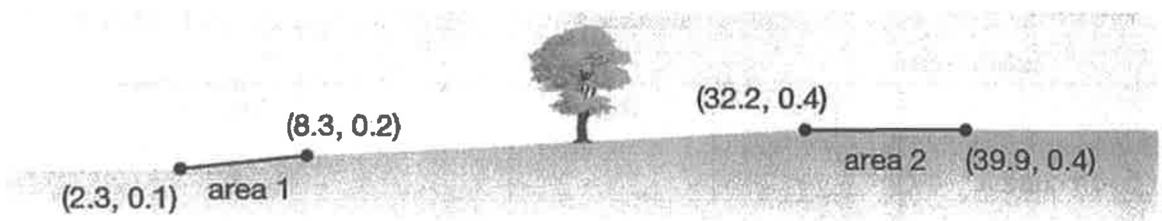
6.3 Vertical and Horizontal Lines

Examples

Ex. 1. Vicki is planning to build an above-ground swimming pool. The pool needs to be built on level, **horizontal** land. Where should she build the pool?



Area 1:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.2 - 0.1}{8.3 - 2.3} = \frac{0.1}{6} = \frac{1}{60}$$

Area 2:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.4 - 0.4}{39.9 - 32.2} = \frac{0}{7.7} = 0$$

Area 2!

*NOTE: All horizontal lines have a slope of 0.
(ie. when y_2 = y_1)

Ex. 2. Using the plan shown for a gazebo, determine if the posts are PLUMB (exactly **vertical**).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7.0 - 0.5}{2.0 - 2.0} = \frac{6.5}{0} = \text{undefined}$$

They are plumb!

*NOTE: All vertical lines have a slope that is UNDEFINED.
(ie. when x_2 = x_1)

p.149 #1-4

6.4 Solving Slope Problems

To solve problems with slope we have to use our **algebra** skills from Unit 1.

- Get the variable to one side of the equation and a number to the other.
- Remember the concept of OPPOSITE operations.
- Remember the GOLDEN RULE OF ALGEBRA.
- You may CROSS-MULTIPLY when a fraction equals a fraction.

Ex. Solve each for x: a. $\frac{5}{x}=\frac{4}{12}$ b. $\frac{x}{3}=\frac{50}{15}$

a.

$4x = 60$

$\frac{4x}{4} = \frac{60}{4}$

$\boxed{x = 15}$

b.

$15x = 150$

$\frac{15x}{15} = \frac{150}{15}$

$\boxed{x = 10}$

Examples

Ex. 1. Sue is installing a pipe in her new house. The minimum grade must be 4% and the vertical drop must be $\frac{1}{2}$ in. What does the horizontal run need to be?

$4\% = 0.04 = \frac{4}{100} = \frac{1}{25}$

$m = \frac{RISE}{RUN} \Rightarrow \frac{1}{25} = \frac{(\frac{1}{2})}{run}$ let x = run

$\frac{1}{25} = \frac{(\frac{1}{2})}{x} \longrightarrow x = (25)(\frac{1}{2}) = \frac{25}{2} = \boxed{12\frac{1}{2} \text{ in.}}$

Ex. 2. Penny knows that the slope of a ski run is $\frac{9}{20}$. The coordinates of the top and bottom of the run are (2m, 0m) and (1250m, y). What is the missing coordinate?

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{9}{20} = \frac{y-0}{1250-2}$

$\frac{9}{20} = \frac{y}{1248}$

$20y = 11232$

$\frac{20y}{20} = \frac{11232}{20}$

$\boxed{y = 561.6 \text{ m}}$

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p.157 #1-3

<u>6.6 Rate of Change</u>

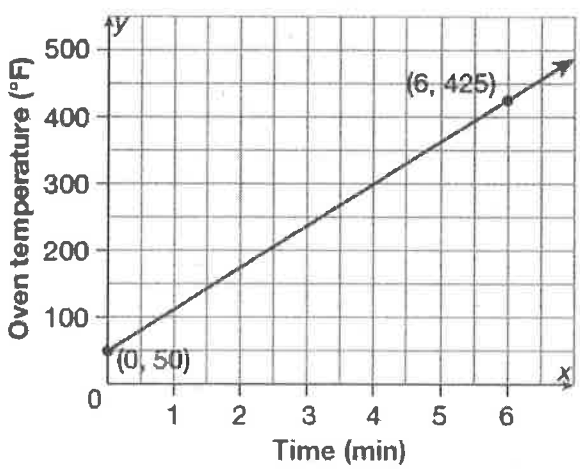
Rate of Change

- The change in one variable relative to the change in another variable.
  - Example: speed (velocity) is the change in DISTANCE relative to the change in *time*. Units for speed tend to have a numerator representing **distance** and a denominator representing **time**:
  - eg: $\frac{km}{h}$, $\frac{m}{s}$, $\frac{mm}{s}$, $\frac{km}{min.}$, etc...
- A negative rate of change indicates a DECREASE.
- A positive rate of change indicates an INCREASE.
- The rate of change can be represented by the SLOPE of a line.

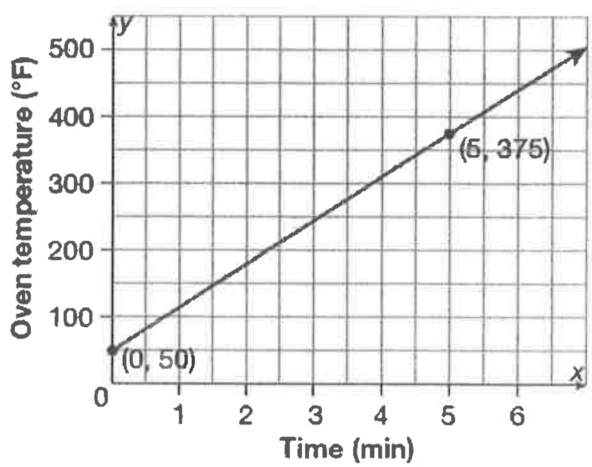
Examples

1. What is the rate of change for each oven's temperature, as represented in the following graphs?

Oven 1



Oven 2



HINT!!! The rate of change is equal to the **slope**.

Oven 1

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{425 - 50}{6 - 0} = \frac{375\,°F}{6\ min} = \boxed{\frac{62.5\,°F}{min}}$$

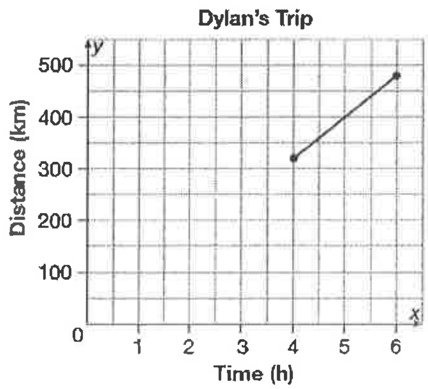
Oven 2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{375 - 50}{5 - 0} = \frac{325\,°F}{5\ min.} = \boxed{\frac{65\,°F}{min.}}$$

Which oven heats up faster?

Oven 2

2. Dylan was driving from Regina to Red Deer. After 4 hours, he had traveled 320 km. After 6 hours, he had traveled 480 km. What was the average rate of change between 4 h and 6 h?



a) Find the slope of the line segment.

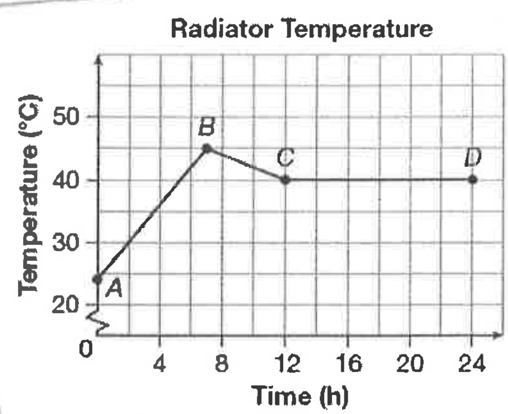
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{480 - 320}{6 - 4} = \frac{160 \text{ km}}{2 \text{ h}}$$

$$\boxed{= 80 \frac{\text{km}}{\text{h}}}$$

b) What is Dylan's average rate of change?

$$\boxed{80 \text{ km/h}}$$

3. Julia is a heating technician and she graphed the water temperature in a radiator system over a 24-hour period. What is the rate of change for each interval? (Note: there are 3 intervals: A to B, B to C, and C to D)



A to B:

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45 - 24}{7 - 0} = \frac{21\,°C}{7 \text{ h}} = \boxed{\frac{3\,°C}{h}}$$

B to C:

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40 - 45}{12 - 7} = \frac{-5\,°C}{5 \text{ h}} = \boxed{-1\,°C/h}$$

C to D:

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40 - 40}{24 - 12} = \frac{0\,°C}{12 \text{ h}} = \boxed{\frac{0\,°C}{h}}$$

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1-6

Ch. 6.7 - Rates and Unit Analysis

Multiplying a rate:

i. Multiply the numbers together;

ii. Cancel any units that match in the numerator and denominator;

iii. Express your answer in the unit that remains.

Eg1: Express the following in one unit:

a. $220\frac{mL}{s} \times 15\ s =$ $\frac{220\ mL}{\cancel{s}} \cdot \frac{15\ \cancel{s}}{} = \boxed{3300\ mL}$

b. $30\frac{km}{h} \times 4\ h =$ $\frac{30\ km}{\cancel{h}} \cdot \frac{4\ \cancel{h}}{} = \boxed{120\ km}$

c. $1.5\frac{^\circ C}{min} \times 100\ min. =$ $\frac{1.5^\circ C}{\cancel{min}} \cdot \frac{100\ \cancel{min}}{} = \boxed{150^\circ C}$

d. $9.4\ ^m/_s \times 1400\ s =$ $\frac{9.4\ m}{\cancel{s}} \cdot \frac{1400\ \cancel{s}}{} = \boxed{13160\ m}$

Canceling Units

There may be occasions where you would need to *reciprocate* the rate factor:

eg: A person traveling 50 km/h will travel 50 km in 1 hour. This can be written in two ways:

$$\frac{50\ km}{1\ h} \quad \text{or} \quad \frac{1\ h}{50\ km}$$

Eg2: Matt is delivering a grizzly bear from Victoria, BC, to Medicine Hat, Alberta. The distance that he has to drive is 2070 km. What is the difference in time between driving at 90 km/h or at 100 km/h?

$$2070\text{ km} \times \frac{1\text{ h}}{90\text{ km}} = 23\text{ h}$$

$$2070\text{ km} \times \frac{1\text{ h}}{100\text{ km}} = 20.7\text{ h}$$

$$23 - 20.7 = \boxed{2.3\text{ h}}$$

$$0.3\text{ h} \times \frac{60\text{ min}}{1\text{ h}} = 18\text{ min}$$

$$\boxed{2\text{ h } 18\text{ min}}$$

Time Conversions

Conversion Factors:

1 week = 7 days   1 day = 24 h   1 h = 60 mins.   1 min. = 60 s

Common mistake: 3 hours and 42 minutes ≠ 3.42 hours!!!

Eg3: Convert the following:

a. 25 minutes to seconds

$$25\,\cancel{\text{min}} \times \frac{60\text{ s}}{1\,\cancel{\text{min}}} = \boxed{1500\text{s}}$$

b. 3 hours to minutes

$$3\,\cancel{\text{h}} \times \frac{60\text{ min}}{1\,\cancel{\text{h}}} = \boxed{180\text{ min.}}$$

c. 4 hours to seconds

$$4\,\cancel{\text{h}} \times \frac{60\,\cancel{\text{min}}}{1\,\cancel{\text{h}}} \times \frac{60\text{ s}}{1\,\cancel{\text{min}}} = \boxed{14400\text{ s}}$$

d. 6 days to minutes

$$6\,\cancel{\text{d}} \times \frac{24\,\cancel{\text{h}}}{1\,\cancel{\text{d}}} \times \frac{60\text{ min.}}{1\,\cancel{\text{h}}} = \boxed{8640\text{ min.}}$$

e. 1 day to seconds

$$1\,\cancel{\text{d}} \times \frac{24\,\cancel{\text{h}}}{1\,\cancel{\text{d}}} \times \frac{60\,\cancel{\text{min.}}}{1\,\cancel{\text{h}}} \times \frac{60\text{ s}}{1\,\cancel{\text{min}}} = \boxed{86400\text{ min}}$$

f. 4 weeks to minutes

$$4\,\cancel{\text{wk.}} \times \frac{7\,\cancel{\text{d}}}{1\,\cancel{\text{wk}}} \times \frac{24\,\cancel{\text{h}}}{1\,\cancel{\text{d}}} \times \frac{60\text{ min.}}{1\,\cancel{\text{h}}} = \boxed{40320\text{ min.}}$$

Eg.4: The slush in a Slurpee™ machine rotates 591 times each hour. How many times does it rotate in a second?

$$\frac{591\text{ times}}{1\,\cancel{\text{h}}} \times \frac{1\,\cancel{\text{h}}}{60\,\cancel{\text{min}}} \times \frac{1\,\cancel{\text{min}}}{60\text{ s}} = \boxed{0.164\text{ times per s.}}$$

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Ch. 6.8 - Conversion within Measurement Systems

All applicable conversion charts are located in the inside of the front cover of your textbook. Your formula sheet has most of these as well.

Simple, One-step Conversions

Use the conversion charts to convert the following:

a. 6 feet to inches

$$\frac{6\text{ ft.}}{} \left| \frac{12\text{ in.}}{1\text{ ft.}} \right. = \boxed{72\text{ in.}}$$

b. 4 gal (US) to quarts (qt.)

$$\frac{4\text{ gal}}{} \left| \frac{4\text{ qt.}}{1\text{ gal}} \right. = \boxed{16\text{ qt.}}$$

c. 4 cups to ounces (US)

$$\frac{4\text{ c.}}{} \left| \frac{8\text{ oz.}}{1\text{ c.}} \right. = \boxed{32\text{ oz.}}$$

d. 350 cm to m

$$\frac{350\text{ cm}}{} \left| \frac{1\text{ m}}{100\text{ cm}} \right. = \boxed{3.5\text{ m}}$$

Two-step Conversions or Greater

For multiple step conversions within the metric system, you should include the base unit in your conversion calculation:

Metric Base Units:

Distance: meter (m); Volume: liter (L); Mass: gram (g)

Use the conversion charts to convert the following:

a. 3 km to cm

$$\frac{3\text{ km}}{} \left| \frac{1000\text{ m}}{1\text{ km}} \right| \frac{100\text{ cm}}{1\text{ m}} = \boxed{300\,000\text{ cm}}$$

b. 7400 mL to kL

$$\frac{7400\text{ mL}}{} \left| \frac{1\text{ L}}{1000\text{ mL}} \right| \frac{1\text{ kL}}{1000\text{ L}} = \boxed{0.0074\text{ kL}}$$

c. 2 miles to feet

$$\frac{2\text{ mi.}}{} \left| \frac{1760\text{ yd}}{1\text{ mi.}} \right| \frac{3\text{ ft.}}{1\text{ yd}} = \boxed{10\,560\text{ ft.}}$$

d. 5.6 gallons to cups (US)

$$\frac{5.6\text{ gal}}{} \left| \frac{4\text{ qt.}}{1\text{ gal.}} \right| \frac{2\text{ pt.}}{1\text{ qt.}} \left| \frac{2\text{ c.}}{1\text{ pt.}} \right. = \boxed{89.6\text{ c.}}$$

More Examples:

1. If you have 200 grams of peanuts, 0.250 kg of raisins, 325 g of M & Ms, 150 grams of almonds, and 0.5 hg of sunflower seeds, what total mass of Trail Mix do you have?

$$\frac{0.250\ \cancel{kg}}{} \cdot \frac{1000\ g}{1\ \cancel{kg}} = 250\ g \text{ raisins}$$

$$\frac{0.5\ \cancel{hg}}{} \cdot \frac{100\ g}{1\ \cancel{hg}} = 50\ g \text{ sunny seeds}$$

$$200\ g + 250\ g + 325\ g + 150\ g + 50\ g = \boxed{975\ g}$$

2. A can of pop contains 355 mL. In Europe, cans are labeled in cL. How many cL in your can of pop?

$$\frac{355\ \cancel{mL}}{} \cdot \frac{1\ \cancel{L}}{1000\ \cancel{mL}} \cdot \frac{1\ cL}{0.01\ \cancel{L}} = \boxed{35.5\ cL}$$

3. Jon can run at a rate of 7.9 m/s. How fast is he running in km/h?

$$\frac{7.9\ \cancel{m}}{1\ \cancel{s}} \cdot \frac{1\ km}{1000\ \cancel{m}} \cdot \frac{60\ \cancel{s}}{1\ \cancel{min}} \cdot \frac{60\ \cancel{min}}{1\ h} = \boxed{28.44\ km/h}$$

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Ch. 6.9 – Conversion between Measurement Systems

Useful conversion factors: (see your Formula Sheet)

1 mile = 1.609 km (1 km = 0.6215 mi)	1 yard = 0.9144 m (1 m = 1.0936 yd)	1 foot = 30.48 cm (1 cm = 0.0328 ft)
1 inch = 2.54 cm (1 cm = 0.3937 in)	1 kg = 2.2 lbs (1 lb = 0.4545 kg)	1 lb = 454 g (1 g = 0.0022 lb)
1 ounce = 28.35 g (1 g = 0.0353 ounce)	1 gallon (US) = 3.79 L (1 L = 0.264 gal (US))	

Eg1: Convert each of the following:

a. 3 miles to km

$$\frac{3\ \cancel{mi.}}{} \Big| \frac{1.609\ km}{1\ \cancel{mi.}} = \boxed{4.827\ km}$$

b. 4 cm to inches

$$\frac{4\ \cancel{cm}}{} \Big| \frac{1\ in.}{2.54\ \cancel{cm}} = \boxed{1.575\ in.}$$

c. 2 gallons (US) to litres

$$\frac{2\ \cancel{gal}}{} \Big| \frac{3.79\ L}{1\ \cancel{gal}} = \boxed{7.58\ L}$$

d. 195 lbs to kg

$$\frac{195\ \cancel{lb.}}{} \Big| \frac{1\ kg}{2.2\ \cancel{lb.}} = \boxed{88.64\ kg}$$

e. 100 yd to meters

$$\frac{100\ \cancel{yd.}}{} \Big| \frac{0.9144\ m}{1\ \cancel{yd.}} = \boxed{91.44\ m}$$

f. 6 ft 4 in to cm

$$4\ in + \frac{6\ \cancel{ft.}}{} \Big| \frac{12\ in.}{1\ \cancel{ft}} = 72\ in = 76\ in.$$

$$\frac{76\ \cancel{in.}}{} \Big| \frac{2.54\ cm}{1\ \cancel{in.}} = \boxed{193.04\ cm}$$

Eg2: The speed of sound is 1116 ft/s. How fast is it in km/h?

$$\frac{1116\ \cancel{ft}}{1\ \cancel{s}} \Big| \frac{30.48\ \cancel{cm}}{1\ \cancel{ft}} \Big| \frac{1\ \cancel{m}}{100\ \cancel{cm}} \Big| \frac{1\ km}{1000\ \cancel{m}} \Big| \frac{60\ \cancel{s}}{1\ \cancel{min}} \Big| \frac{60\ \cancel{min}}{1\ h}$$

$$= \boxed{1224.6\ km/h}$$

Eg3: Rainfall data is collected and presented in a table below. Which location had the most amount of rainfall?

Area A	Area B
0.7 in.	19 mm
1.4 in.	38 mm
2.1 in.	57 mm

4.2 in. 114 mm

$$\frac{4.2\ \cancel{in.}}{} \cdot \frac{2.54\ \cancel{cm}}{1\ \cancel{in}} \cdot \frac{10\ mm}{1\ \cancel{cm}}$$

= 106.7 mm

$\boxed{\text{Area B}}$

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