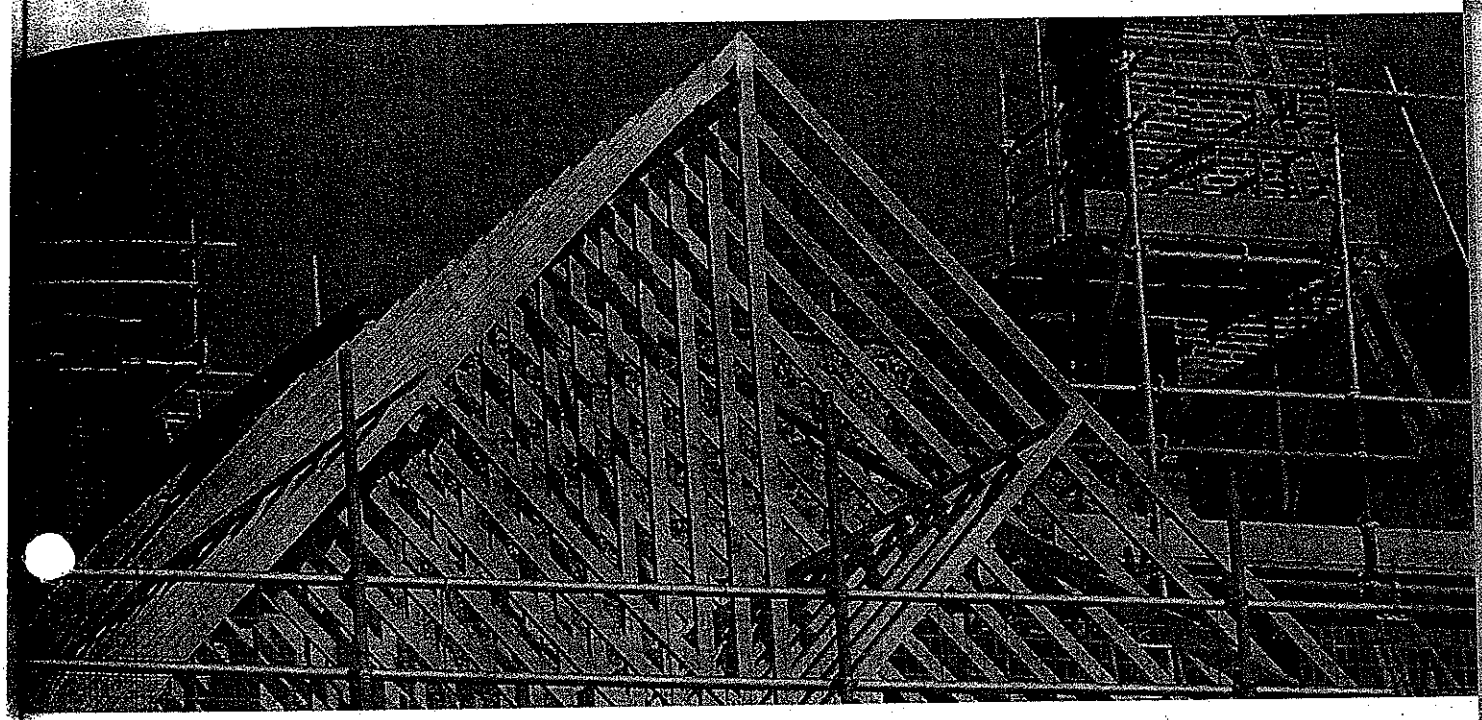
Slope and Rates 6



José and Marjorie are working on the roof of a house in Estevan. They had to build the roof trusses at an angle that matches the building design.

A. Why might one build a roof with an incline instead of a flat roof?

e.g., A roof with an incline lets debris and rainwater drain off. Melting snow and ice can drain off a roof with an incline. On a flat roof, snow and ice would pile up too much.

B. What are some other places where an incline would be important?

e.g., An incline is important for ramps, ski runs, positions of pipes, or mountain roads.

6 Getting Started

Hint
Divide by the greatest number that divides into both the numerator and denominator.

1. Write each fraction in lowest terms. The first one is started for you.

a) $\frac{6}{8} = \frac{\boxed{3}}{4}$ **b)** $\frac{12}{15} = \frac{4}{5}$ **c)** $\frac{10}{4} = \frac{5}{2}$

2. Write each decimal as a fraction in lowest terms. The first one is started for you.

a) $0.5 = \frac{\boxed{5}}{10}$, or $\frac{\boxed{1}}{2}$ **c)** $0.24 = \frac{24}{100}$, or $\frac{6}{25}$

b) $0.68 = \frac{68}{100}$, or $\frac{17}{25}$ **d)** $-0.8 = \frac{-8}{10}$, or $\frac{-4}{5}$

3. Write each ratio in lowest terms. The first one is started for you.

a) $2:6 = 1:\boxed{3}$ **c)** $12:18 = 2:3$

b) $28:21 = 4:3$ **d)** $-6:4 = -3:2$

4. Multiply or divide.

a) $-6 \div 1 = -6$ **c)** $-4 \times 7 = -28$

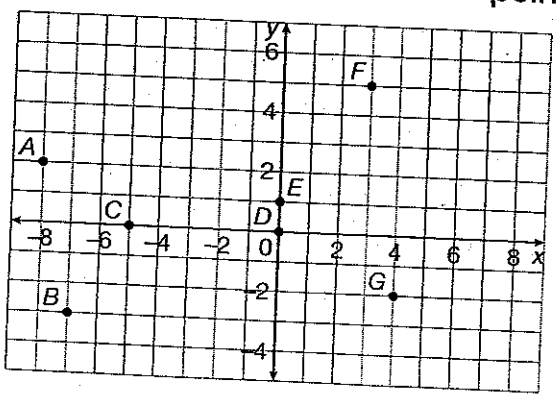
b) $-5 \times (-6) = 30$ **d)** $-15 \div (-5) = 3$

5. Subtract.

a) $-9 - (-4) = -5$ **c)** $-10 - 5 = -15$

b) $7 - (-7) = 14$ **d)** $-8 - (-8) = 0$

6. What are the coordinates of each point?



A (−8, 2)
B (−7, −3)
C (−5, 0)
D (0, 0)
E (0, 1)
F (3, 5)
G (4, −2)

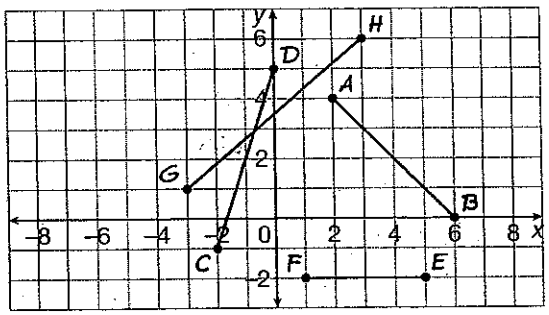
7. Plot each pair of points. Connect the two points in each pair.

a) $A(2, 4)$ $B(6, 0)$

b) $C(-2, -1)$ $D(0, 5)$

c) $E(5, -2)$ $F(1, -2)$

d) $G(-3, 1)$ $H(3, 6)$



8. Solve for x. The first one is done for you.

a) $3x = 12$

$3x \div 3 = 12 \div 3$

$x = 4$

b) $\frac{x}{6} = 10$

e.g., $\frac{6x}{6} = 6(10)$

$x = 60$

c) $x - 9 = 14$

e.g., $x - 9 + 9 = 14 + 9$

$x = 23$

d) $x + 30 = 75$

e.g., $x + 30 - 30 = 75 - 30$

$x = 45$

9. Calculate the **tangent** for each angle.

a) $\tan 83° =$ 8.1443... **b)** $\tan 5° =$ 0.0874...

10. What is the measure of each angle, to the nearest degree?

a) $\tan^{-1}(0.5317) =$ 28° **b)** $\tan^{-1}(3.4874) =$ 74°

Tech Tip

Tangent

Make sure your calculator is in degree mode.

Try calculating $x°$ for $\tan x° = 1.0000$ by pressing 2nd tan 1.0000 = or 1.0000 2nd tan =

11. Solve each equation. The first two have been started for you.

a) $\frac{y}{15} = \frac{3}{5}$

$15 \times \frac{y}{15} = 15 \times \frac{3}{5}$

$y =$ 9

b) $\frac{7}{y} = \frac{1}{4}$

$y \times \frac{7}{y} = y \times \frac{1}{4}$

4 $\times 7 =$ 4 $\times y \times \frac{1}{4}$

28 = y

c) $\frac{y}{3} = 1.5$

e.g., $3 \times \frac{y}{3} = 3 \times 1.5$

$y = 4.5$

d) $\frac{25}{y} = 5$

e.g., $y \times \frac{25}{y} = y \times 5$

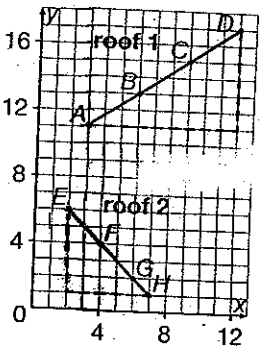
$25 = 5y$

$5 = y$

E. How are the slopes different? What does this show?

e.g., One is positive. One is negative. This shows that the parts of the roof are slanted in opposite directions.

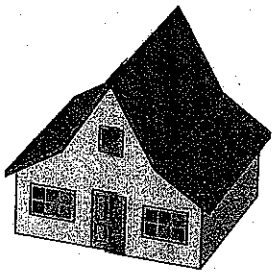
Practice



1. a) Xavier is making plans for a building project. He drew the roof sections on the left. He needs to check that the pitch of each roof section is constant. Complete the chart. Are any changes needed? Explain.

Line segment	Rise	Run	Slope
AD	6	9	$\frac{6}{9} = \frac{2}{3}$
BC	2	3	$\frac{2}{3}$
EH	−5	5	−1
FG	−2	2	−1

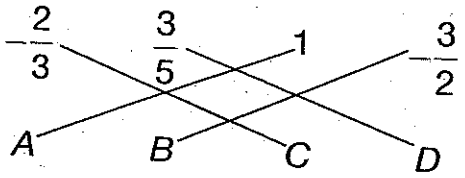
No. e.g., All slopes for roof 1 are the same. All slopes for roof 2 are the same.

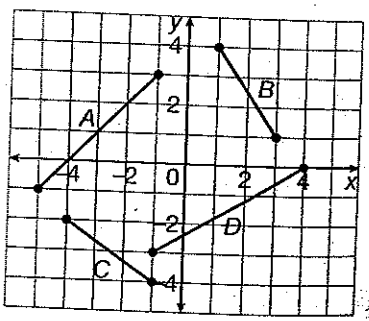


b) The pitch of the roof on the left is not constant. What might happen to debris on the roof?

e.g., The debris might become stuck.

2. Match each slope with a line segment.





3. What is the slope of the line segment that joins each pair of points?

a) (4, 1) and (−6, −2)

$\frac{-2-1}{-6-4} = \frac{-3}{-10}$, or $\frac{3}{10}$

b) (−10, −25) and (−16, −55)

$\frac{-55-(-25)}{-16-(-10)} = \frac{-30}{-6}$, or 5

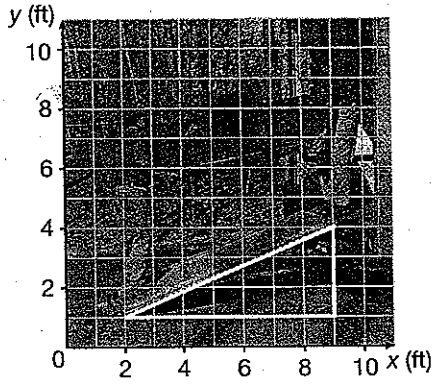
c) (−3, 3) and (7, −1)

$\frac{-1-3}{7-(-3)} = \frac{-4}{10}$, or $-\frac{2}{5}$

d) (12, 8) and (2, 10)

$\frac{10-8}{2-12} = \frac{2}{-10}$, or $-\frac{1}{5}$

4. a) What is the slope of this ramp? What is the grade?



Rise: 3 ft Run: 7 ft

Slope: $\frac{3 \text{ ft}}{7 \text{ ft}} = \frac{3}{7}$, or 0.428... as a decimal

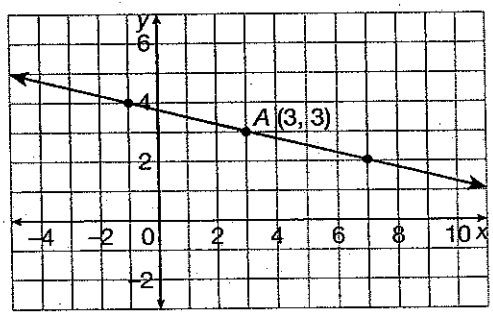
Grade: 0.428... × 100% ≈ 43 %

The slope of the ramp is $\frac{3}{7}$, or 0.428.... The grade is about 43%.

b) Verify the slope of the ramp for Part a). Use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

e.g., $m = \frac{4 - 1}{9 - 2} = \frac{3}{7}$, or about 0.43. This verifies the slope for Part a).

5. Draw a straight line through point $A(3, 3)$ with slope $\frac{-1}{4}$. Mark another point on this line. Repeat these steps using the slope $\frac{1}{-4}$.



a) Do all three points lie on the same straight line? yes

b) What is the slope of the line joining the two points you plotted?

e.g., $m = \frac{y_2 - y_1}{x_2 - x_1}$

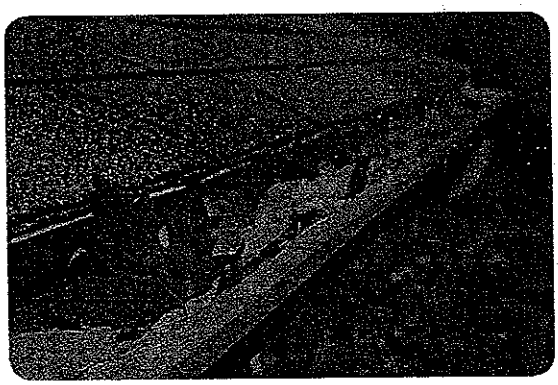
$= \frac{2 - 4}{7 - (-1)}$

$= \frac{-2}{8}$, or $-\frac{1}{4}$ The slope is $\frac{-1}{4} = \frac{1}{-4}$.

REFLECTING

When might someone use a slope formula at work?

6. Ray is installing an eavestrough. He needs the eavestrough to drop 1 in. for every 60 in. length so the water will drain properly.



a) What is the slope of the eavestrough?

Slope: $\frac{\text{rise}}{\text{run}} = \frac{-1 \text{ in.}}{60 \text{ in.}}$, or $-\frac{1}{60}$

The slope of the eavestrough is $-\frac{1}{60}$.

b) Ray found standing water in the eavestrough. How can he fix this?

e.g., He can make the eavestrough slope more steeply by increasing the drop.

Practice

1. Circle the steeper slope or grade for each pair.

a) 22%, (−0.43)

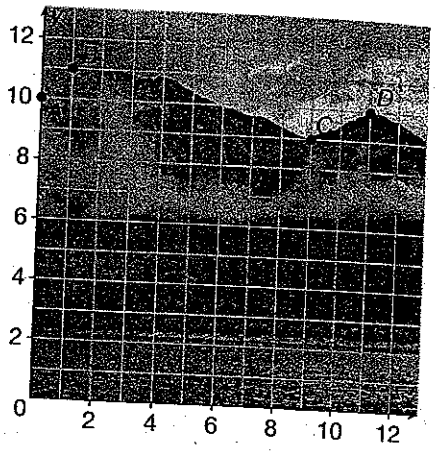
$\frac{-0.43}{}$ × 100% = −43%

b) Rise = 0.3 m and run = 0.5 m,

(Rise = 15 in. and run = 24 in.)

$\frac{0.3\text{ m}}{0.5\text{ m}} = \frac{0.6}{}$, $\frac{15\text{ in.}}{24\text{ in.}} = 0.625$

2. Tien has this photograph of Pyramid Mountain near Jasper. He claims that section *AB* is steeper than *CD*. Show that he is correct.



e.g., *AB*: slope is $\frac{11-10}{1-0} = \frac{1}{1}$, or 1

CD: slope is $\frac{10-9}{11-9} = \frac{1}{2}$

$1 > \frac{1}{2}$ Section *AB* is steeper.

3. Doris pours concrete to make driveways. A customer wants a driveway with a grade between 3% and 3.5% and a rise of 9 in.

Hint
Use the charts inside the back cover.

a) What is the minimum run of the driveway, in feet and inches? What is the maximum run?

e.g., $\frac{\boxed{3}}{100} = \frac{\boxed{9\text{ in.}}}{x}$

$100x\frac{\boxed{3}}{100} = 100x\frac{\boxed{9\text{ in.}}}{x}$

$\frac{3x}{\boxed{3}} = \frac{\boxed{900\text{ in.}}}{\boxed{3}}$

$x =$ 300 in.,

or 25 ft

$\frac{\boxed{3.5}}{100} = \frac{\boxed{9\text{ in.}}}{x}$

$\frac{100x\frac{3.5}{100}}{} = \frac{100x\frac{9\text{ in.}}{x}}{}$

$\frac{3.5x}{3.5} = \frac{900\text{ in.}}{3.5}$

$x =$ 257.142... in.,

or about 21 ft 5 in.

REFLECTING
For Question 3, does the slope with the maximum grade have the minimum or maximum run? Why?

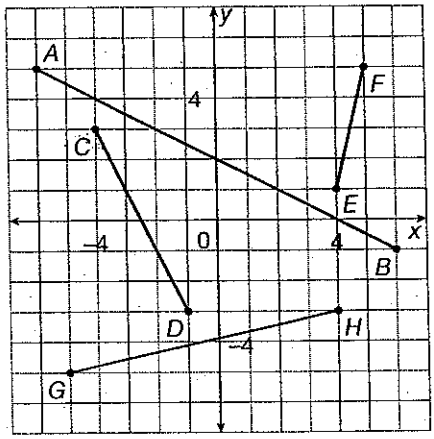
The minimum run is about 21 ft 5 in. The maximum run is 25 ft.

b) What is the difference between the two runs in Part a)?

Difference in runs: about 300 in. − 257 in. = 43 in. The difference is about 3 ft 7 in.

4. **a)** Predict which line segments have negative slopes.

AB and *CD* have negative slopes.



b) What is the slope of each line segment on the grid?

AB: slope = $\frac{6}{12}$, or $\frac{1}{2}$

CD: slope = $\frac{6}{-3}$, or -2

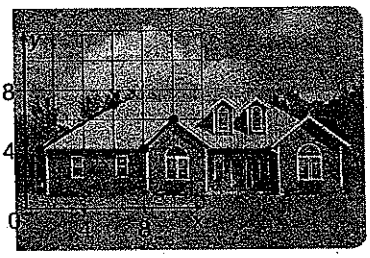
EF: slope = $\frac{4}{1}$, or 4

GH: slope = $\frac{2}{9}$

c) Which line segment is steepest? Explain why.

EF is steepest. e.g., Because the rise divided by run is the greatest.

5. Compare the grade of the main roof section to the grade of the smaller roof section. What is the difference between the percents?



Main roof section:

$m = \frac{7 - 4}{6 - 1}$

$= \frac{3}{5}$, or 60%

Smaller roof section:

$m = \frac{6 - 4}{10 - 8}$

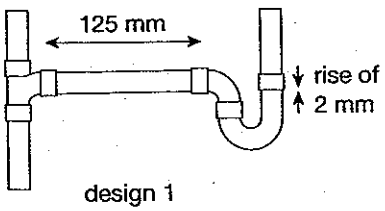
$= 1$, or 100%

The main roof section is less steep. The difference is 40%.

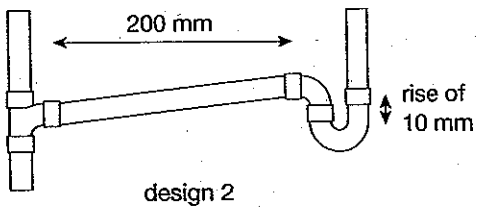
REFLECTING

What are the different ways you can express a slope? When might you use each?

6. Dwayne is a plumber. He is installing a new drain system. The drop must be at least 6 mm for every 300 mm of pipe length. This will leave a little water in the trap to prevent methane gas from flowing through the pipes and into the building.



design 1



design 2

a) Calculate the slopes of the designs. Which is steeper?

Design 1: $\frac{2 \text{ mm}}{125 \text{ mm}} = 0.016$ Design 2: $\frac{10 \text{ mm}}{200 \text{ mm}} = 0.05$ Design 2 has a steeper slope.

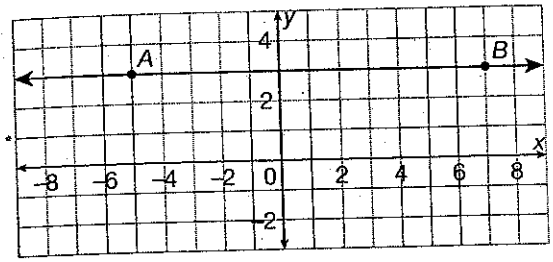
b) Which design meets Dwayne's requirements?

Minimum slope: $\frac{6 \text{ mm}}{300 \text{ mm}} = 0.02$ Design 2 meets Dwayne's minimum requirements.

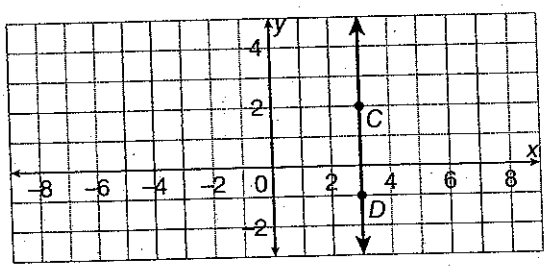
Practice

1. a) Which line looks as if it has a slope of 0? Which line looks as if it has an undefined slope?

Slope: 0



Slope: undefined



b) Use the slope formula to verify the slope of each line above.

e.g., AB: $\frac{3-3}{7-(-5)} = \frac{0}{12}$, or 0 CD: $\frac{-1-2}{3-3} = \frac{-3}{0}$, or undefined

2. Is the line joining each pair of points vertical, horizontal, or neither? Justify your answers.

a) (34.5, 2.6) and (45.0, 2.6)

$\frac{2.6-2.6}{45.0-34.5} = \frac{0.0}{10.5}$, or 0

Horizontal; slope is zero.

c) (68, 10) and (52, 12)

$\frac{12-10}{52-68} = \frac{2}{-16}$, or $-\frac{1}{8}$

Neither; slope is not zero or undefined.

b) (40, 100) and (40, 200)

$\frac{200-100}{40-40} = \frac{100}{0}$

Vertical; slope is undefined.

d) (31, 5) and (62, 12)

$\frac{12-5}{62-31} = \frac{7}{31}$

Neither; slope is not zero or undefined.

REFLECTING

How do coordinates show that a slope is zero? How do they show that a slope is undefined?

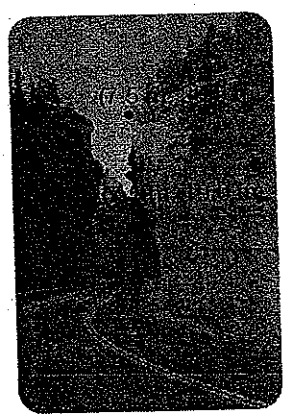
3. Order the slopes in Question 2 from flattest to steepest.

Slope of Part c): $-\frac{1}{8} = -0.125$ Slope of Part d): $\frac{7}{31} = 0.225...$ Parts a), c), d), b)

4. This mountain road passes through the West Gate Canyon in Kootenay National Park. The centre of the road is the origin.

- The top of one section of canyon wall is (7.5 m, 28.5 m).
- The base of this section of canyon wall is (7.5 m, 19.0 m).

What is the slope of this section of canyon wall?

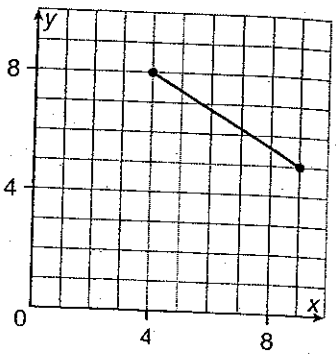


e.g., $\frac{28.5\text{ m} - 19.0\text{ m}}{7.5\text{ m} - 7.5\text{ m}} = \frac{9.5\text{ m}}{0.0\text{ m}}$ The slope is undefined.

Practice

1. A line connecting the points $(x, 5)$ and $(4, 8)$ has slope -0.6.

a) What is the missing coordinate, x?



$$-0.6 = \frac{8 - \boxed{5}}{\boxed{4} - x}$$

$$-0.6(\boxed{4} - x) = \frac{8 - \boxed{5}}{\boxed{4} - x}(\boxed{4} - x)$$

$$-2.4 + 0.6x + 2.4 = 3 + 2.4$$

$$\frac{0.6x}{0.6} = \frac{5.4}{0.6}$$

$$x = 9$$

b) Plot both points on the grid. Connect the points. Count squares to verify that the slope is correct.

2. Use the map for Example 2. What is the grade of Mount White along the line drawn on the map?

e.g., Horizontal distance: 1.6 cm $\times \frac{100\text{ m}}{1\text{ cm}}$ = 160 m

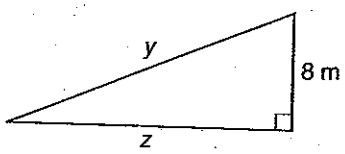
Vertical distance: 1480 m − 800 m = 680 m

Grade: $\frac{\boxed{680}\text{ m}}{\boxed{160}\text{ m}} \times 100\%$ = 425 %

The grade of Mount White is 425 %.

3. Nathan designs dirt bike ramps. He wants the slope of a ramp to be $\frac{4}{11}$. He needs the ramp to be 8 m high.

a) How long should the base of the ramp be?



e.g., $\frac{4}{11} = \frac{8\text{ m}}{z}$

$$11z \times \frac{4}{11} = 11z \times \frac{8\text{ m}}{z}$$

$$\frac{4z}{4} = \frac{88\text{ m}}{4}$$

$$z = 22\text{ m}$$

The base of the ramp should be 22 m long.

b) How long should the ramp be, to the nearest tenth of a metre?

Hint
Use the Pythagorean theorem.

$(22\text{ m})^2 + (8\text{ m})^2 = y^2$

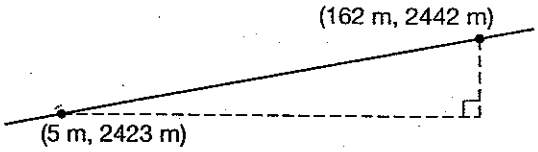
$484\text{ m}^2 + 64\text{ m}^2 = y^2$

$\sqrt{548\text{ m}^2} = y$

$23.409...\text{ m} = y$

The ramp should be 23.4 m long, to the nearest tenth of a metre.

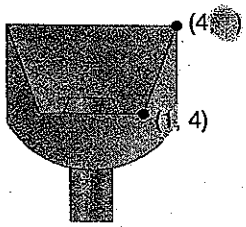
4. Simon owns a logging company. He will use helicopter logging for any hill with a grade greater than 10%. Will Simon use helicopter logging for the terrain on the right?



e.g., Grade: $\frac{2442\text{ m} - 2423\text{ m}}{162\text{ m} - 5\text{ m}} = \frac{19\text{ m}}{157\text{ m}} \times 100\%$, or about 12.1%

Simon will use helicopter logging.

5. Victoria spilled coffee on her plans while designing a new fountain. She knows that the water running out of the fountain should have a slope of $\frac{7}{3}$. What is the missing coordinate?



e.g., $\frac{7}{3} = \frac{y - 4}{4 - 1}$

$(3)\frac{7}{3} = (3)\frac{y - 4}{3}$

$7 = y - 4$

$11 = y$ The missing coordinate is 11.

6. Fatima is a roofer. She can use asphalt shingles only when the pitch of the roof is 17% or greater. The horizontal run of this roof is 13 ft. What is the minimum rise required, to the nearest inch, to use asphalt shingles?

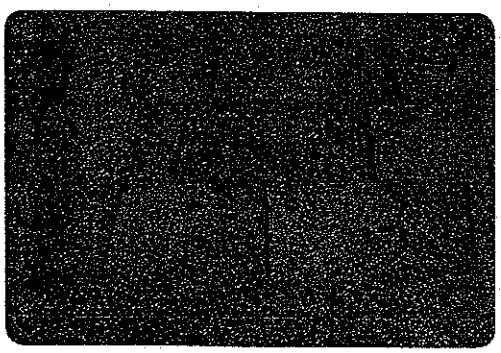
e.g., $\frac{17}{100} = \frac{x}{13\text{ ft}}$

$(13\text{ ft})\frac{17}{100} = (13\text{ ft})\frac{x}{13\text{ ft}}$

$2.21\text{ ft} = x$

$0.21\text{ ft} \times \frac{12\text{ in.}}{1\text{ ft}} = 2.52\text{ in.}$, or about 3 in.

The minimum rise is 2 ft 3 in., to the nearest inch.



Solution

A. What is the slope of Bev's line of sight?

Slope: $\frac{\text{rise}}{\text{run}} = \frac{\boxed{91}\text{ ft}}{250\text{ ft}}$, or 0.364

B. What is the decimal value of tan 20°?

tan 20° = 0.3639...

C. How are the slope and the angle of elevation related?

The slope equals the tangent of the angle of elevation.

Hint

Use a protractor. Check that the angle of elevation is 20°.

REFLECTING

Why does the relationship between the slope of a line and the tangent of its angle of elevation make sense?

Practice

1. Complete this chart. Round angles to the nearest degree and measurements to the nearest unit. The first row is done for you.

Slope	Angle of elevation	Rise	Run
tan 30° = 0.5773 ...	30°	$\frac{x}{48} = 0.577\ldots$ $x \doteq 28$	48
2.36	tan⁻¹(2.36) = 67°	$\frac{x}{13\text{ m}} = 2.36$ $x = 31$	13
$\frac{3}{8}$	$\tan^{-1}\left(\frac{3}{8}\right) \doteq 21°$	3	8

2. A surveyor is sighting to the top of a building. The horizontal distance to the building is 67 m. The sight line forms an angle of elevation of 32°.

a) How high above the surveyor's instrument is the top of the building?

Slope = tan 32°

$\frac{x}{\boxed{67}} = 0.6248...$

$x = 41.866...$

It is about 42 m above the surveyor's instrument.

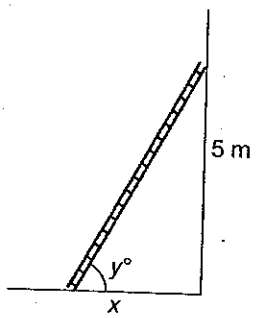
b) The surveyor is sighting from a height of 1.4 m. How tall is the building?

e.g., 41.866... m + 1.4 m = 43.266... m The building is about 43 m tall.

3. Johanna and Noel are roofers. Do they agree?

- Johanna measures roof pitch in degrees. She says the angle of elevation of a new roof should be 26.5°.
- Noel measures roof pitch as a grade. He says the grade of the roof should be 50%.

e.g., Noel: 50% = 0.5; Johanna: tan 26.5° = 0.4985..., or about 0.5
Johanna and Noel agree.



4. Claire is putting up patio lights. She rests a ladder against the house. For safety, the distance from the foot of the ladder to the house should be about one quarter of the ladder's height.

a) What should the slope of the ladder be?

$m = \frac{4}{1}$

b) How far away should the foot of the ladder be from the house?

e.g., $\frac{1}{4}(5\text{ m}) = \frac{5}{4}\text{ m}$ or 1.25 m

The foot of the ladder should be about 1.25 m from the house.

c) What angle of elevation does the ladder make with the ground, to the nearest degree?

e.g., $\tan y^\circ = 4$

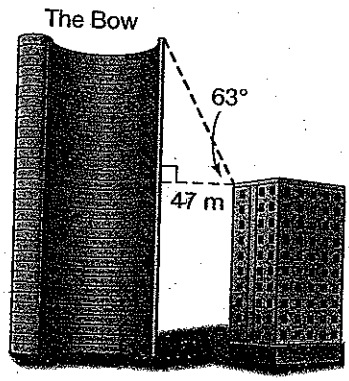
$y^\circ = \tan^{-1}(4)$

$y^\circ = 75.963...^\circ$ The angle of elevation is 76°, to the nearest degree.

REFLECTING

Pam said that The Bow is about 24 m taller. What mistake might she have made?

5. a) The Bow in Calgary was the tallest building in Western Canada when it was completed. How much taller is The Bow than the other building in this diagram?



e.g., $\tan 63^\circ = \text{slope}$

$\tan 63^\circ = \frac{x}{47}$

$1.9626... = \frac{x}{47}$

$92.242... = x$ The Bow is about 92 m taller.

b) Does the side opposite the 63° angle match the rise or the run of the triangle in Part a)?

rise

Mid-Chapter Review

1. a) For safety, a moving van's ramp should have a maximum slope of $\frac{3}{10}$. Explain what this means.

e.g., The ramp must rise at most 3 units for every 10 units of run.

b) Is a ramp with a rise of 2.5 ft and a run of 9 ft safe?

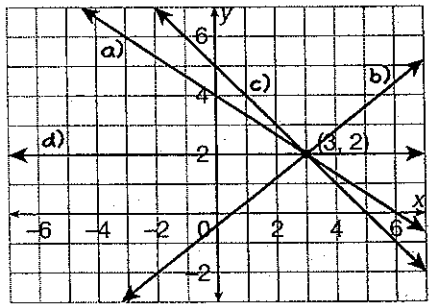
e.g., Slope: $\frac{2.5 \text{ ft}}{9 \text{ ft}} = 0.277...$ Maximum slope: $\frac{3}{10} = 0.3$

$0.277... < 0.3$ So the ramp is safe.

c) What is the grade of the ramp in Part b)?

The grade is $0.277... \times 100\%$, or about 28%.

2. Draw a line through the point (3, 2) with each slope.



a) $-\frac{2}{3}$ **b)** 0.8 **c)** -1 **d)** 0

3. A sign is needed for any road with a grade greater than 6%. A scale drawing of the cross-section of a road has points with coordinates (45, 143) and (145, 150). Is a sign needed?

Grade: $\frac{150 - 143}{145 - 45} \times 100\% = 7\%$ A sign is needed.

4. Mikka is a conservationist. She looked up to the top of a sitka spruce tree. She measured the angle of elevation as 50°.

a) What is the slope of the line Mikka is looking along?

The slope is $\tan 50° = 1.1917...$

b) Mikka's eyes are 1.52 m above the ground. She is standing 100 m from the tree. What is the height of the sitka, to the nearest metre?

$$1.1917... = \frac{x}{100 \text{ m}}$$

$119.175... \text{ m} = x$

Height: $119.175... \text{ m} + 1.52 \text{ m} = 120.695... \text{ m}$

The sitka is 121 m tall, to the nearest metre.

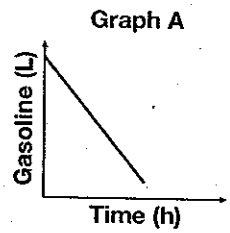
Practice

1. Circle the slopes that show an increase. Underline the slopes that show a decrease. Cross out the slopes that show no change.

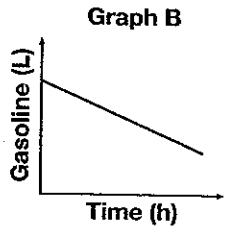
$\frac{6}{5}$ $\frac{1}{4}$ 0 $\frac{5}{8}$ 0 $\frac{4}{9}$ $\frac{10}{3}$

2. Graphs A, B, and C show the amount of gasoline in a car's tank over time.

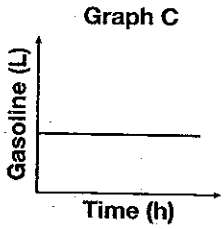
- Describe the rate of change for each graph.
- Tell what it could represent.



Graph A: e.g., has a steep negative slope or rate of change. So the amount of gas is decreasing rapidly over time. It could represent the car moving at a high speed.



Graph B: e.g., has a less-steep negative slope or rate of change. So the amount of gas is decreasing slowly over time. It could represent the car moving at a low speed.

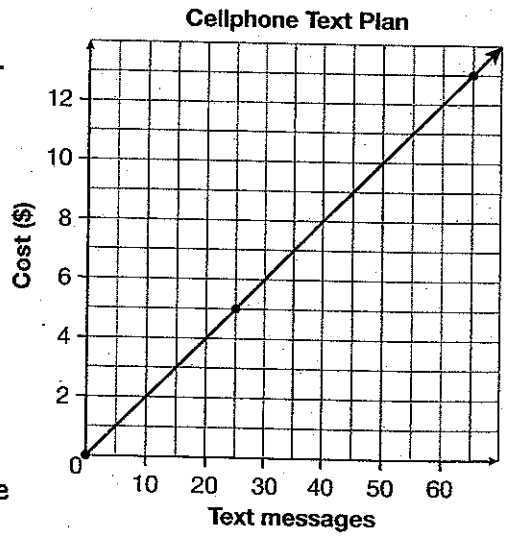


Graph C: e.g., has a rate of change, or slope, of zero. This means no gas is being used. So the car is stopped.

3. Andy's text plan for his cellphone is shown in the chart.

Text messages	Cost ($)
0	0
25	5
65	13

a) Graph the data. Draw a line segment through the points.



b) What is the slope of the line segment? What does the slope represent?

Slope: $\frac{5 - 0}{25 - 0} = 0.20$

e.g., The slope represents the cost per text message.

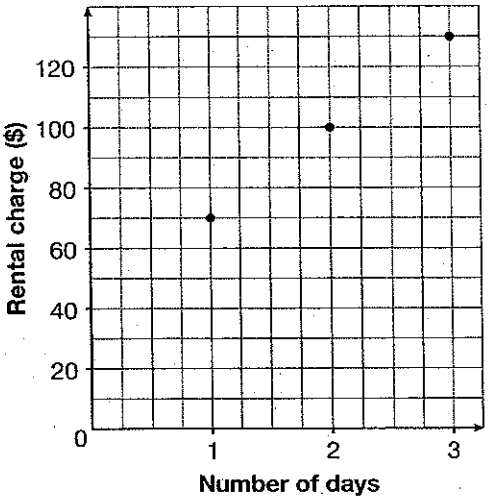
c) Andy is offered a plan of 100 text messages for $18. Should he take the plan?

e.g., Old plan: $0.20 \times 100 = \$20$ 100 text messages would cost $20 under the old plan. $\$20 > \$18$. So Andy should take the new plan.

4. Nahla is renting a vehicle for a business trip.

Number of days	Rental charge ($)
1	70
2	100
3	130

Car Rental Rates



a) Nahla calculated the rate of change in rental cost:

Slope is $\frac{\text{rise}}{\text{run}} = \frac{\$130}{3\ d}$, or $\$43.33/d$.

What error did Nahla make?

e.g., She used (0, 0) as a value in the rate of change.

b) Plot the data in the chart on the grid. What is the rate of change?

e.g., $\frac{\$100 - \$70}{2\ d - 1\ d} = \$30/d$

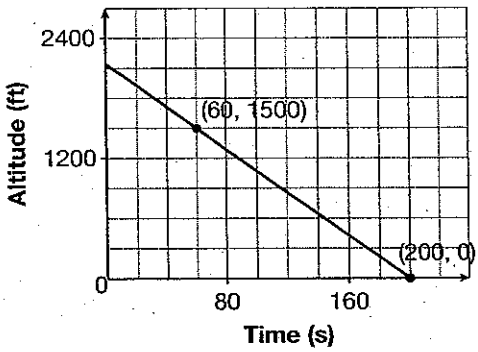
The correct rate of change is $\$30/d$.

c) What does this rate of change mean?

e.g., Each additional day the car is rented costs another $30.

5. Use the graph on the right. At what rate does the airplane descend into the airport? Answer to the nearest tenth.

Descent Altitude

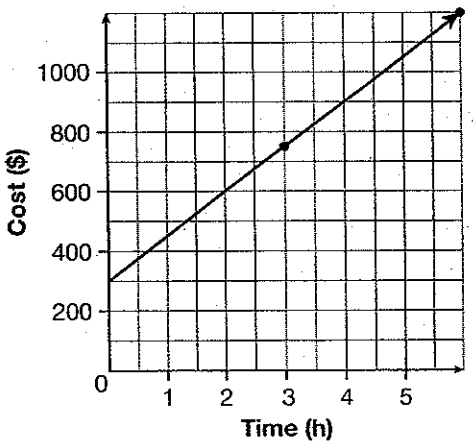


e.g., Slope: $\frac{0\ ft - 1500\ ft}{200\ s - 60\ s} = -10.714...\ ft/s$

The plane descends at 10.7 ft/s, to the nearest tenth.

6. Ambrose is a wedding planner. He has hired a band that charges $750 for 3 h of music or $1200 for 6 h of music.

Band Rental Rates



a) Graph this information on the grid. Draw a line segment through the points.

b) What is the slope of the line segment you drew? What does this slope represent?

e.g., Slope: $\frac{\$1200 - \$750}{6\ h - 3\ h} = \$150/h$

The slope represents the band's rate per hour.

c) Extend the graph to the vertical axis. What is the band's flat rate, before charging an hourly wage?

e.g., The flat rate is the cost for 0 h, or $300.

d) Ambrose wants the band for 5 h. How much will it cost?

e.g., Cost: $\$300 + 5 \times \$150 = \$1050$ It will cost $1050.

C. How long would the trip take at 100 km/h?

$$2070 \text{ km} \times \frac{1 \text{ h}}{\boxed{100} \text{ km}} = \underline{20.7} \text{ h}$$

REFLECTING

When might you use unit analysis? When might someone use unit analysis for work?

D. Difference between times: __23__ h − __20.7__ h = __2.3__ h, or __2__ h __18__ min

The difference is __2__ h __18__ min.

Hint

1 h = 60 min

Practice

1. Calculate.

a) Distance travelled at 110 km/h for 5 h:

$$\frac{\boxed{110} \text{ km}}{1 \text{ h}} \times \underline{5} \text{ h} = \underline{550} \text{ km}$$

b) Volume of water running at 85 gal/min for 12 min:

$$\frac{\boxed{85} \text{ gal}}{\boxed{1} \text{ min}} \times \boxed{12} \text{ min} = \boxed{1020} \text{ gal}$$

REFLECTING

Create your own problem involving unit analysis. Then, trade with a partner. Solve each other's problem.

2. Wray installs wind turbines that convert wind power into electricity. On a typical day, a wind turbine completes 960 rotations every hour. How many seconds does one rotation take?

Hint

1 h = 60 min
1 min = 60 s

$$\frac{\boxed{60} \text{ s}}{1 \text{ min}} \times \frac{\boxed{60} \text{ min}}{1 \text{ h}} \div \frac{\boxed{960} \text{ rotations}}{1 \text{ h}} = \frac{\boxed{3.75} \text{ s}}{1 \text{ rotation}}$$

One rotation takes __3.75__ s.

3. Chloe is taking a train to apply for a job. The train travels 6 m per $\frac{1}{5}$ s. How far does the train travel in 3 s?

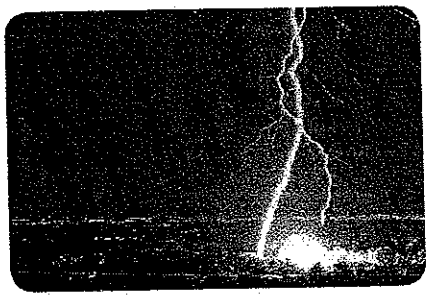
Hint

Start by calculating how far the train travels in 1 s.

e.g., The train travels 6 m in $\frac{1}{5}$ s. So it travels $5 \times 6 \text{ m} = 30 \text{ m}$ in $5 \times \frac{1}{5} \text{ s} = 1 \text{ s}$

$3 \text{ s} \times \frac{30 \text{ m}}{1 \text{ s}} = 90 \text{ m}$ The train travels 90 m in 3 s.

4. You can estimate how far away a lightning strike is by counting the number of seconds between the lightning and the thunder. For every 5 s, the distance is 1 mi. Yolly saw a lightning strike. She heard the thunder 24 s later. How far away was the lightning?



e.g., $24 \text{ s} \times \frac{1 \text{ mi}}{5 \text{ s}} = 4.8 \text{ mi}$

The lightning was 4.8 mi away.

Solution

A. What is the horizontal distance in metres for each route?

Route 1: 5.2 km $\times \frac{1000 \text{ m}}{1 \text{ km}} =$ 5200 m

Route 2: 7.4 km $\times \frac{1000 \text{ m}}{1 \text{ km}} =$ 7400 m

B. What is the grade of the road for each route?

Route 1 grade: $\frac{490 \text{ m}}{5200 \text{ m}} \times 100\% \doteq$ 9.4 %

Route 2 grade: $\frac{660 \text{ m}}{7400 \text{ m}} \times 100\% \doteq$ 8.9 %

REFLECTING

Could you solve the problem by converting the vertical distance to kilometres? Explain.

C. Botan should choose Route 2, because it is less than the maximum grade.

Practice

1. Yaris is a farmer in Moose Jaw, Saskatchewan. He drives his combine harvester at 6.0 km/h.

a) Yaris will drive 29.0 km to harvest his crop. How many hours and minutes will this take Yaris?

e.g., $\frac{1 \text{ h}}{6.0 \text{ km}} \times 29.0 \text{ km} = 4.833... \text{ h}$

$0.833... \text{ h} \times \frac{60 \text{ min}}{1 \text{ h}} = 50 \text{ min}$

It will take Yaris 4 h 50 min.

b) When the wheat is tough, Yaris slows down to 4.5 km/h. How much longer, in hours and minutes, will this take Yaris?

e.g., $\frac{1 \text{ h}}{4.5 \text{ km}} \times 29.0 \text{ km} = 6.444... \text{ h}$

$6.444... \text{ h} - 4.833... \text{ h} = 1.611... \text{ h more}$

$0.611... \text{ h} \times \frac{60 \text{ min}}{1 \text{ h}} = 36.666... \text{ min}$

It would take about 1 h 37 min longer.

2. The Corkscrew roller coaster at the Pacific National Exhibition in Vancouver has a maximum speed of 64 km/h. How far does it travel in 7.5 s, to the nearest metre?

e.g., $64 \text{ km/h} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times 7.5 \text{ s} = 0.133... \text{ km}$, or about 133 m

The roller coaster travels 133 m in 7.5 s, to the nearest metre.

3. Usain Bolt set a new world record for the 100 m dash in 2009 in Berlin. He ran 100 m in 9.58 s.

a) What was his average speed in metres per second?

e.g., $\frac{100 \text{ m}}{9.58 \text{ s}} = 10.438... \text{ m/s}$

Bolt's average speed was about 10.44 m/s.

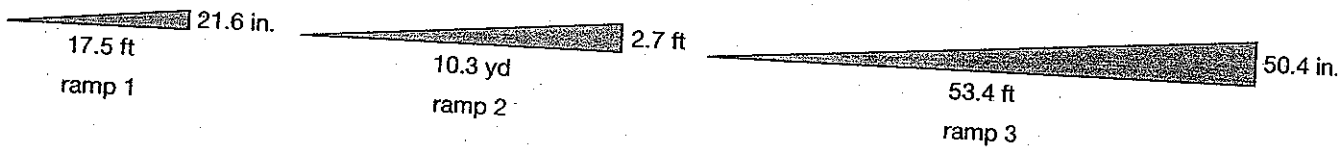
b) What was Bolt's average speed in kilometres per hour?

e.g., $10.438... \text{ m/s} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 37.578... \text{ km/h}$

Bolt's average speed was about 37.6 km/h.

Hint
Start by converting to get the dimensions in the same units.

4. a) The code for wheelchair ramps states that they must not have a slope greater than $\frac{1}{12}$. Which of these ramps fit the code?



e.g., Code: $\frac{1}{12} \times 100\% = 8.333...\%$

ramp 1: $21.6 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 1.8 \text{ ft}$

$\frac{1.8 \text{ ft}}{17.5 \text{ ft}} \times 100\% = 10.285...\%$

ramp 2: $2.7 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} = 0.9 \text{ yd}$

$\frac{0.9 \text{ yd}}{10.3 \text{ yd}} \times 100\% = 8.737\%$

ramp 3: $50.4 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 4.2 \text{ ft}$

$\frac{4.2 \text{ ft}}{53.4 \text{ ft}} \times 100\% = 7.865\%$ Only ramp 3 meets code.

REFLECTING

In Question 4, how did you decide which units to convert? What is a different way you could have converted units?

b) What changes to the horizontal measurements would result in all the ramps in Part a) meeting the code?

e.g., ramp 1:

$$\frac{1.8 \text{ ft}}{x} = \frac{1}{12}$$

$$\frac{1.8 \text{ ft}}{x} \times 12x = \frac{1}{12} \times 12x$$

$$21.6 \text{ ft} = x$$

ramp 2:

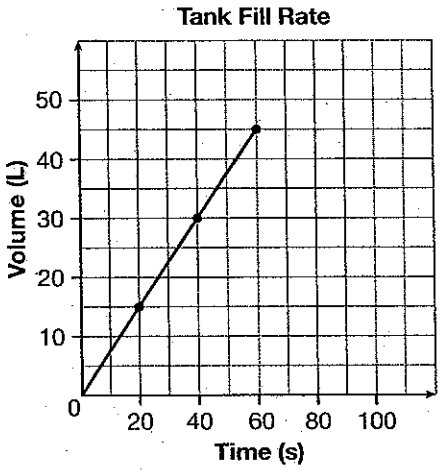
$$\frac{0.9 \text{ yd}}{y} = \frac{1}{12}$$

$$\frac{0.9 \text{ yd}}{y} \times 12y = \frac{1}{12} \times 12y$$

$$10.8 \text{ yd} = y$$

Extend the horizontal lengths of ramp 1 to 21.6 ft and ramp 2 to 10.8 yd.

5. Callum is filling a rectangular water storage tank. The graph shows the volume of water in the tank as he fills it.



a) What is the rate of change in the volume of water in the tank, to the nearest hundredth?

e.g., Rate of change: $\frac{45 \text{ L} - 15 \text{ L}}{60 \text{ s} - 20 \text{ s}} = 0.75 \text{ L/s}$

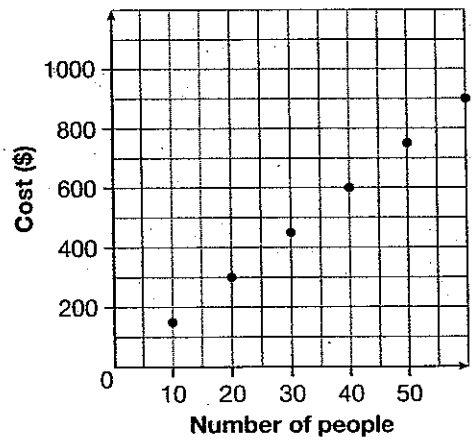
The rate of change of the volume of water is 0.75 L/s.

b) What is this rate of change in millilitres per minute?

e.g., $0.75 \text{ L/s} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 45\,000 \text{ mL/min}$

The rate of change is 45 000 mL/min.

6. Betty runs a catering business in Prince Rupert. The cost per person is represented in the graph.



a) How much would Betty charge for 200 guests?

e.g., $\frac{\$150 - \$0}{10 \text{ people} - 0 \text{ people}} = \frac{\$15}{1 \text{ person}}$

$$\frac{\$15}{1 \text{ person}} \times 200 \text{ people} = \$3000$$

Betty would charge $3000 for 200 guests.

b) Does Betty charge a flat fee? Explain.

No. e.g., If I place a clear ruler along the points, the ruler passes through the origin. So she does not charge a flat fee.

B. What is the rate of change for Braydon between 1 h and 3 h?

$$\frac{57 \text{ mm} - \boxed{19 \text{ mm}}}{\boxed{3 \text{ h}} - 1 \text{ h}} = \underline{19} \text{ mm/h}$$

C. What is the rate in millimetres per hour for Ramo's rainfall?

$$\underline{0.7} \text{ in./h} \times \frac{\boxed{25.4} \text{ mm}}{1 \text{ in.}} \doteq \underline{17.8} \text{ mm/h}$$

Hint: 1 in. = 25.4 mm

REFLECTING

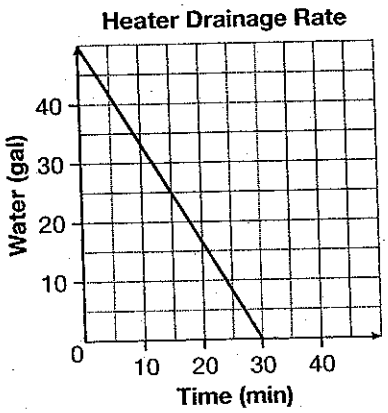
How could you use 1 mm ≐ 0.039 in. to solve the problem?

D. Who had the greater rate of rainfall?

Braydon had the greater rate of rainfall.

Example 2

Venci is draining a water heater so he can repair it. The graph on the right shows the rate. How fast is the water draining, in millilitres per second?



Solution

A. What is the rate of change in gallons per minute?

$$\frac{0 \text{ gal} - \boxed{50} \text{ gal}}{\boxed{30} \text{ min} - 0 \text{ min}} = \underline{-1.666...} \text{ gal/min}$$

The rate of change is about $\underline{-1.67}$ gal/min.

B. What is this rate of change in millilitres per second?

Hint: 1 gal ≐ 3.79 L

$$\underline{-1.666...} \text{ gal/min} \times \frac{\boxed{3.79} \text{ L}}{1 \text{ gal}} \doteq \underline{-6.316...} \text{ L/min}$$

Hint: A negative rate of change means the volume of water is decreasing.

$$\underline{-6.316...} \text{ L/min} \times \frac{\boxed{1000} \text{ mL}}{1 \text{ L}} \times \frac{\boxed{1} \text{ min}}{\boxed{60} \text{ s}} \doteq \underline{-105.277...} \text{ mL/s}$$

The water is draining at about $\underline{105}$ mL/s.

Practice

Hint: 1 mi ≐ 1.61 km

1. Mariah is a mechanic at a theme park. She is repairing the hydraulic launcher on the roller coaster. The top speed of the roller coaster is 84 mi/h. How fast is the roller coaster's top speed in kilometres per hour?

e.g., $84 \text{ mi/h} \times \frac{1.61 \text{ km}}{1 \text{ mi}} \doteq 135.24 \text{ km/h}$

The top speed is about 135.24 km/h.

Hint

Formula to convert °F to °C:

$C = \frac{5}{9}(F - 32)$

2. a) Hayden is finishing the plumbing in a new house. He is installing a thermostat so that the water temperature never goes above 120 °F. What is this temperature in degrees Celsius, to the nearest degree?

$\frac{5}{9}(\underline{120} - 32)\ °C = \underline{48.888...°C}$ This temperature is 49°C.

b) At 131°F, water will burn a child in 1 s of contact.

What is this temperature in degrees Celsius, to the nearest degree?

e.g., $\frac{5}{9}(131 - 32)\ °C = 55°C$ This temperature is 55°C.

Hint

1 ft ≐ 0.30 m

3. Ivana works with a highway repair crew. She operates a heat lancer that blasts hot air at 3000 ft/s to dry moisture from overnight dew or rain. How fast does the hot air move in metres per second, to the nearest tenth?

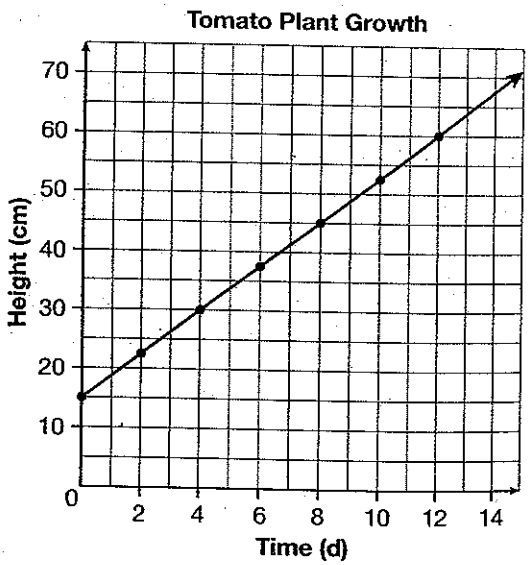
e.g., $3000\ ft/s \times \frac{0.30\ m}{1\ ft} \doteq 900\ m/s$

The hot air moves at about 900 m/s.

REFLECTING

Why are calculations in Question 4 approximate? Give more than one reason.

4. Sylvia works in a gardening store. She graphed the growth of a tomato plant.



a) What is the rate of change in height of the tomato plant in centimetres per day?

e.g., Slope: $\frac{60\ cm - 15\ cm}{12\ d - 0\ d} \doteq 3.75\ cm/d$

The plant is growing at about 3.75 cm/d.

b) What is the rate of change in height of the tomato plant in inches per week?

Hint

1 in. = 25.40 mm
What is 1 in. in centimetres?

e.g., Slope: $\frac{3.75\ cm}{1\ d} \times \frac{1\ in.}{2.54\ cm} \times \frac{7\ d}{1\ wk} = 10.334...\ in./wk$

The plant is growing at about 10.3 in./wk.

Chapter Review

1. Explain what you think this road sign means.

e.g., The grade is 18%. Every 100 m of horizontal distance drops a vertical distance of 18 m.

2. Baldwin Street in Dunedin, New Zealand, is one of the steepest streets in the world.

- The street runs for a horizontal distance of 350 ft.
- It rises from 30 ft to 153 ft over this distance.

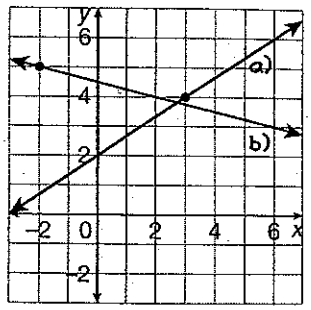
What is the grade of Baldwin Street?

e.g., $\frac{153 \text{ ft} - 30 \text{ ft}}{350 \text{ ft}} \times 100\% = 0.351...\%$, or about 35% The grade is about 35%.

3. Graph each line.

a) (3, 4), $m = 2:3$

b) (−2, 5), $m = -1:4$



4. a) What is the slope of the line segment that joins the points?

$A(5, -8)$ to $B(-6, -14)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-14 - (-8)}{-6 - 5}$$

$$= \frac{-6}{-11}, \text{ or } \frac{6}{11}$$

$C(2, 1)$ to $D(4, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - 1}{4 - 2}$$

$$= \frac{6}{2}, \text{ or } 3$$

b) Which line segment in Part a) is steeper? Explain.

CD is steeper. e.g., Its slope is 3. That is greater than the slope of AB.

5. Explain how $\frac{\text{rise}}{\text{run}}$ and $\frac{y_2 - y_1}{x_2 - x_1}$ both represent the slope formula.

e.g., Rise is the vertical distance or the change between y-values. Run is the horizontal distance or the change between x-values.

6. Ayaka is a real estate agent. She sold a house for $190 000. It sold for $268 000 11 yr later. What is the average rate of change in price, to the nearest dollar?

e.g., $\frac{\$268\,000 - \$190\,000}{11\text{ yr} - 0\text{ yr}} \doteq \$7091/\text{yr}$

The average rate of change was $7091/yr, to the nearest dollar.

7. Suppose the price of the house in Question 6 continues to rise at the same rate. What would it be worth in 5 yr?

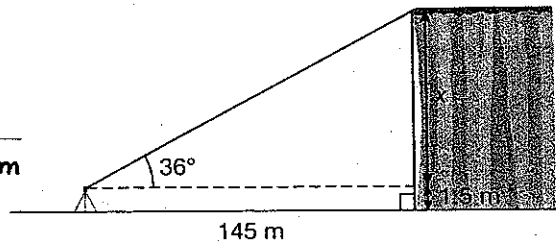
e.g., $7091 × 5 + $268 000 = $303 455 It would be worth $303 455 in 5 yr.

8. Ruth is a surveyor. She is using a sight to measure the height of a cliff. What is the height of the cliff, to the nearest metre?

e.g., $\tan 36° = \frac{x}{145\text{ m}}$

$145\text{ m} \times \tan 36° = 145\text{ m} \times \frac{x}{145\text{ m}}$

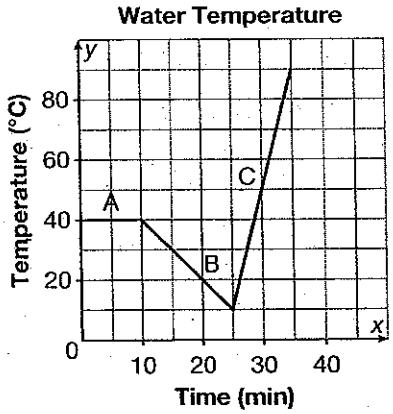
$105.348...\text{ m} = x$



Height: 105.348... m + 1.5 m = 106.848... m

The height of the cliff is about 107 m.

9. Hayvn is conducting a science experiment. He plotted the following data. What is the rate of change for each section? What does each rate of change mean?

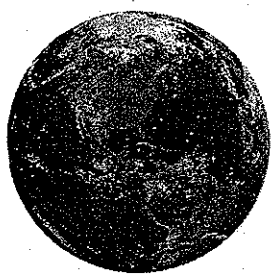


Section A: 0 °C/min; no change in temperature

Section B: −2 °C/min; temperature decreases by 2 °C/min

Section C: 8 °C/min; temperature increases by 8 °C/min

10. Earth rotates at about 465 m/s at the equator.



a) What is this speed in kilometres per hour?

e.g., $465\text{ m/s} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{3600\text{ s}}{1\text{ h}} = 1674\text{ km/h}$

Earth rotates at about 1674 km/h at the equator.

b) What is this speed in miles per hour?

e.g., $1674\text{ km/h} \times \frac{1\text{ mi}}{1.61\text{ km}} = 1039.751...\text{ mi/h}$

Earth rotates at about 1040 mi/h at the equator.

Hint
1 mi ≐ 1.61 km

Chapter Test

1. Which line segment is steeper?

AB: $A(3, 2)$, $B(1, 4)$ *CD*: $C(-1, -2)$, $D(2, 3)$

Slope *AB*: $\frac{4-2}{1-3} = \frac{2}{-2}$, or -1 Slope *CD*: $\frac{3-(-2)}{2-(-1)} = \frac{5}{3}$

$\frac{5}{3} > 1$, so *CD* is steeper.

2. Alberto is a personal trainer. He tells clients to walk at a 12% grade on a treadmill. The length of the treadmill is 75 in. Each level on the treadmill means 1 in. in rise. What level should the treadmill be on for Alberto's clients?

e.g., $12\% = \frac{12}{100}$, or 0.12, so $\frac{x}{75 \text{ in.}} = 0.12$

$\frac{x}{75 \text{ in.}}(75 \text{ in.}) = 0.12(75 \text{ in.})$

$x = 9$ in. The treadmill should be on level 9.

3. a) A hawk can fly 35 m in 5.2 s. What speed is this?

e.g., Speed: $\frac{35 \text{ m}}{5.2 \text{ s}} = 6.730...$ m/s The hawk can fly about 6.7 m/s.

b) At this rate, how long would a hawk take to fly 52 m?

e.g., Unit analysis: m ÷ m/s = m × s/m, or s

52 m ÷ 6.730... m/s = 7.725... s It would take about 7.7 s.

4. Ina had 6 gal of gas in her motorcycle tank at the start of a trip. After driving 150 mi, she had 4 gal left.

a) How many miles per gallon did Ina's motorcycle get?

e.g., $\frac{150 \text{ mi} - 0 \text{ mi}}{4 \text{ gal} - 6 \text{ gal}} = -75$ mi/gal It got 75 mi/gal.

Hint
1 mi ≐ 1.61 km

b) How many kilometres per litre did it get?

e.g., $75 \text{ mi/gal} \times \frac{1.61 \text{ km}}{\text{mi}} \times \frac{1 \text{ gal}}{3.79 \text{ L}} = 31.860...$ km/L It got about 32 km/L.