CONICS REVIEW 'CUT 'N' PASTE'

4. Graph each relation and identify the curve.

a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$ b) $x^2 = 4y$ c) $x^2 + y^2 = 25$

d) $x^2 - y^2 = 9$ e) $4x^2 + 4y^2 = 49$ f) $9x^2 - 16y^2 = -144$

5. A rectangular hyperbola with centre (0,0) and vertices on the y-axis passes through $(6, -9)$. Is $(-2,7)$ on the hyperbola?

6. An ellipse has centre (0,0) and major axis on the y-axis. Write the equation of the ellipse if:
   a) the x- and y-intercepts are ± 3 and ± 5 respectively
   b) the major axis is 12 units and the minor axis is 6 units.

7. Find the equation of the hyperbola, centre (0,0), conjugate axis on the y-axis, and:
   a) vertices at $(\pm 4,0)$, and an asymptote defined by $y = 2x$
   b) a transverse axis of 8 units, and a conjugate axis of 14 units.

8. A stone thrown horizontally from a bridge 25 m above the river splashes in the water 40 m from the base of the bridge. If the stone falls in a parabolic path, find its equation.

9. One of the supports in a retractable roof of a sports complex is semi-elliptical. If it is 25 m high and spans 60 m, find its equation.

10. The base of a bridge arch is 80 m wide and 25 m high. Find its equation if the arch is in the shape of a rectangular hyperbola.

11. Describe and sketch the graph of each relation.

a) $y + 3 = \frac{1}{2}(x + 2)^2$ b) $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$

c) $\frac{(x+5)^2}{25} - \frac{(y-2)^2}{16} = 1$ d) $(x - 2)^2 - 3(y + 1)^2 = -27$

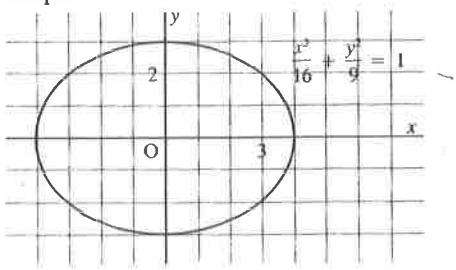
12. Describe the graph of each relation.

a) $x^2 + y^2 + 4x - 10y - 20 = 0$ b) $9x^2 + 16y^2 - 18x + 96y + 9 = 0$

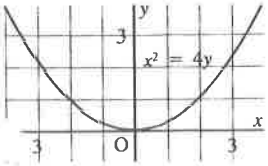
c) $y^2 - 4y - 6x = 38$ d) $5x^2 - 4y^2 - 30x - 16y + 49 = 0$

Answers:

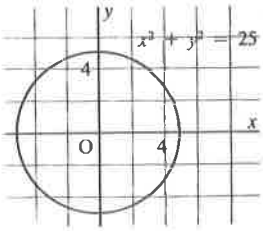
4. a) Ellipse



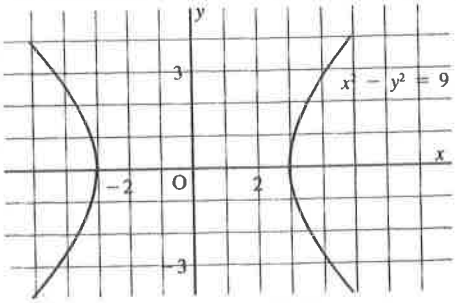
b) Parabola



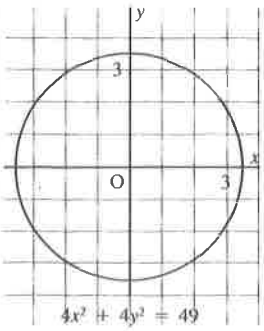
c) Circle



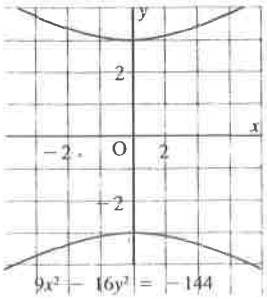
d) Hyperbola



e) Circle



f) Hyperbola



More answers over page →

5. Yes

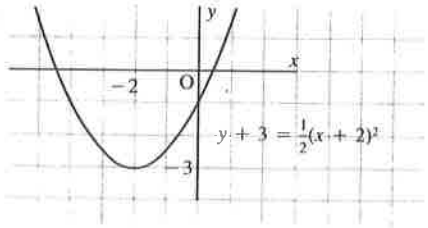
6. a) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ b) $\frac{x^2}{9} + \frac{y^2}{36} = 1$

7. a) $\frac{x^2}{16} - \frac{y^2}{64} = 1$ b) $\frac{x^2}{16} - \frac{y^2}{49} = 1$

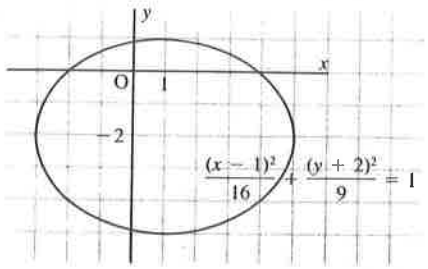
8. $y = -\frac{1}{64}x^2$

9. $\frac{x^2}{900} + \frac{y^2}{625} = 1$ 10. $x^2 - y^2 = -380.25$

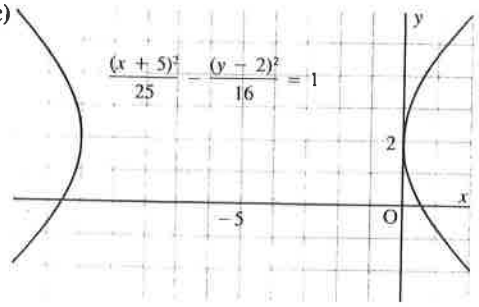
11. a)



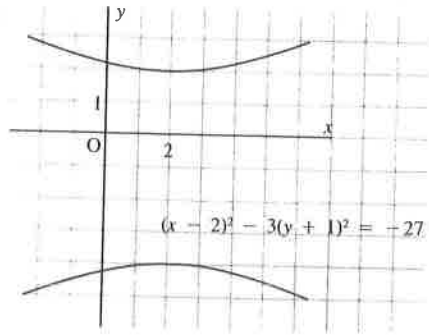
b)



c)



d)



12. a) $(x+2)^2 + (y-5)^2 = 49$; a circle, centre $(-2,5)$, radius 7

b) $\frac{(x-1)^2}{16} + \frac{(y+3)^2}{9} = 1$; an ellipse, centre $(1,-3)$, major axis length 8, minor axis length 6

c) $x + 7 = \frac{1}{6}(y-4)^2$; a parabola, vertex $(-7,4)$, opening to the right, congruent to $x = \frac{1}{6}y^2$

d) $\frac{(x-3)^2}{4} - \frac{(y+2)^2}{5} = -1$; a hyperbola, centre $(3,-2)$, transverse axis is vertical with length $2\sqrt{5}$, conjugate axis length 4

Ⓐ

1. Only one of these equations represents a conic with axes parallel to the coordinate axes. Which equation is this?
$x^2 - 2xy + y^2 - 6x - 14y + 19 = 0$
$xy + 2x - 2y + 4 = 0$
$3x^2 + 2y^2 - 6x + 8y - 1 = 0$
$3x^2 - 4xy + 16x - 8y + 16 = 0$

2. Only one of these equations represents an ellipse. Which equation is this?
$7x^2 + 6xy - y^2 - 54x - 14y + 63 = 0$
$x^2 - 6xy + 9y^2 + 130x + 10y - 575 = 0$
$4xy + 3y^2 - 8x - 16y + 4 = 0$
$5x^2 - 4xy + 8y^2 + 2x - 44y + 29 = 0$

Ⓑ

3. State which equations could represent each conic.
i) an ellipse ii) a hyperbola iii) a parabola
a) $9x^2 - 24xy + 16y^2 - 125y + 355 = 0$
b) $8x^2 + 12xy + 17y^2 - 4x + 22y - 7 = 0$
c) $x^2 - 16xy - 11y^2 + 135 = 0$
d) $4x^2 - 36xy + 31y^2 + 28x - 26y - 21 = 0$
e) $16x^2 + 24xy + 9y^2 - 120x + 160y + 600 = 0$
f) $41x^2 + 4xy + 44y^2 - 720 = 0$

Answers:

1. $3x^2 + 2y^2 - 6x + 8y - 1 = 0$
2. $5x^2 - 4xy + 8y^2 + 2x - 44y + 29 = 0$
3. a) Parabola b) Ellipse c) Hyperbola d) Hyperbola e) Parabola f) Ellipse

CIRCLE STUFF

Write the standard equation for each circle with the given center and radius.

1. Center $(0, 3)$, radius 5
2. Center $(2, 0)$, radius 3
3. Center $(1, -2)$, radius 9
4. Center $(-3, 5)$, radius 4
5. Center $(0, 0)$, radius $\sqrt{3}$
6. Center $(0, 0)$, radius $\sqrt{2}$
7. Center $(-6, -3)$, radius $\frac{1}{2}$
8. Center $(-3, -5)$, radius $\frac{1}{4}$
9. Center $\left(\frac{1}{2}, \frac{1}{3}\right)$, radius 0.1
10. Center $\left(-\frac{1}{2}, 3\right)$, radius 0.2

Find the center and radius for each circle.

11. $(x-3)^2 + (y-5)^2 = 2$
12. $(x+3)^2 + (y-7)^2 = 6$
13. $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$
14. $5x^2 + 5y^2 = 5$
15. $4x^2 + 4y^2 = 9$
16. $9x^2 + 9y^2 = 49$
17. $3 - y^2 = (x-2)^2$
18. $9 - x^2 = (y+1)^2$

Sketch the graph of each equation.

19. $x^2 + y^2 = 9$
20. $x^2 + y^2 = 16$
21. $x^2 + (y-3)^2 = 9$
22. $(x-4)^2 + y^2 = 16$
23. $(x+1)^2 + (y-1)^2 = 2$
24. $(x-2)^2 + (y+2)^2 = 8$
25. $(x-4)^2 + (y+3)^2 = 16$
26. $(x-3)^2 + (y-7)^2 = 25$
27. $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$
28. $\left(x + \frac{1}{3}\right)^2 + y^2 = \frac{1}{9}$

Rewrite each equation in the standard form for the equation of a circle, and identify its center and radius.

29. $x^2 + 4x + y^2 + 6y = 0$
30. $x^2 - 10x + y^2 + 8y = 0$
31. $x^2 - 2x + y^2 - 4y - 3 = 0$
32. $x^2 - 6x + y^2 - 2y + 9 = 0$
33. $x^2 + y^2 = 8y + 10x - 32$
34. $x^2 + y^2 = 8x - 10y$
35. $x^2 - x + y^2 + y = 0$
36. $x^2 - 3x + y^2 = 0$
37. $x^2 - 3x + y^2 - y = 1$
38. $x^2 - 5x + y^2 + 3y = 2$
39. $x^2 - \frac{2}{3}x + y^2 + \frac{3}{2}y = 0$
40. $x^2 + \frac{1}{3}x + y^2 - \frac{2}{3}y = \frac{1}{9}$

Graph both equations of each system on the same coordinate axes. Solve the system by elimination of variables to find all points of intersection of the graphs

41. $x^2 + y^2 = 10$
    $y = 3x$
42. $x^2 + y^2 = 4$
    $y = x - 2$
43. $x^2 + y^2 = 9$
    $y = x^2 - 3$
44. $x^2 + y^2 = 4$
    $y = x^2 - 2$
45. $(x-2)^2 + (y+3)^2 = 4$
    $y = x - 3$
46. $(x+1)^2 + (y-4)^2 = 17$
    $y = x + 2$

In Exercises 47–56, solve each problem.

47. Determine all points of intersection of the circle $(x-1)^2 + (y-2)^2 = 4$ with the y-axis.
48. Determine the points of intersection of the circle $x^2 + (y-3)^2 = 25$ with the x-axis.
49. Find the radius of the circle that has center $(2, -5)$ and passes through the origin.
50. Find the radius of the circle that has center $(-2, 3)$ and passes through $(3, -1)$.
51. Determine the equation of the circle that is centered at $(2, 3)$ and passes through $(-2, -1)$.
52. Determine the equation of the circle that is centered at $(3, 4)$ and passes through the origin.
53. Find all points of intersection of the circles $x^2 + y^2 = 9$ and $(x-5)^2 + y^2 = 9$.
54. A donkey is tied at the point $(2, -3)$ on a rope of length 12. Turnips are growing at the point $(6, 7)$. Can the donkey reach them?

Answers to odd # questions over the page

→

Odd # Answers

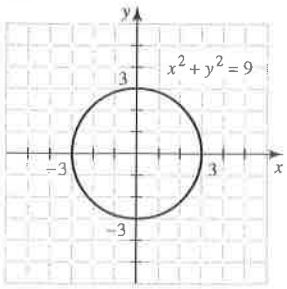
1. $x^2 + (y-3)^2 = 25$ **3.** $(x-1)^2 + (y+2)^2 = 81$

5. $x^2 + y^2 = 3$ **7.** $(x+6)^2 + (y+3)^2 = \frac{1}{4}$

9. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{3}\right)^2 = 0.01$ **11.** $(3, 5), \sqrt{2}$

13. $\left(0, \frac{1}{2}\right), \frac{\sqrt{2}}{2}$ **15.** $(0, 0), \frac{3}{2}$ **17.** $(2, 0), \sqrt{3}$

19.



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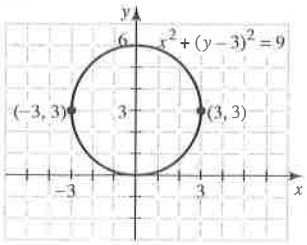

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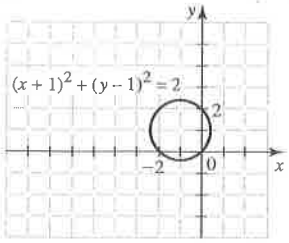

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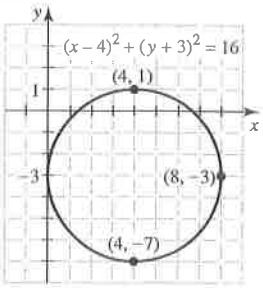

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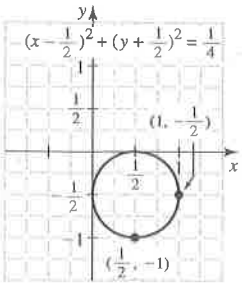


29. $(x+2)^2 + (y+3)^2 = 13$, $(-2, -3)$, $\sqrt{13}$

31. $(x-1)^2 + (y-2)^2 = 8$, $(1, 2)$, $2\sqrt{2}$

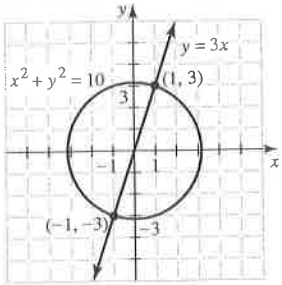
33. $(x-5)^2 + (y-4)^2 = 9$, $(5, 4)$, 3

35. $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{2}$, $\left(\frac{1}{2}, -\frac{1}{2}\right)$, $\frac{\sqrt{2}}{2}$

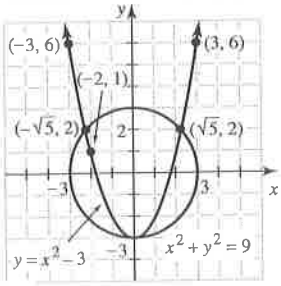
37. $\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{7}{2}$, $\left(\frac{3}{2}, \frac{1}{2}\right)$, $\frac{\sqrt{14}}{2}$

39. $\left(x - \frac{1}{3}\right)^2 + \left(y + \frac{3}{4}\right)^2 = \frac{97}{144}$, $\left(\frac{1}{3}, -\frac{3}{4}\right)$, $\frac{\sqrt{97}}{12}$

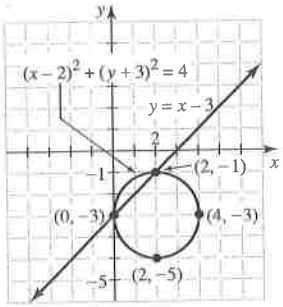
41. $\{(1, 3), (-1, -3)\}$



43. $\{(0, -3), (\sqrt{5}, 2), (-\sqrt{5}, 2)\}$



45. $\{(0, -3), (2, -1)\}$



47. $(0, 2 + \sqrt{3})$ and $(0, 2 - \sqrt{3})$

49. $\sqrt{29}$ **51.** $(x-2)^2 + (y-3)^2 = 32$

53. $\left(\frac{5}{2}, -\frac{\sqrt{11}}{2}\right)$ and $\left(\frac{5}{2}, \frac{\sqrt{11}}{2}\right)$

ELLIPSE and HYPERBOLA STUFF

Sketch the graph of each ellipse.

1. $\frac{x^2}{9} + \frac{y^2}{4} = 1$
2. $\frac{x^2}{9} + \frac{y^2}{16} = 1$
3. $\frac{x^2}{9} + y^2 = 1$
4. $x^2 + \frac{y^2}{4} = 1$
5. $\frac{x^2}{36} + \frac{y^2}{25} = 1$
6. $\frac{x^2}{25} + \frac{y^2}{49} = 1$
7. $\frac{x^2}{24} + \frac{y^2}{5} = 1$
8. $\frac{x^2}{6} + \frac{y^2}{17} = 1$
9. $9x^2 + 16y^2 = 144$
10. $9x^2 + 25y^2 = 225$
11. $25x^2 + y^2 = 25$
12. $x^2 + 16y^2 = 16$
13. $4x^2 + 9y^2 = 1$
14. $25x^2 + 16y^2 = 1$

Sketch the graph of each ellipse.

15. $\frac{(x-3)^2}{4} + \frac{(y-1)^2}{9} = 1$
16. $\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$
17. $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{25} = 1$
18. $\frac{(x-3)^2}{36} + \frac{(y+4)^2}{64} = 1$
19. $(x-2)^2 + \frac{(y+1)^2}{36} = 1$
20. $\frac{(x+3)^2}{9} + (y+1)^2 = 1$

Sketch the graph of each hyperbola and write the equations of its asymptotes.

21. $\frac{x^2}{4} - \frac{y^2}{9} = 1$
22. $\frac{x^2}{16} - \frac{y^2}{9} = 1$
23. $\frac{y^2}{4} - \frac{x^2}{25} = 1$
24. $\frac{y^2}{9} - \frac{x^2}{16} = 1$
25. $\frac{x^2}{25} - y^2 = 1$
26. $x^2 - \frac{y^2}{9} = 1$
27. $x^2 - \frac{y^2}{25} = 1$
28. $\frac{x^2}{9} - y^2 = 1$
29. $9x^2 - 16y^2 = 144$
30. $9x^2 - 25y^2 = 225$
31. $x^2 - y^2 = 1$
32. $y^2 - x^2 = 1$

Graph both equations of each system on the same coordinate axes. Use elimination of variables (as in Section 10.4) to find all points of intersection.

33. $\frac{x^2}{4} + \frac{y^2}{9} = 1$
$x^2 - \frac{y^2}{9} = 1$

34. $x^2 - \frac{y^2}{4} = 1$
$\frac{x^2}{9} + \frac{y^2}{4} = 1$

35. $\frac{x^2}{4} + \frac{y^2}{16} = 1$
$x^2 + y^2 = 1$

36. $x^2 + \frac{y^2}{9} = 1$
$x^2 + y^2 = 4$

37. ~~$x^2 + y^2 = 4$~~
~~$x^2 - y^2 = 1$~~

38. ~~$x^2 + y^2 = 16$~~
~~$x^2 - y^2 = 4$~~

39. ~~$x^2 + 9y^2 = 9$~~
~~$x^2 + y^2 = 4$~~

40. ~~$x^2 + y^2 = 25$~~
~~$x^2 + 25y^2 = 25$~~

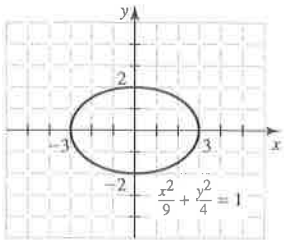
41. ~~$x^2 + 9y^2 = 9$~~
~~$y = x^2 - 1$~~

42. ~~$4x^2 + y^2 = 4$~~
~~$y = 2x^2 - 2$~~

43. ~~$9x^2 - 4y^2 = 36$~~
~~$2y = x - 2$~~

ODD # answers:

1.



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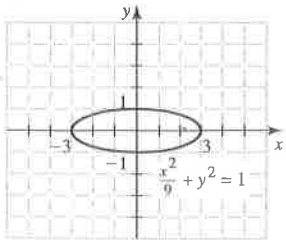

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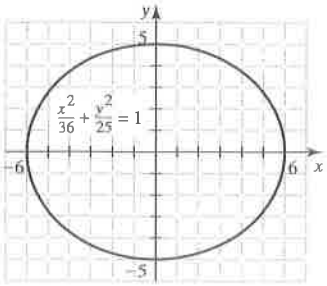

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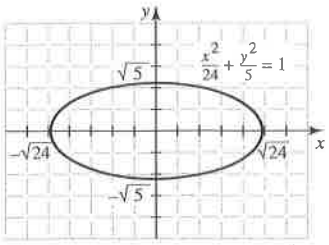

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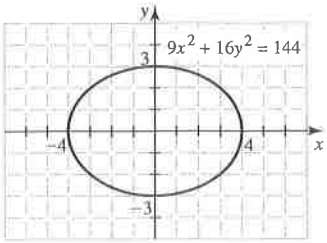

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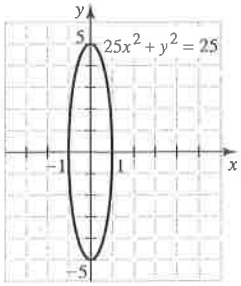

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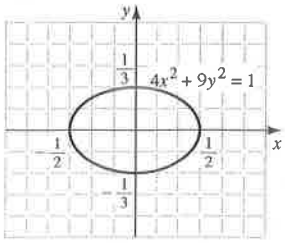

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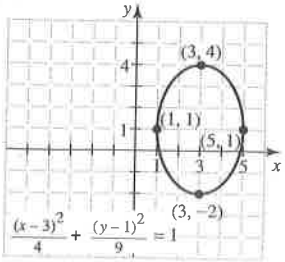
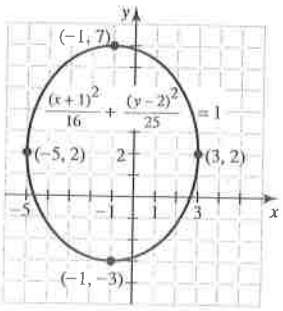
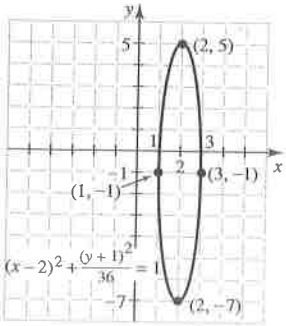


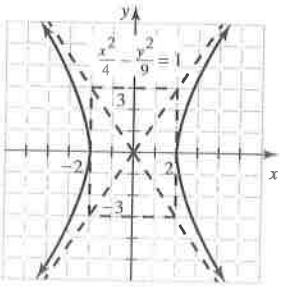
17.



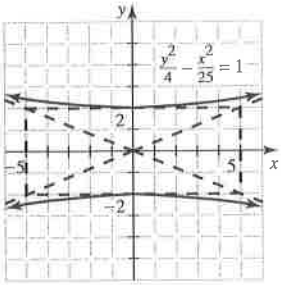
19.



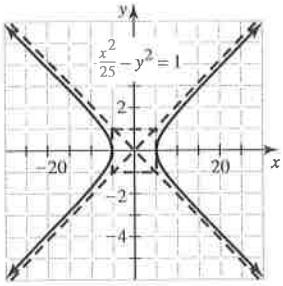
21. $y = \pm\frac{3}{2}x$



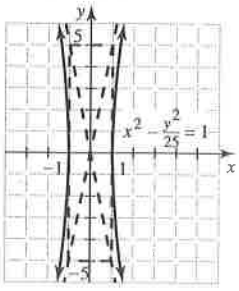
23. $y = \pm\frac{2}{5}x$



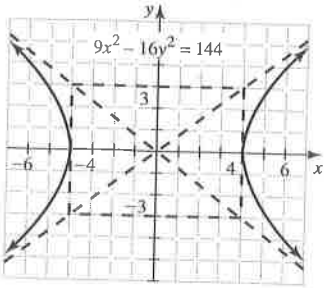
25. $y = \pm\frac{1}{5}x$



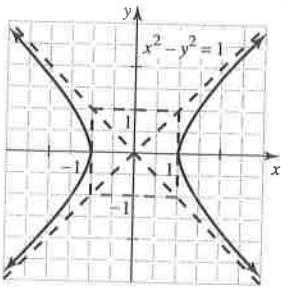
27. $y = \pm 5x$



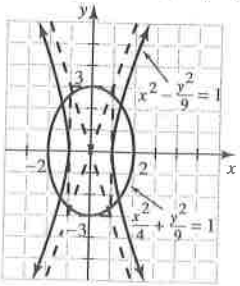
29. $y = \pm\frac{3}{4}x$



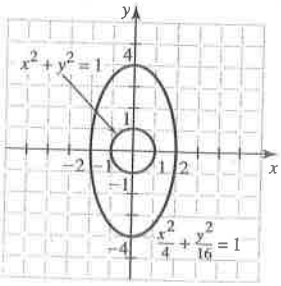
31. $y = \pm x$



33. $\left(\frac{2\sqrt{10}}{5}, \frac{3\sqrt{15}}{5}\right), \left(\frac{2\sqrt{10}}{5}, -\frac{3\sqrt{15}}{5}\right), \left(-\frac{2\sqrt{10}}{5}, \frac{3\sqrt{15}}{5}\right), \left(-\frac{2\sqrt{10}}{5}, -\frac{3\sqrt{15}}{5}\right)$



35. No points of intersection



In Exercises 7–22, find the center, foci, vertices, of the ellipse, and sketch its graph.

7. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

8. $\frac{x^2}{144} + \frac{y^2}{169} = 1$

9. $\frac{x^2}{16} + \frac{y^2}{25} = 1$

10. $\frac{x^2}{169} + \frac{y^2}{144} = 1$

11. $\frac{x^2}{9} + \frac{y^2}{5} = 1$

12. $\frac{x^2}{28} + \frac{y^2}{64} = 1$

13. $x^2 + 4y^2 = 4$

14. $5x^2 + 3y^2 = 15$

15. $3x^2 + 2y^2 = 6$

16. $5x^2 + 7y^2 = 70$

17. $\frac{(x-1)^2}{9} + \frac{(y-5)^2}{25} = 1$

18. $(x+2)^2 + \frac{(y+4)^2}{1/4} = 1$

19. $9x^2 + 4y^2 + 36x - 24y + 36 = 0$

20. $9x^2 + 4y^2 - 36x + 8y + 31 = 0$

21. $16x^2 + 25y^2 - 32x + 50y + 31 = 0$

C **In Exercises 23–28, find the center, foci, and vertices of the ellipse.**

23. $4x^2 + y^2 = 1$

24. $16x^2 + 25y^2 = 1$

25. $12x^2 + 20y^2 - 12x + 40y - 37 = 0$

26. $36x^2 + 9y^2 + 48x - 36y + 43 = 0$

27. $x^2 + 2y^2 - 3x + 4y + 0.25 = 0$

In Exercises 29–38, find an equation of the ellipse.

29. Center: (0, 0)
Focus: (2, 0)
Vertex: (3, 0)

30. Center: (0, 0)
Vertex: (2, 0)
Minor axis length: 3

33. Vertices: (3, 1), (3, 9)
Minor axis length: 6

34. Vertices: (0, 2), (4, 2)
Minor axis length: 2

35. Foci: (0, ±5)
Major axis length: 14

36. Foci: (±2, 0)
Major axis length: 8

37. Center: (0, 0)
Major axis: horizontal
Points on ellipse:
(3, 1), (4, 0)

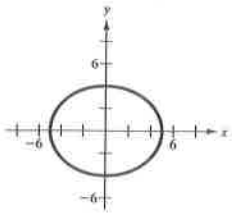
38. Center: (1, 2)
Major axis: vertical
Points on ellipse:
(1, 6), (3, 2)

47. *Mountain Tunnel* A semielliptical arch over a tunnel for a road through a mountain has a major axis of 100 feet and a height at the center of 30 feet. Determine the height of the arch 5 feet from the edge of the tunnel.

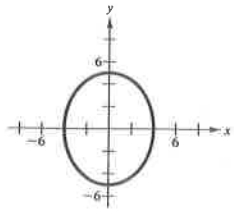
ODD # answers

1. e 3. c 5. f

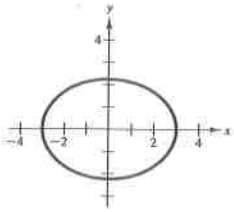
7. Center: (0, 0)
Foci: (±3, 0)
Vertices: (±5, 0)



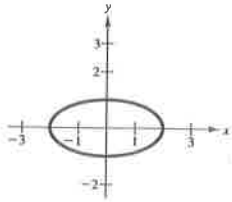
9. Center: (0, 0)
Foci: (0, ±3)
Vertices: (0, ±5)



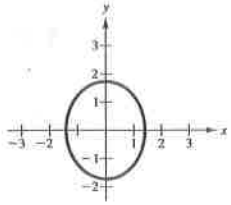
11. Center: (0, 0)
Foci: (±2, 0)
Vertices: (±3, 0)



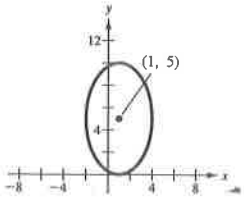
13. Center: (0, 0)
Foci: $(\pm\sqrt{3}, 0)$
Vertices: (±2, 0)



15. Center: (0, 0)
Foci: (0, ±1)
Vertices: $(0, \pm\sqrt{3})$

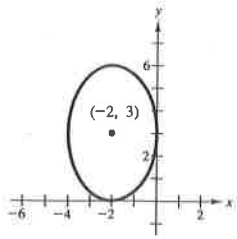


17. Center: (1, 5)
Foci: (1, 9), (1, 1)
Vertices: (1, 10), (1, 0)

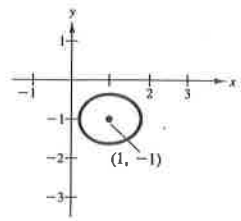


More answers over page →

19. Center: $(-2, 3)$
Foci: $(-2, 3 \pm \sqrt{5})$
Vertices: $(-2, 6), (-2, 0)$
$e = \dfrac{\sqrt{5}}{3}$



21. Center: $(1, -1)$
Foci: $\left(1 \pm \dfrac{3\sqrt{10}}{20}, -1\right)$
Vertices: $\left(1 \pm \dfrac{\sqrt{10}}{4}, -1\right)$
$e = \dfrac{3}{5}$



23. Center: $(0, 0)$
Foci: $\left(0, \pm\dfrac{\sqrt{3}}{2}\right)$
Vertices: $(0, \pm 1)$

25. Center: $\left(\dfrac{1}{2}, -1\right)$
Foci: $\left(\dfrac{1}{2} \pm \sqrt{2}, -1\right)$
Vertices: $\left(\dfrac{1}{2} \pm \sqrt{5}, -1\right)$

27. Center: $\left(\dfrac{3}{2}, -1\right)$
Foci: $\left(\dfrac{3}{2} - \sqrt{2}, -1\right), \left(\dfrac{3}{2} + \sqrt{2}, -1\right)$
Vertices: $\left(-\dfrac{1}{2}, -1\right), \left(\dfrac{7}{2}, -1\right)$

29. $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$

33. $\dfrac{(x-3)^2}{9} + \dfrac{(y-5)^2}{16} = 1$

35. $\dfrac{x^2}{24} + \dfrac{y^2}{49} = 1$

37. $\dfrac{x^2}{16} + \dfrac{7y^2}{16} = 1$

47. $3\sqrt{19} \approx 13$ ft

In Exercises 7–22, find the center, foci, and vertices of the hyperbola and sketch its graph using asymptotes as an aid.

7. $x^2 - y^2 = 1$

8. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

9. $y^2 - \dfrac{x^2}{4} = 1$

10. $\dfrac{y^2}{9} - \dfrac{x^2}{1} = 1$

11. $\dfrac{y^2}{25} - \dfrac{x^2}{144} = 1$

12. $\dfrac{x^2}{36} - \dfrac{y^2}{4} = 1$

13. $5y^2 = 4x^2 + 20$

14. $7x^2 - 3y^2 = 21$

15. $\dfrac{(x-1)^2}{4} - \dfrac{(y+2)^2}{1} = 1$

16. $\dfrac{(x+1)^2}{144} - \dfrac{(y-4)^2}{25} = 1$

17. $(y+6)^2 - (x-2)^2 = 1$

18. $\dfrac{(y-1)^2}{1/4} - \dfrac{(x+3)^2}{1/9} = 1$

19. $9x^2 - y^2 - 36x - 6y + 18 = 0$

20. $x^2 - 9y^2 + 36y - 72 = 0$

In Exercises 23–28, find the center, foci, and vertices of the hyperbola

23. $2x^2 - 3y^2 = 6$

24. $3y^2 = 5x^2 + 15$

25. $9y^2 - x^2 + 2x + 54y + 62 = 0$

26. $9x^2 - y^2 + 54x + 10y + 55 = 0$

27. $3x^2 - 2y^2 - 6x - 12y - 27 = 0$

28. $3y^2 - x^2 + 6x - 12y = 0$

In Exercises 29–38, find an equation of the specified hyperbola.

29. Center: (0, 0)
Vertex: (0, 2)
Focus: (0, 4)

30. Center: (0, 0)
Vertex: (3, 0)
Focus: (5, 0)

31. Vertices $(\pm 1, 0)$
Asymptotes: $y = \pm 3x$

32. Vertices: $(0, \pm 3)$
Asymptotes: $y = \pm 3x$

33. Vertices: (0, 2), (6, 2)
Asymptotes: $y = \frac{2}{3}x$
$y = 4 - \frac{2}{3}x$

34. Focus: (10, 0)
Asymptotes: $y = \pm\frac{3}{4}x$
Center: (0, 0)

35. Vertices: $(2, \pm 3)$
Point on graph: (0, 5)

36. Vertices: $(2, \pm 3)$
Foci: $(2, \pm 5)$

37. Find an equation of the hyperbola such that for any point on the hyperbola, the difference between its distances from the points (2, 2) and (10, 2) is 6.

38. Find an equation of the hyperbola such that for any point on the hyperbola, the difference between its distances from the points (−3, 0) and (−3, 3) is 2.

More answers

27. Center: (1, −3)
Vertices: (−1, −3), (3, −3)
Foci: $(1 \pm \sqrt{10}, -3)$

29. $\dfrac{y^2}{4} - \dfrac{x^2}{12} = 1$

31. $\dfrac{x^2}{1} - \dfrac{y^2}{9} = 1$

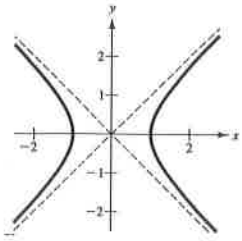
33. $\dfrac{(x-3)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

35. $\dfrac{y^2}{9} - \dfrac{(x-2)^2}{9/4} = 1$

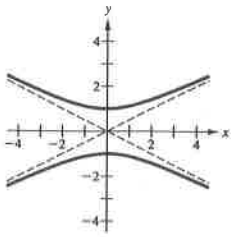
37. $\dfrac{(x-6)^2}{9} - \dfrac{(y-2)^2}{7} = 1$

ODD # answers

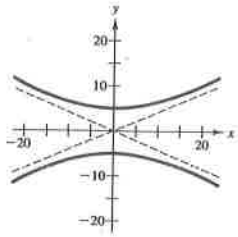
7. Center: (0, 0)
Vertices: $(\pm 1, 0)$
Foci: $(\pm\sqrt{2}, 0)$



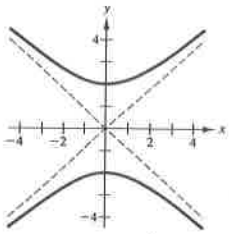
9. Center: (0, 0)
Vertices: $(0, \pm 1)$
Foci: $(0, \pm\sqrt{5})$



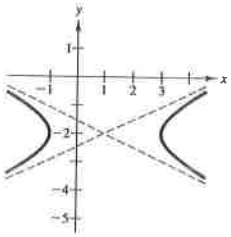
11. Center: (0, 0)
Vertices: $(0, \pm 5)$
Foci: $(0, \pm 13)$



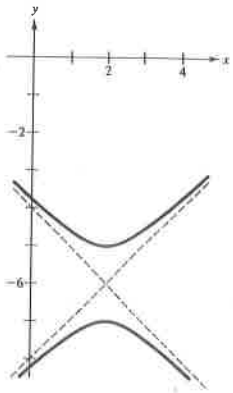
13. Center: (0, 0)
Vertices: $(0, \pm 2)$
Foci: $(0, \pm 3)$



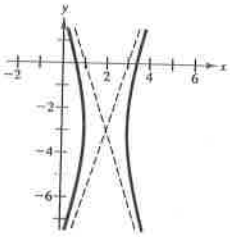
15. Center: (1, −2)
Vertices: (−1, −2), (3, −2)
Foci: $(1 \pm \sqrt{5}, -2)$



17. Center: (2, −6)
Vertices: (2, −5), (2, −7)
Foci: $(2, -6 \pm \sqrt{2})$



19. Center: (2, −3)
Vertices: (1, −3), (3, −3)
Foci: $(2 \pm \sqrt{10}, -3)$



23. Center: (0, 0)
Vertices: $(\pm\sqrt{3}, 0)$
Foci: $(\pm\sqrt{5}, 0)$

25. Center: (1, −3)
Vertices: $(1, -3 \pm \sqrt{2})$
Foci: $(1, -3 \pm 2\sqrt{5})$

PARABOLA STUFF

Find the vertex, focus, and directrix for each parabola.

1. $y = 2x^2$
2. $y = \frac{1}{2}x^2$
3. $y = -\frac{1}{4}x^2$
4. $y = -\frac{1}{12}x^2$
5. $y = \frac{1}{2}(x-3)^2 + 2$
6. $y = \frac{1}{4}(x+2)^2 - 5$
7. $y = -(x+1)^2 + 6$
8. $y = -3(x-4)^2 + 1$

Find the equation of the parabola with the given focus and directrix.

9. Focus $(0, 2)$, directrix $y = -2$
10. Focus $(0, -3)$, directrix $y = 3$
11. Focus $\left(0, -\frac{1}{2}\right)$, directrix $y = \frac{1}{2}$
12. Focus $\left(0, \frac{1}{8}\right)$, directrix $y = -\frac{1}{8}$
13. Focus $(3, 2)$, directrix $y = 1$
14. Focus $(-4, 5)$, directrix $y = 4$
15. Focus $(1, -2)$, directrix $y = 2$
16. Focus $(2, -3)$, directrix $y = 1$
17. Focus $(-3, 1.25)$, directrix $y = 0.75$
18. Focus $\left(5, \frac{17}{8}\right)$, directrix $y = \frac{15}{8}$

Write each equation in the form $y = a(x-h)^2 + k$. Identify the vertex, focus, and directrix of each parabola.

19. $y = x^2 - 6x + 1$
20. $y = x^2 + 4x - 7$
21. $y = 2x^2 + 12x + 5$
22. $y = 3x^2 + 6x - 7$
23. $y = -2x^2 + 16x + 1$
24. $y = -3x^2 - 6x + 7$
25. $y = 5x^2 + 40x$
26. $y = -2x^2 + 10x$

Find the vertex, focus, and directrix of each parabola and determine whether the parabola opens upward or downward.

27. $y = x^2 - 4x + 1$
28. $y = x^2 - 6x - 7$
29. $y = -x^2 + 2x - 3$
30. $y = -x^2 + 4x + 9$
31. $y = 3x^2 - 6x + 1$
32. $y = 2x^2 + 4x - 3$
33. $y = -x^2 - 3x + 2$
34. $y = -x^2 + 3x - 1$
35. $y = 3x^2 + 5$
36. $y = -2x^2 - 6$

Find the vertex, axis of symmetry, x-intercepts, and y-intercept for each parabola. Find several additional points on the parabola, and then sketch its graph

37. $y = x^2 - 3x + 2$
38. $y = x^2 + 6x + 8$
39. $y = -x^2 - 2x + 8$
40. $y = -x^2 - 2x + 15$
41. $y = (x+2)^2 + 1$
42. $y = -2(x+1)^2 + 3$
43. $y = x^2 + 2x + 1$
44. $y = x^2 - 6x + 9$
45. $y = -4x^2 + 4x - 1$
46. $y = -4x^2 + 12x - 9$
47. $y = x^2 - 5x$
48. $y = 3x^2 - 9x$
49. $y = 3x^2 + 5$
50. $y = -2x^2 + 3$
51. $v = x^2 - 2x - 1$
52. $y = x^2 - 4x + 1$

ODD # answers

1. Vertex $(0, 0)$, focus $\left(0, \frac{1}{8}\right)$, directrix $y = -\frac{1}{8}$
3. Vertex $(0, 0)$, focus $(0, -1)$, directrix $y = 1$
5. Vertex $(3, 2)$, focus $(3, 2.5)$, directrix $y = 1.5$
7. Vertex $(-1, 6)$, focus $(-1, 5.75)$, directrix $y = 6.25$

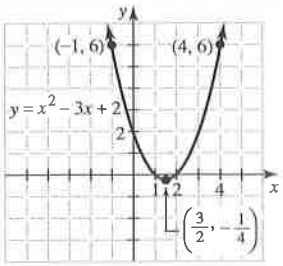
9. $y = \frac{1}{8}x^2$ 11. $y = -\frac{1}{2}x^2$ 13. $y = \frac{1}{2}x^2 - 3x + 6$

15. $y = -\frac{1}{8}x^2 + \frac{1}{4}x - \frac{1}{8}$ 17. $y = x^2 + 6x + 10$

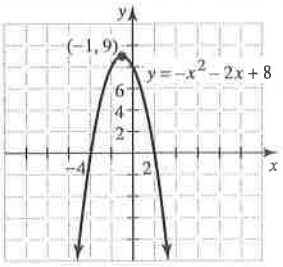
19. $y = (x-3)^2 - 8$, vertex $(3, -8)$, focus $(3, -7.75)$, directrix $y = -8.25$
21. $y = 2(x+3)^2 - 13$, vertex $(-3, -13)$, focus $(-3, -12.875)$, directrix $y = -13.125$
23. $y = -2(x-4)^2 + 33$, vertex $(4, 33)$, focus $\left(4, 32\frac{7}{8}\right)$, directrix $y = 33\frac{1}{8}$
25. $y = 5(x+4)^2 - 80$, vertex $(-4, -80)$, focus $\left(-4, -79\frac{19}{20}\right)$, directrix $y = -80\frac{1}{20}$
27. Vertex $(2, -3)$, focus $\left(2, -2\frac{3}{4}\right)$, directrix $y = -3\frac{1}{4}$, upward
29. Vertex $(1, -2)$, focus $\left(1, -2\frac{1}{4}\right)$, directrix $y = -1\frac{3}{4}$, downward
31. Vertex $(1, -2)$, focus $\left(1, -1\frac{11}{12}\right)$, directrix $y = -2\frac{1}{12}$, upward
33. Vertex $\left(-\frac{3}{2}, \frac{17}{4}\right)$, focus $\left(-\frac{3}{2}, 4\right)$, directrix $y = \frac{9}{2}$, downward
35. Vertex $(0, 5)$, focus $\left(0, 5\frac{1}{12}\right)$, directrix $y = 4\frac{11}{12}$, upward

more answers →

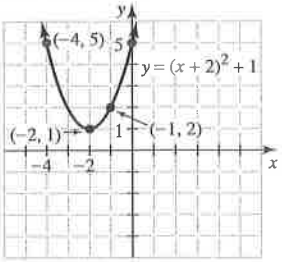
37. Vertex $\left(\frac{3}{2}, -\frac{1}{4}\right)$, axis of symmetry $x = \frac{3}{2}$, intercepts $(0, 2)$, $(1, 0)$, $(2, 0)$



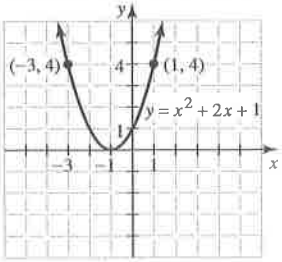
39. Vertex $(-1, 9)$, axis of symmetry $x = -1$, intercepts $(0, 8)$, $(-4, 0)$, $(2, 0)$



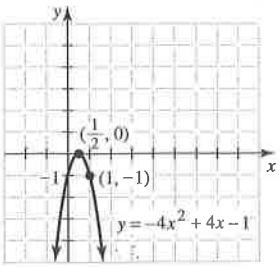
41. Vertex $(-2, 1)$, axis of symmetry $x = -2$, intercept $(0, 5)$



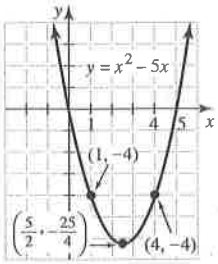
43. Vertex $(-1, 0)$, axis of symmetry $x = -1$, intercepts $(0, 1)$, $(-1, 0)$



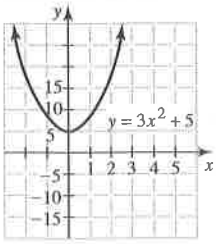
45. Vertex $\left(\frac{1}{2}, 0\right)$, axis of symmetry $x = \frac{1}{2}$, intercepts $(0, -1)$, $\left(\frac{1}{2}, 0\right)$



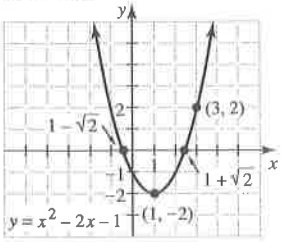
47. Vertex is $\left(\frac{5}{2}, -\frac{25}{4}\right)$, axis of symmetry $x = \frac{5}{2}$, intercepts $(0, 0)$, $(5, 0)$



49. Vertex $(0, 5)$, axis of symmetry $x = 0$, intercept $(0, 5)$



51. Vertex $(1, -2)$, axis of symmetry $x = 1$, intercepts $(0, -1)$, $(1 + \sqrt{2}, 0)$, $(1 - \sqrt{2}, 0)$



Conics Review Cut-and-Paste

Even-numbered Solutions

Circle Stuff

2. $(x-2)^2 + y^2 = 9$

4. $(x+3)^2 + (y-5)^2 = 16$

6. $x^2 + y^2 = 2$

8. $(x+3)^2 + (y+5)^2 = \frac{1}{16}$

10. $(x+\frac{1}{2})^2 + (y-3)^2 = 0.04$

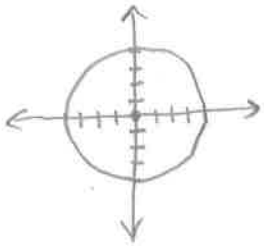
12. $C = (-3, 7)$ $r = \sqrt{6}$

14. $C = (0, 0)$ $r = 1$

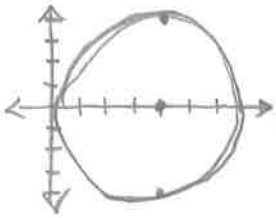
16. $C = (0, 0)$ $r = \frac{7}{3}$

18. $C = (0, -1)$ $r = 3$

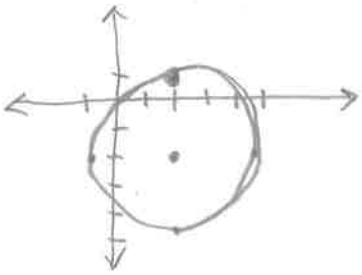
20.



22.



24.



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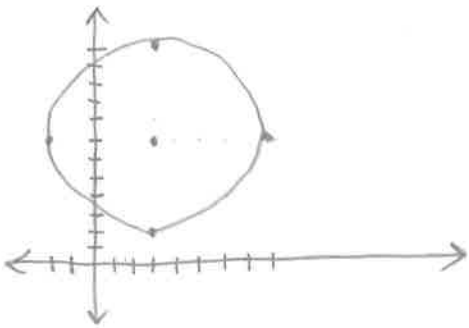
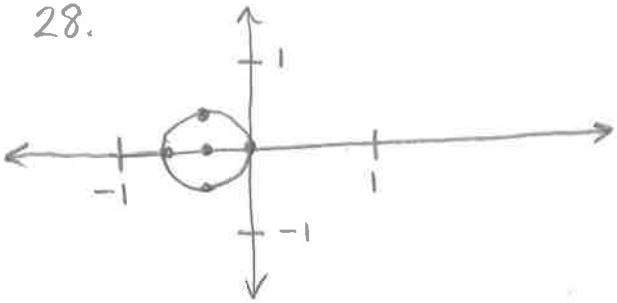


28.



30. $(x-5)^2 + (y+4)^2 = 41$

32. $(x-3)^2 + (y-1)^2 = 13$

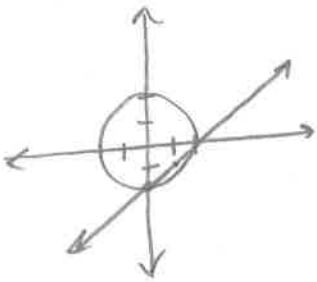
34. $(x-4)^2 + (y+5)^2 = 41$

36. $\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$

38. $\left(x - \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{21}{2}$

40. $\left(x + \frac{1}{6}\right)^2 + \left(y - \frac{1}{3}\right)^2 = \frac{1}{4}$

42.



$x^2 + (x-2)^2 = 4$

$2x^2 - 4x + 4 = 4$

$2x^2 - 4x = 0$

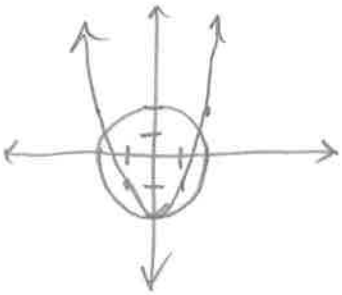
$2x(x-2) = 0$

$x = 0, 2$

$y = -2, 0$

$\boxed{(0,-2), (2,0)}$

44.



$x^2 + (x^2-2)^2 = 4$

$x^4 - 3x^2 + 4 = 4$

$x^4 - 3x^2 = 0$

$x^2(x^2-3) = 0$

$x = 0, \pm\sqrt{3}$

$y = -2, 1$

$(0,-2), (\sqrt{3},1), (-\sqrt{3},1)$

46. $(x+1)^2 + ((x+2)-4)^2 = 17$

$(x^2+2x+1) + (x-2)^2 = 17$

$x^2+2x+1 + x^2-4x+4 = 17$

$2x^2-2x-12 = 0$

$x^2-x-6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$

$y = 5, 0$

$\boxed{(3,5), (-2,0)}$

48. $(4,0), (-4,0)$

50. $\sqrt{41}$

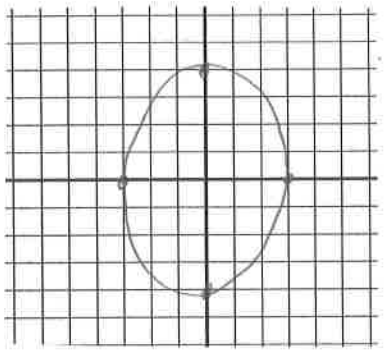
52. $(x-3)^2 + (y-4)^2 = 25$

54. $\sqrt{(6-2)^2 + 7-(-3)^2} = \sqrt{16+100} = \sqrt{116} < 12$

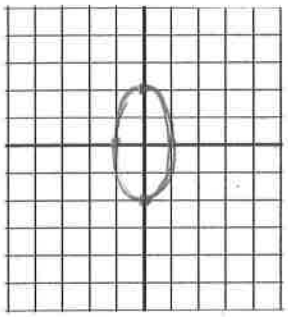
Yes!

Ellipse and Hyperbola Stuff

2.



4.



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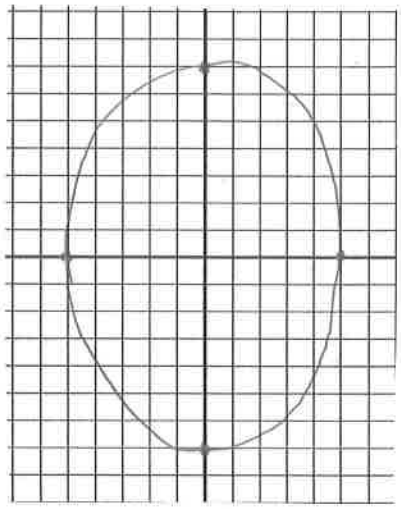
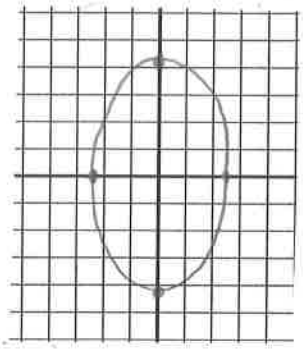


8.



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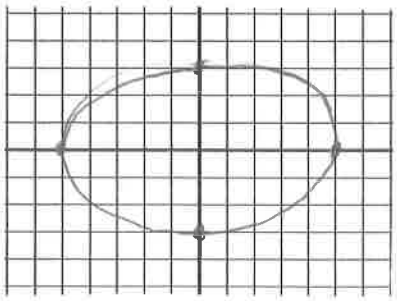
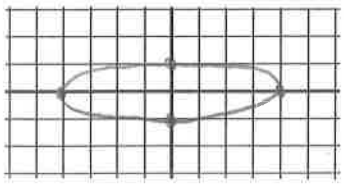


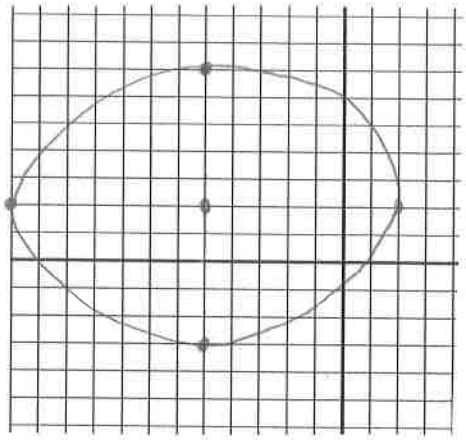
12.



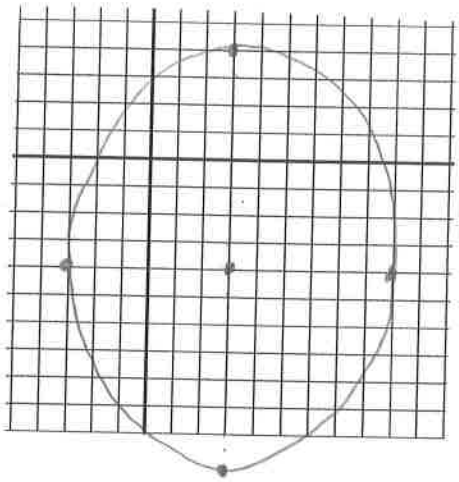
14.

$$\left(\frac{x^2}{\left(\frac{1}{25}\right)} + \frac{y^2}{\left(\frac{1}{16}\right)} = 1\right)$$

16.



18.



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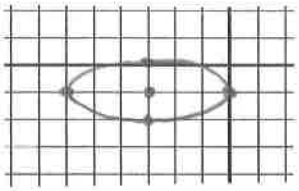

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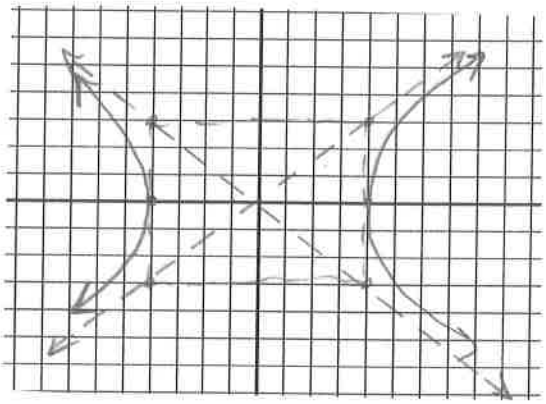
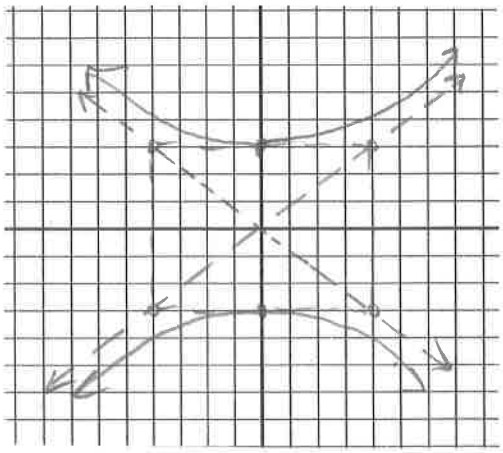


24.



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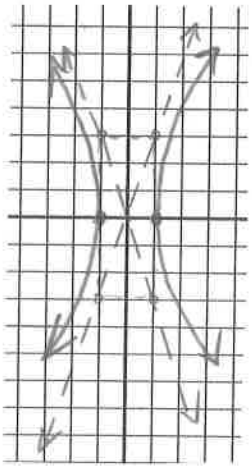

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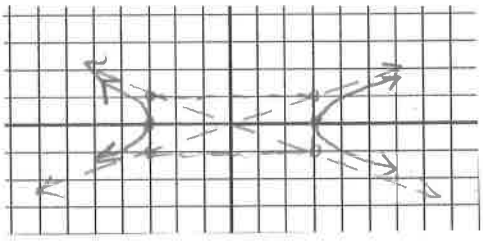
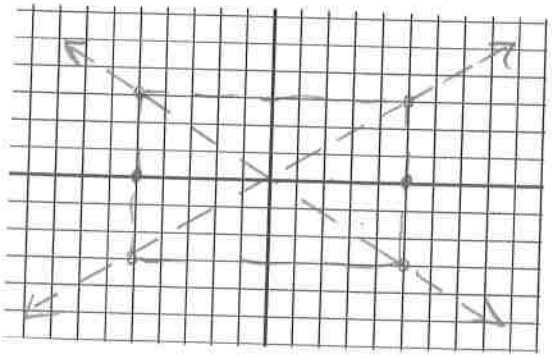
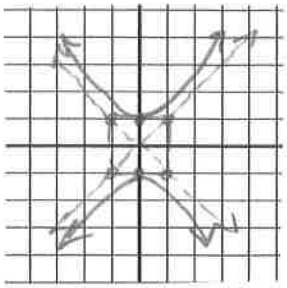


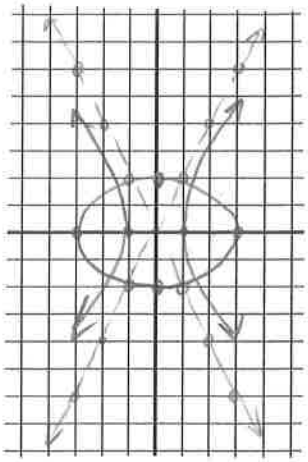
30.



32.



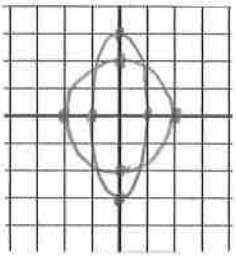
34.



4 points:

$\left(\pm\frac{3\sqrt{5}}{5}, \pm\frac{4\sqrt{5}}{5}\right)$

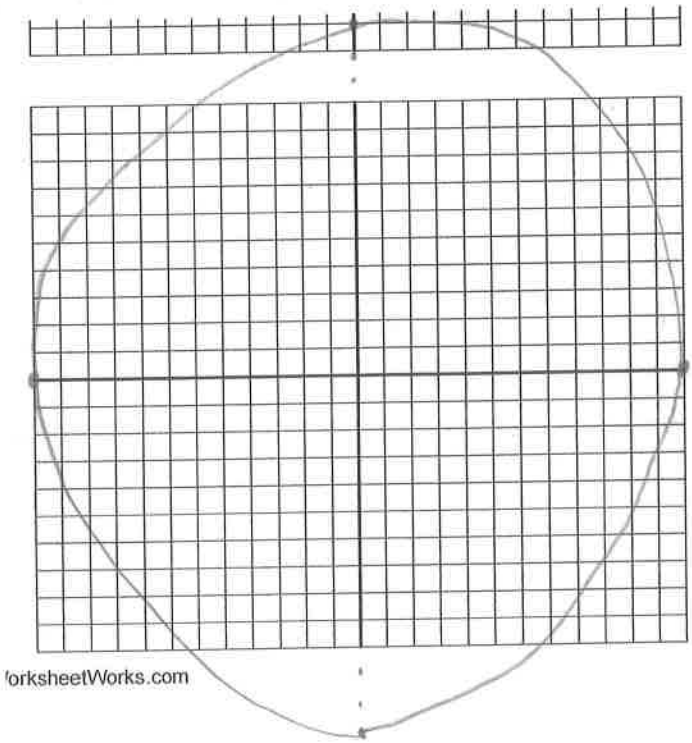
36.



4 points:

$\left(\pm\frac{\sqrt{10}}{4}, \pm\frac{3\sqrt{6}}{4}\right)$

8.



Center: $(0,0)$

Foci: $b^2 + c^2 = a^2$

$a^2 = 169$ $\quad c^2 = 25$

$b^2 = 144$ $\quad c = \pm 5$

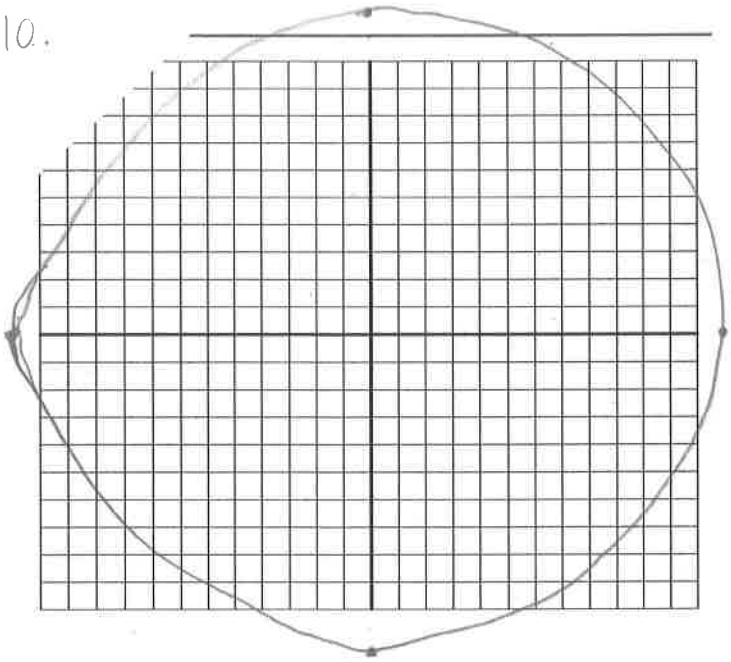
Foci @ $(0,5), (0,-5)$

Vertices:

$(12,0), (-12,0),$

$(0,13), (0,-13)$

10.



Center: $(0,0)$

Foci: $b^2 + c^2 = a^2$

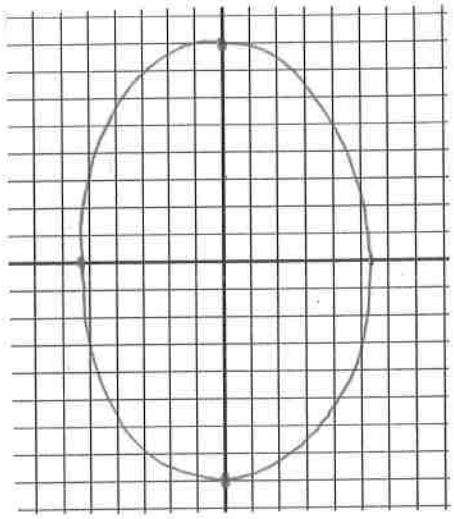
$b^2 = 144$ $\quad c^2 = 25$

$a^2 = 169$ $\quad c = \pm 5$

Foci @ $(5,0)$, $(-5,0)$

Vertices: $(0,12)$, $(0,-12)$, $(13,0)$, $(-13,0)$

12.



Center: $(0,0)$

Foci: $b^2 + c^2 = a^2$

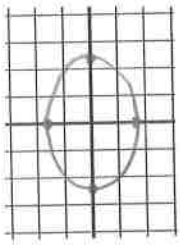
$b^2 = 28$ $\quad c^2 = 36$

$a^2 = 64$ $\quad c = \pm 6$

Foci @ $(0,6)$, $(0,-6)$

Vertices: $(0,8)$, $(0,-8)$, $(2\sqrt{7},0)$, $(-2\sqrt{7},0)$

14.



Center: $(0,0)$

Foci: $b^2 + c^2 = a^2$

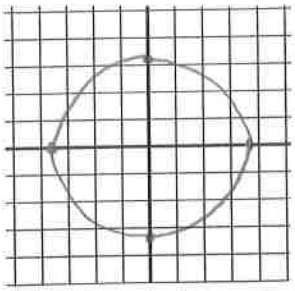
$b^2 = 3$ $\quad c^2 = 2$

$a^2 = 5$ $\quad c = \pm\sqrt{2}$

Foci @ $(0,\sqrt{2})$, $(0,-\sqrt{2})$

Vertices: $(0,\sqrt{5})$, $(0,-\sqrt{5})$, $(\sqrt{3},0)$, $(-\sqrt{3},0)$

16.



Center: $(0,0)$

Foci: $b^2 + c^2 = a^2$

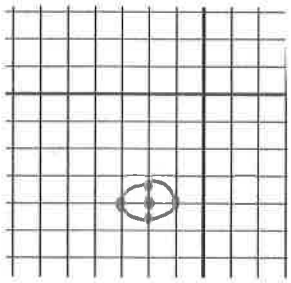
$b^2 = 10$ $\quad$ $c^2 = 4$

$a^2 = 14$ $\quad$ $c = \pm 2$

Foci @ $(2,0)$, $(-2,0)$

Vertices: $(0,\sqrt{10})$, $(0,-\sqrt{10})$,
$(\sqrt{14},0)$, $(-\sqrt{14},0)$

18.



Center: $(-2,-4)$

Foci: $b^2 + c^2 = a^2$

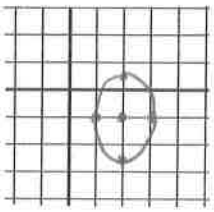
$b^2 = \frac{1}{4}$ $\quad$ $a^2 = \frac{3}{4}$

$c^2 = 1$ $\quad$ $a = \frac{\pm\sqrt{3}}{2}$

Foci @ $\left(\frac{-4+\sqrt{3}}{2}, -4\right)$, $\left(\frac{-4-\sqrt{3}}{2}, -4\right)$

Vertices: $(-1,-4)$, $(-3,-4)$,
$(-2,-\frac{7}{2})$, $(-2,-\frac{9}{2})$.

20.



Center: $(2,-1)$

Foci: $b^2 + c^2 = a^2$

$b^2 = 1$

$c^2 = \frac{9}{4}$ $\quad$ $a^2 = \frac{5}{4}$

$a = \frac{\pm\sqrt{5}}{2}$

Foci @ $\left(2, \frac{-2+\sqrt{5}}{2}\right)$, $\left(2, \frac{-2-\sqrt{5}}{2}\right)$

Vertices: $(3,-1)$, $(1,-1)$,
$(2,\frac{1}{2})$, $(2,-\frac{5}{2})$.

24. $\dfrac{x^2}{\left(\frac{1}{16}\right)} + \dfrac{y^2}{\left(\frac{1}{25}\right)}$

C: $(0,0)$

Foci: $a^2 = \frac{1}{16}$ $\quad c^2 = \frac{9}{400}$

$b^2 = \frac{1}{25}$ $\quad c = \pm \frac{3}{20}$

Foci @ $\left(\frac{3}{20}, 0\right), \left(\frac{-3}{20}, 0\right)$

Vertices: $\left(\frac{1}{4}, 0\right), \left(-\frac{1}{4}, 0\right),$
$\left(0, \frac{1}{5}\right), \left(0, -\frac{1}{5}\right)$

26. $\dfrac{\left(x + \frac{2}{3}\right)^2}{\left(\frac{1}{4}\right)} + (y-2)^2 = 1$

C: $\left(-\frac{2}{3}, 2\right)$

Foci: $a^2 = 1$ $\quad c^2 = \frac{3}{4}$

$b^2 = \frac{1}{4}$ $\quad c = \pm \frac{\sqrt{3}}{2}$

Foci @ $\left(-\frac{2}{3}, \frac{4+\sqrt{3}}{2}\right), \left(-\frac{2}{3}, \frac{4-\sqrt{3}}{2}\right)$

Vertices: $\left(-\frac{1}{6}, 2\right), \left(-\frac{7}{6}, 2\right),$
$\left(-\frac{2}{3}, 3\right), \left(-\frac{2}{3}, 1\right)$

30. $\dfrac{x^2}{4} + \dfrac{y^2}{\left(\frac{9}{4}\right)} = 1$

34. $\dfrac{(x-2)^2}{4} + (y-2)^2 = 1$

36. $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$

38. $\dfrac{(x-1)^2}{4} + \dfrac{(y-2)^2}{16} = 1$

8. Center: $(0,0)$

Foci: $a^2 + b^2 = c^2$

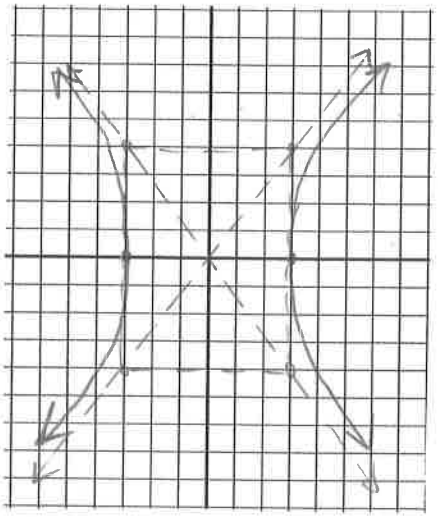
$9 + 16 = c^2$

$c^2 = 25$

$c = \pm 5$

Foci @ $(5,0)$, $(-5,0)$

Vertices: $(3,0)$, $(-3,0)$



10. Center: $(0,0)$

Foci: $a^2 + b^2 = c^2$

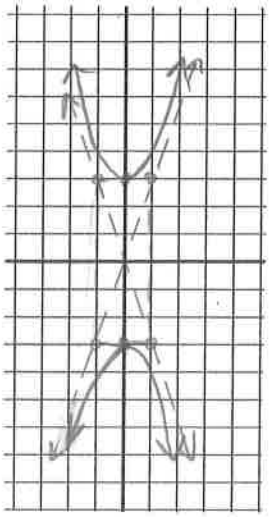
$9 + 1 = c^2$

$c^2 = 10$

$c = \pm\sqrt{10}$

Foci @ $(0,\sqrt{10})$, $(0,-\sqrt{10})$

Vertices: $(0,3)$, $(0,-3)$



12. Center: $(0,0)$

Foci: $a^2 + b^2 = c^2$

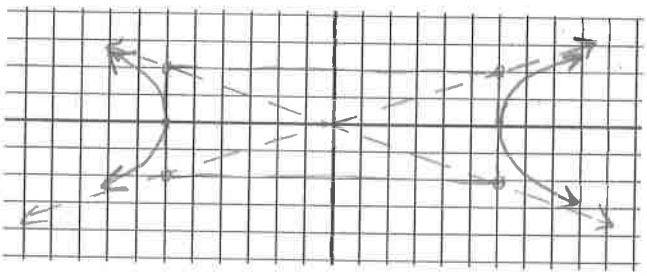
$36 + 4 = c^2$

$c^2 = 40$

$c = \pm 2\sqrt{10}$

Foci @ $(2\sqrt{10},0)$, $(-2\sqrt{10},0)$

Vertices: $(6,0)$, $(-6,0)$



14. Center: $(0,0)$

Foci: $a^2 + b^2 = c^2$

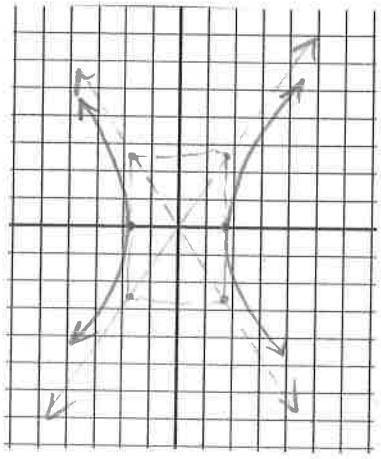
$3 + 7 = c^2$

$c^2 = 10$

$c = \pm\sqrt{10}$

Foci @ $(\sqrt{10}, 0)$, $(-\sqrt{10}, 0)$

Vertices: $(\sqrt{3}, 0)$, $(-\sqrt{3}, 0)$



16. Center: $(-1, 4)$

Foci: $a^2 + b^2 = c^2$

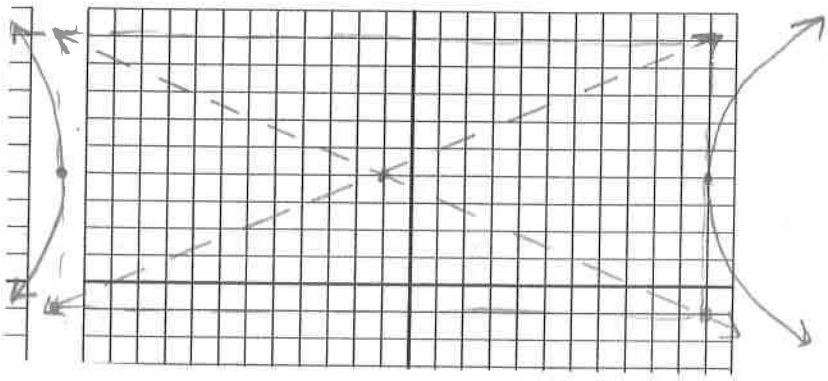
$144 + 25 = c^2$

$c^2 = 169$

$c = \pm 13$

Foci @ $(12, 4)$, $(-14, 4)$

Vertices: $(11, 4)$, $(-13, 4)$



18. Center: $(-3, 1)$

Foci: $a^2 + b^2 = c^2$

$\frac{1}{4} + \frac{1}{9} = c^2$

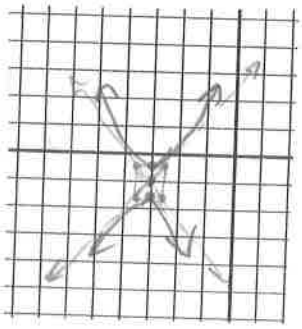
$\frac{13}{36} = c^2$

$c = \pm\frac{\sqrt{13}}{36}$

Foci @ $\left(-3, \frac{36 + \sqrt{13}}{36}\right)$,

$\left(-3, \frac{36 - \sqrt{13}}{36}\right)$

Vertices: $\left(-3, \frac{3}{2}\right)$, $\left(-3, \frac{1}{2}\right)$



20. Center: $(0,2)$

Foci: $a^2+b^2=c^2$

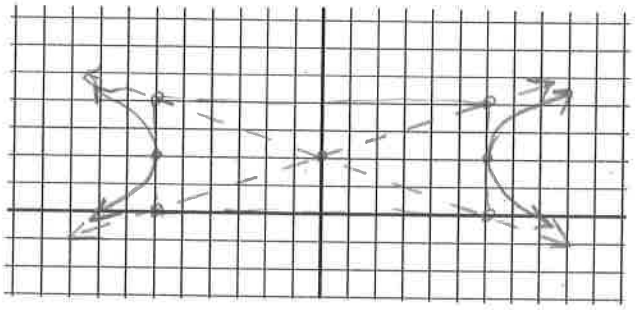
$36+4=c^2$

$c^2=40$

$c=\pm 2\sqrt{10}$

Foci @ $(2\sqrt{10},2)$, $(-2\sqrt{10},2)$

Vertices: $(6,2)$, $(-6,2)$



24. $\frac{y^2}{5}-\frac{x^2}{3}=1$

Center: $(0,0)$

Foci @ $(0,2\sqrt{2})$, $(0,-2\sqrt{2})$

Vertices: $(0,\sqrt{5})$, $(0,-\sqrt{5})$

26. $\frac{(x+3)^2}{\left(\frac{1}{9}\right)}-(y-5)^2=1$

Center: $(-3,5)$

Foci @ $\left(\frac{-9+\sqrt{10}}{3},5\right)$, $\left(\frac{-9-\sqrt{10}}{3},5\right)$

Vertices: $\left(\frac{-10}{3},5\right)$, $\left(\frac{-8}{3},5\right)$

28. $(y-2)^2-\frac{(x-3)^2}{3}=1$

Center: $(3,2)$

Foci @: $(3,0)$, $(3,4)$

Vertices: $(3,1)$, $(3,3)$

30. $\frac{x^2}{9}-\frac{y^2}{16}=1$

32. $\frac{y^2}{9}-x^2=1$

34. $\frac{x^2}{64}-\frac{y^2}{36}=1$

36. $\frac{y^2}{9}-\frac{(x-2)^2}{16}=1$

38. $2a=2$ $\quad$ $2c=3$

$a=1$ $\quad$ $c=\frac{3}{2}$ $\quad$ so, $b^2=\frac{5}{4}$

$a^2=1$ $\quad$ $c^2=\frac{9}{4}$

$$\boxed{\left(y-\frac{3}{2}\right)^2-\frac{(x+3)^2}{\left(\frac{5}{4}\right)}=1}$$

Parabola Stuff

$$(x-h)^2 = 4p(y-k)$$

2. Vertex: $(0,0)$
   Focus: $(0, \frac{1}{2})$
   Directrix: $y = -\frac{1}{2}$

4. Vertex $(0,0)$
   Focus $(0,-3)$
   Directrix: $y = 3$

6. Vertex: $(-2,-5)$
   Focus: $(-2,-4)$
   Directrix: $y = -6$

8. Vertex: $(4,1)$
   Focus: $(4, \frac{11}{12})$
   Directrix: $y = \frac{13}{12}$

10. $x^2 = -3y$

12. $x^2 = \frac{1}{8}y$

14. $(x+4)^2 = \frac{1}{2}(y - \frac{9}{2})$

16. $(x-2)^2 = -2(y+1)$

18. $(x-5)^2 = \frac{1}{8}(y-2)$

20. $y = (x+2)^2 - 11$
    Vertex: $(-2,-11)$
    Focus: $(-2, -\frac{43}{4})$
    Directrix: $y = -\frac{45}{4}$

22. $y = 3(x+1)^2 - 10$
    Vertex: $(-1,-10)$
    Focus: $(-1, -\frac{119}{12})$
    Directrix: $y = \frac{-121}{12}$

24. $y = -3(x+3)^2 + 34$
    Vertex: $(-3,-34)$
    Focus: $(-3, \frac{-409}{12})$
    Directrix: $y = \frac{-407}{12}$

26. $y = -2(x-5)^2 + 50$
    Vertex: $(5, 50)$
    Focus: $(5, \frac{399}{8})$
    Directrix: $y = \frac{401}{8}$

28. Vertex: $(3, -16)$
Focus: $(3, -\frac{63}{4})$
Directrix: $y = -\frac{65}{4}$
$\boxed{\text{UP}}$

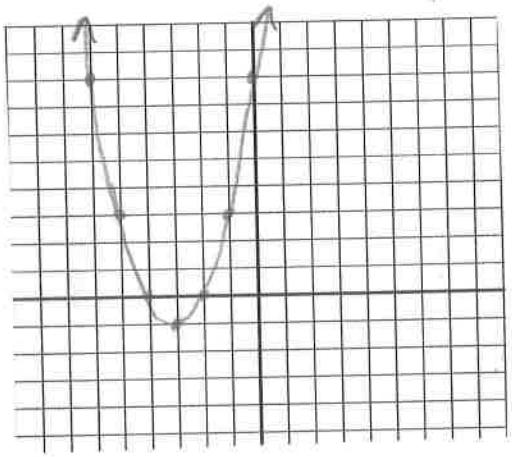
30. Vertex: $(2, 13)$
Focus: $(2, \frac{51}{4})$
Directrix: $y = \frac{53}{4}$
$\boxed{\text{DOWN}}$

32. Vertex: $(-1, -5)$
Focus: $(-1, -\frac{39}{8})$
Directrix: $y = -\frac{41}{8}$
$\boxed{\text{UP}}$

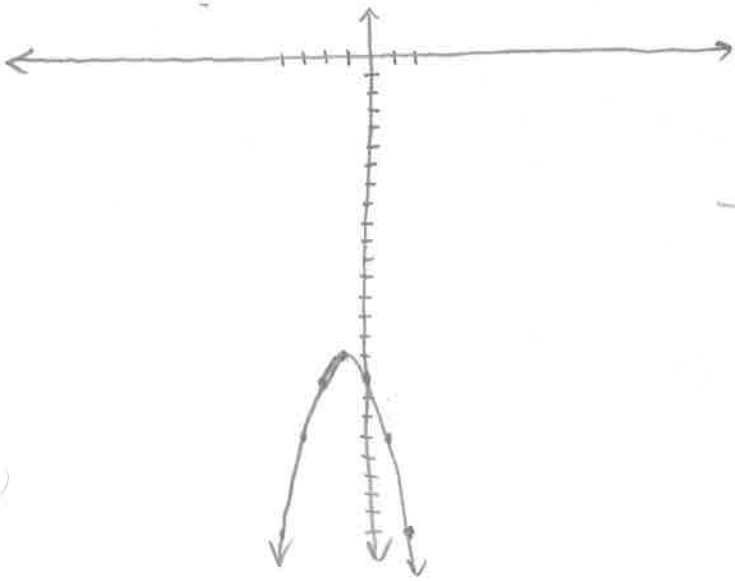
34. Vertex: $(\frac{3}{2}, \frac{5}{4})$
Focus: $(\frac{3}{2}, 1)$
Directrix: $(\frac{3}{2}, \frac{3}{2})$
$\boxed{\text{DOWN}}$

36. Vertex: $(0, -6)$
Focus: $(0, -\frac{49}{8})$
Directrix: $y = -\frac{47}{8}$
$\boxed{\text{DOWN}}$

38. Vertex: $(-3, -1)$
Axis of Sym.: $x = -3$
x-ints.: $x = -2, -4$
y-int.: $y = 8$



40. Vertex: $(-1, -16)$
Axis of Sym.: $x = -1$
x-ints.: $x = \emptyset$ (none)
y-int.: $y = -17$

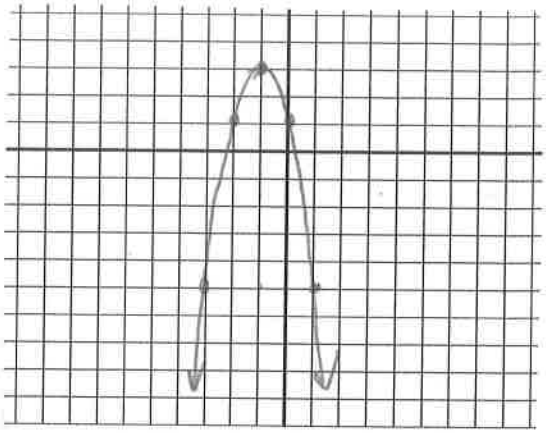


42. Vertex: $(-1, 3)$

Axis of Sym.: $x = -1$

x-ints.: $x = \boxed{\frac{5}{4}, \frac{-13}{4}}$

y-int.: $y = \boxed{1}$

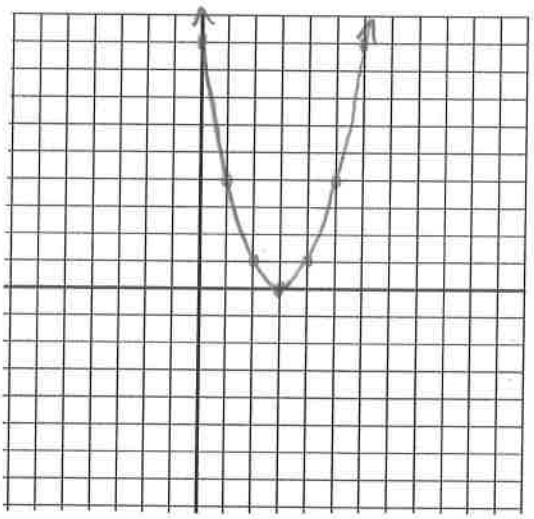


44. Vertex: $(3, 0)$

Axis of Sym.: $x = 3$

x-int.: $x = \boxed{3}$

y-int.: $y = \boxed{9}$

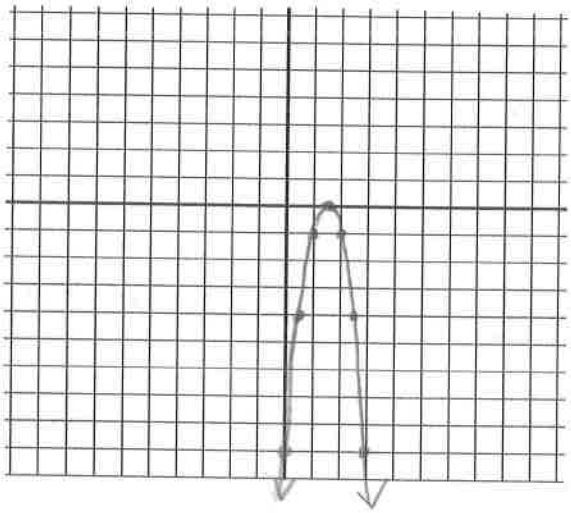


46. Vertex: $(\frac{3}{2}, 0)$

Axis of Sym.: $x = \frac{3}{2}$

x-int.: $x = \boxed{\frac{3}{2}}$

y-int.: $y = \boxed{-9}$

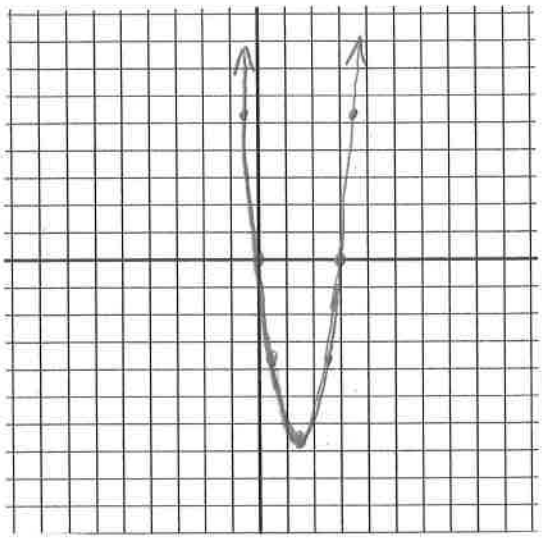


48. Vertex: $\left(\frac{3}{2}, \frac{-27}{4}\right)$

Axis of Sym.: $x = \frac{3}{2}$

x-ints.: $x = \boxed{0, 3}$

y-int.: $y = \boxed{0}$

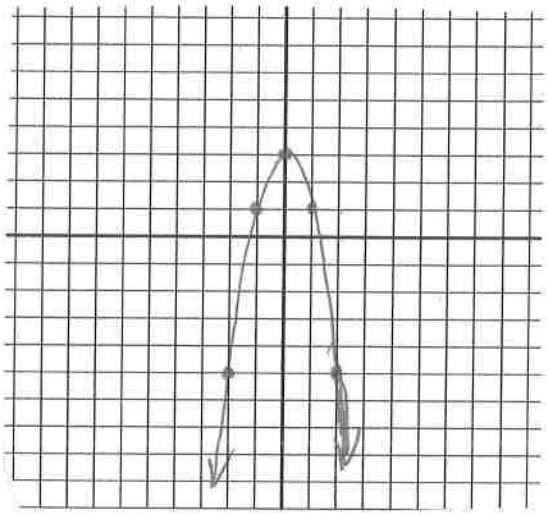


50. Vertex: $(0, 3)$

Axis of Sym.: $x = 0$

x-ints.: $x = \boxed{\frac{\pm\sqrt{6}}{2}}$

y-int.: $y = \boxed{3}$



52. Vertex: $(2, -3)$

Axis of Sym.: $x = 2$

x-ints.: $x = \boxed{2+\sqrt{3},\ 2-\sqrt{3}}$

y-int.: $y = \boxed{1}$