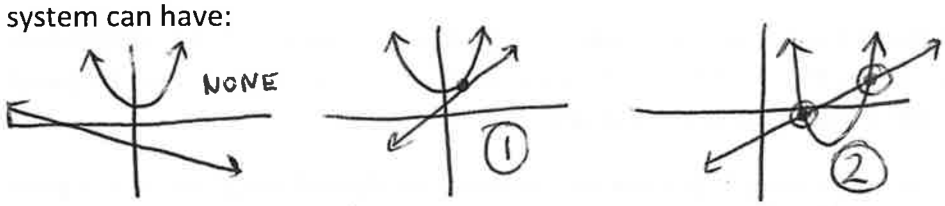
4.1 – Solving Systems of Equations Graphically

Linear-Quadratic

A Linear-Quadratic System of Equations is a linear equation and a quadratic equation involving the same two variables. The solution(s) to this system are the point(s) where the line intersects the parabola (if it does at all).

Draw pictures to represent the possible number of solutions that a linear-quadratic system can have:



Example 1 – Solve the following system of equations graphically:

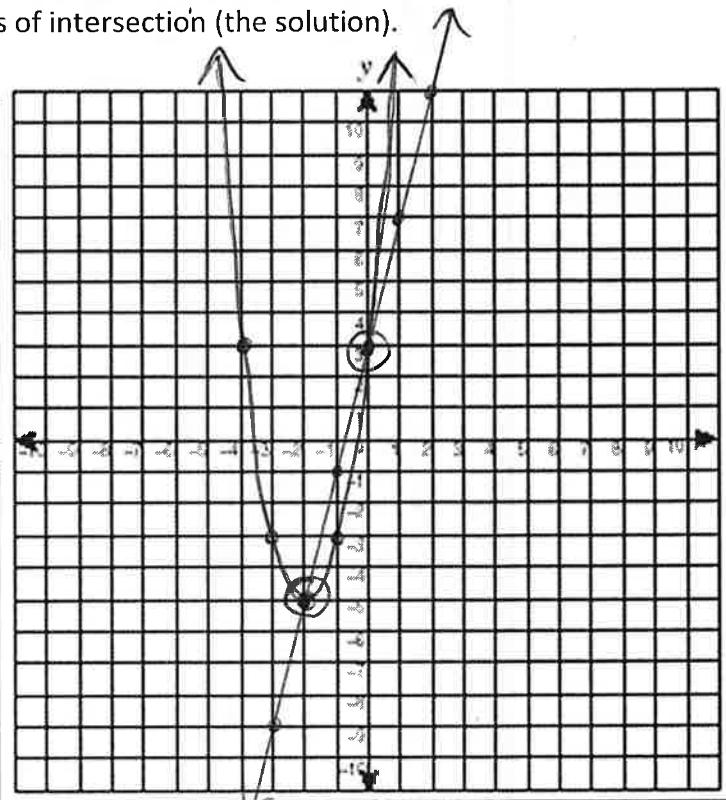
1) $4x - y + 3 = 0$
2) $2x^2 + 8x - y + 3 = 0$

a) Get the linear equation into $y = mx \pm b$ form and graph.
b) Complete the square and graph the quadratic equation.
c) Identify and write down the points of intersection (the solution).
d) Verify the solution by checks.

a) $4x - y + 3 = 0$
$+y \qquad +y$
$y = 4x + 3$
$b = 3$
$m = \frac{4}{1}$ ← up, ← right

b) $2x^2 + 8x - y + 3 = 0$
$+y \qquad +y$
$y = 2x^2 + 8x + 3$
$y = 2(x^2 + 4x) + 3$ $\qquad b = 4$, $\frac{4}{2}$, 4
$y = 2(x^2 + 4x + 4 - 4) + 3$
$y = 2(x^2 + 4x + 4) - 8 + 3$
$y = 2(x+2)^2 - 5$
Vertex $(-2, -5)$
$a = 2$

c) $(-2, -5)$ & $(0, 3)$



d)

$(-2,-5)$	$(0,3)$
$4(-2) - (-5) + 3 = 0$ $-8 + 5 + 3 = 0$ ✓	$4(0) - 3 + 3 = 0$ $0 - 3 + 3 = 0$ ✓
$2(-2)^2 + 8(-2) - (-5) + 3 = 0$ $8 - 16 + 5 + 3 = 0$ ✓	$2(0)^2 + 8(0) - 3 + 3 = 0$ $0 + 0 - 3 + 3 = 0$ ✓

Example 2 – Is (5, 7) a solution to the system 1) $3x^2 - 10y = 5$ and 2) $-y = x - 11$?

Check!

① $3(5)^2 - 10(7) = 5$
$75 - 70 = 5$
✓

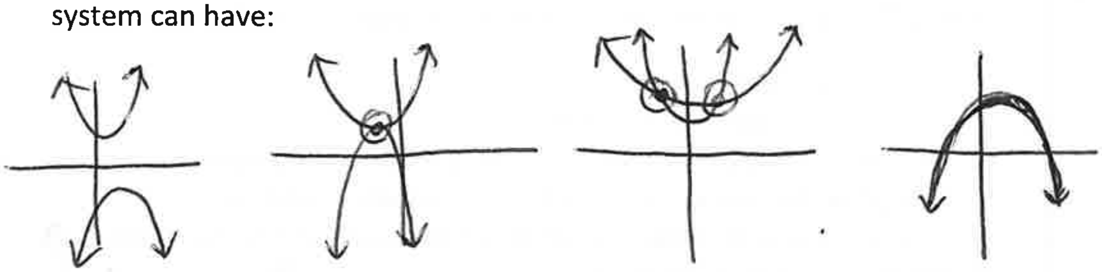
② $-7 = 5 - 11$
$-7 = -6$
✗

No, (5,7) is not a solution to the system

Quadratic-Quadratic

A Quadratic-Quadratic System of Equations is two quadratic equations involving the same variables. The solution(s) to this system are the point(s) where the parabola intersects the other parabola (if it does at all).

Draw pictures to represent the possible number of solutions that a quadratic-quadratic system can have:



NONE ONE TWO INFINITE

Quadratic-Quadratic

Example 3 – Solve 1) $2x^2 - 8x + 7 - y = 0$ and 2) $y + x^2 - 4x + 2 = 0$

① $y = 2x^2 - 8x + 7$

$y = 2(x^2 - 4x) + 7$ $b = -4$, -2, 4

$y = 2(x^2 - 4x + 4 - 4) + 7$

$y = 2(x^2 - 4x + 4) - 8 + 7$

$y = 2(x-2)^2 - 1$

vertex $(2, -1)$

$a = 2$

② $y = -x^2 + 4x - 2$

$y = -1(x^2 - 4x) - 2$ $b = -4$, -2, 4

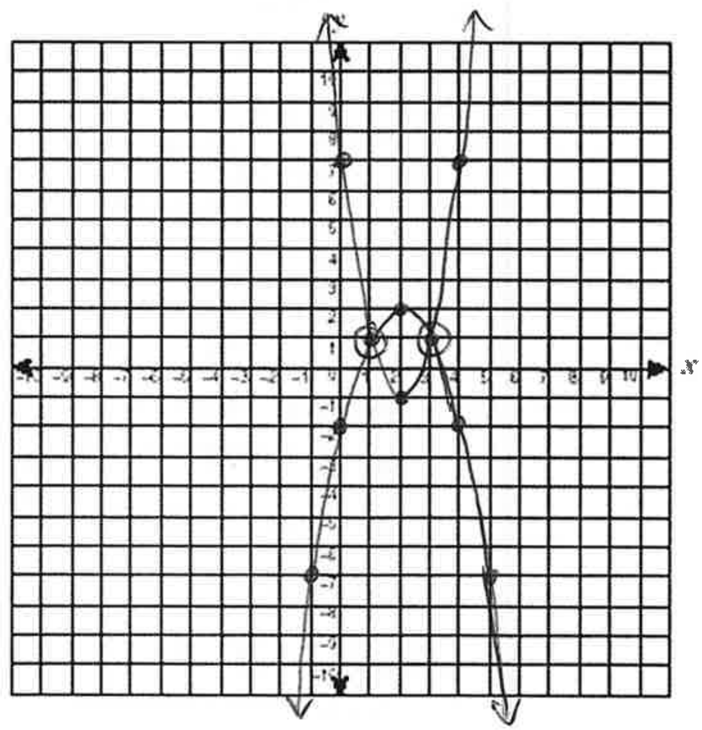
$y = -(x^2 - 4x + 4 - 4) - 2$

$y = -(x^2 - 4x + 4) + 4 - 2$

$y = -(x-2)^2 + 2$

vertex $(2, 2)$

$a = -1$



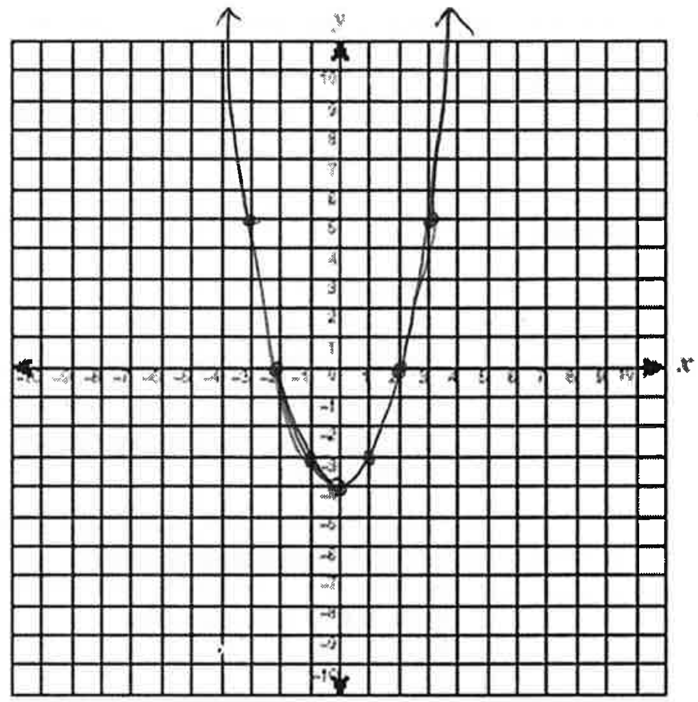
Solutions: (1,1) & (3,1)

Example 4 – Solve the system $y - x^2 + 4 = 0$ and $-2y + 2x^2 - 8 = 0$

① $y = x^2 - 4$

vertex $(0, -4)$

$a = 1$



② $-2y + 2x^2 - 8 = 0$

$\frac{2x^2}{2} - \frac{8}{2} = \frac{2y}{2}$

$y = x^2 - 4$

same as eqn ①

Therefore,

INFINITE SOLUTIONS!

4.2 – Solving Systems of Equations Algebraically

Linear-Quadratic

For a Linear-Quadratic System of Equations, what are all the possible # of solutions?

0, 1, 2

Solutions can be found graphically, as in Section 4.1, or algebraically, using either substitution or elimination.

substitution

Example 1 – Solve the following linear-quadratic system using **substitution**:

1) $3x + y = -9$
2) $4x^2 - x + y = -9$

a) Solve the linear equation for y.
b) Substitute the linear equation for y in the quadratic equation.
c) Solve the quadratic equation by factoring (if you cannot factor, use the quadratic formula).
d) Substitute the resulting x value(s) into the original linear equation to determine the corresponding y values.

a) $y = -3x - 9$

b &c) $y = -4x^2 + x - 9$

$-3x - 9 = -4x^2 + x - 9$

$4x^2 - 4x = 0$

$4x(x-1) = 0$

$x = 0, 1$

d) $3x + y = -9$

$x = 0$

$3(0) + y = -9$

$y = -9$

$(0, -9)$

$x = 1$

$3(1) + y = -9$

$y = -12$

$(1, -12)$

Example 2 – Solve by substitution: 1) $5x - y = 10$ and 2) $x^2 + x - 2y = 0$

① $y = 5x - 10$

② $x^2 + x - 2y = 0$

$x^2 + x - 2(5x - 10) = 0$

$x^2 + x - 10x + 20 = 0$

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$x = 5, 4$

$x = 5$

① $5x - y = 10$

$5(5) - y = 10$

$y = 15$

$(5, 15)$

$x = 4$

② $5x - y = 10$

$5(4) - y = 10$

$10 = y$

$(4, 10)$

elimination

Now, solve the same system using **elimination**:

1) $5x - y = 10$
2) $x^2 + x - 2y = 0$

a) Align the terms with the same degree. Since the squared term is the variable x, eliminate the y-term.
b) Multiply one or more of the equations if necessary to have the same coefficient for y.
c) Add or subtract the two equations to eliminate y.
d) Solve the resulting quadratic equation by factoring or the quadratic formula to find the x coordinates of the solution(s).
e) Substitute the resulting x value(s) into the original linear equation to determine the corresponding y values.

② $x^2 + x - 2y = 0$
① $(5x - y = 10) \times 2$

② $x^2 + x - 2y = 0$
① $-\ (10x - 2y = 20)$

$x^2 - 9x = -20$

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$x = 5, 4$

① $5x - y = 10$
$5(5) - y = 10$
$y = 15$
$(5, 15)$

① $5x - y = 10$
$5(4) - y = 10$
$y = 10$
$(4, 10)$

Quadratic-Quadratic

For a Quadratic-Quadratic Systems of Equations, what are all the possible # of solutions?

0, 1, 2, infinite

Example 3 – Solve the following system first by substitution, then by elimination.

1) $6x^2 - x - y = -1$
2) $4x^2 - 4x - y = -6$

Substitution:

① $y = 6x^2 - x + 1$

② $4x^2 - 4x - y = -6$

$4x^2 - 4x - (6x^2 - x + 1) = -6$

$4x^2 - 4x - 6x^2 + x - 1 = -6$

$-2x^2 - 3x + 5 = 0$

$2x^2 + 3x - 5 = 0$

$2x^2 - 2x + 5x - 5 = 0$

$2x(x-1) + 5(x-1) = 0$

$(x-1)(2x+5) = 0$

$x = 1, -\frac{5}{2}$

① $6(1)^2 - 1 - y = -1$
$6 - 1 - y = -1$
$5 - y = -1$
$y = 6$
$(1, 6)$

① $6\left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right) - y = -1$
$6\left(\frac{25}{4}\right) + \frac{5}{2} + 1 = y$
$\frac{75}{2} + \frac{5}{2} + \frac{2}{2} = y$
$= \frac{82}{2} = 41$

$\left(-\frac{5}{2}, 41\right)$

Elimination:

$6x^2 - x - y = -1$
$-\ (4x^2 - 4x - y = -6)$

$2x^2 + 3x = 5$

$2x^2 + 3x - 5 = 0$

etc...

Example 4 – A Canadian cargo plane drops a crate of emergency supplies to aid-workers on the ground. The crate drops freely at first before a parachute opens to bring the crate gently to the ground. The crate's height, h, in metres, above the ground t seconds after leaving the aircraft is given by the following two equations. $h = -4.9t^2 + 900$ represents the height of the crate during freefall. $h = -4t + 500$ represents the height of the crate with the parachute open.

a) How long after the crate leaves the aircraft does the parachute open? Express your answer to the nearest hundredth of a second.
b) What height above the ground is the crate when the parachute opens? Express your answer to the nearest metre.
c) Verify your solution.

The answers to a and b are the intersection of the two equations.

① $h = -4.9t^2 + 900$

② $h = -4t + 500$

$-4.9t^2 + 900 = -4t + 500$

$4.9t^2 - 4t - 400 = 0$

$a = 4.9,\ b = -4,\ c = -400$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{4 \pm \sqrt{(-4)^2 - 4(4.9)(-400)}}{2(4.9)}$$

$$t = \frac{4 \pm \sqrt{16 + 7840}}{9.8}$$

$$t = \frac{4 \pm 88.634}{9.8}$$

$t = 9.4525,\ \cancel{-8.636}$ reject

$h = -4t + 500$

$h = -4(9.4525) + 500$

$h = 462$

ⓐ The parachute opens 9.45s after leaving the aircraft

ⓑ The crate will be 462m off the ground

ⓒ (9.4525, 462.19)

check:

① $h = -4.9t^2 + 900$

$462.19 = -4.9(9.4525)^2 + 900$

$462.19 = -437.81 + 900$ ✓

② $h = -4t + 500$

$462.19 = -4(9.4525) + 500$

$462.19 = -37.81 + 500$ ✓

4.3A – Linear Inequalities in Two Variables

Warmup

How do we read these inequalities (from left to right)? $5 > 2$ $-3 < -1$

5 is greater than 2 -3 is less than -1

What does each symbol mean? $>$ $<$ $\geq$ $\leq$

greater than less than greater than or equal to less than or equal to

How do you say this aloud? $x \geq 4$

x is greater than or equal to 4

What are some possible answers?

4, 5, 6, 28, etc..

What is the primary difference between an **equation** and an **inequality**?

An equation has one (or two) solutions whereas an inequality has a range or solutions.

Example 1 - Solve the following inequality: $3x - 7 < -5$

$+7 \quad +7$

$3x < 2$

$\frac{\cancel{3}x}{\cancel{3}} < \frac{2}{3}$

$x < \frac{2}{3}$

Example 2 – What are some possible answers to $-2x < 6$?

-2, -1, 0, 1, 2, 3, 4, etc..

$\frac{-2x}{-2} < \frac{6}{-2}$

Flip!

$x > -3$

When multiplying or dividing both sides by a negative, flip the inequality sign

How is solving an inequality like solving an equation? How is it different?

Solve an inequality just like you would solve an equation except if mult or dividing both sides by a negative number, flip the sign.

Find some solutions to $3y - 2x \geq 6$

(2, 10) (-2, 1) and so on...

(7, 8) (-10, 0)

There is a more efficient way to find the range of solutions for the inequality above.

steps

1. Rearrange the inequality so it's in $mx \pm b$ form. Don't forget to flip the inequality if you multiply or divide by a negative number.

2. Decide whether to use a solid line or dotted line:

- If the inequality is ≤ or ≥, points on the line are included in the solution (due to the 'equals to' line under the sign), so we keep the line solid.
- If the inequality is < or >, points on the line are not included in the inequality, so we draw a dotted line.

3. Graph the line using slope and *y*-intercept. The line is called the **boundary**.

4. For y > mx + b or y ≥ mx + b, solutions to the inequality are all of the points **above** the line, so shade above. For y < mx + b or y ≤ mx + b, shade **below** the line. The shading represents the **solution region**: all of the points that satisfy the inequality.

5. CHECK: Pick a **test point** in the shaded region and test its coordinates in the inequality. If it satisfies the inequality, you've been successful. If it doesn't satisfy the inequality, you either shaded the incorrect region, or the boundary line has been graphed incorrectly.

Example 1 – Solve the inequality by graphing $3y - 2x \geq 6$.

$$3y - 2x \geq 6$$
$$\quad + 2x \qquad + 2x$$

$$\frac{3y}{3} \geq \frac{2x}{3} + \frac{6}{3}$$

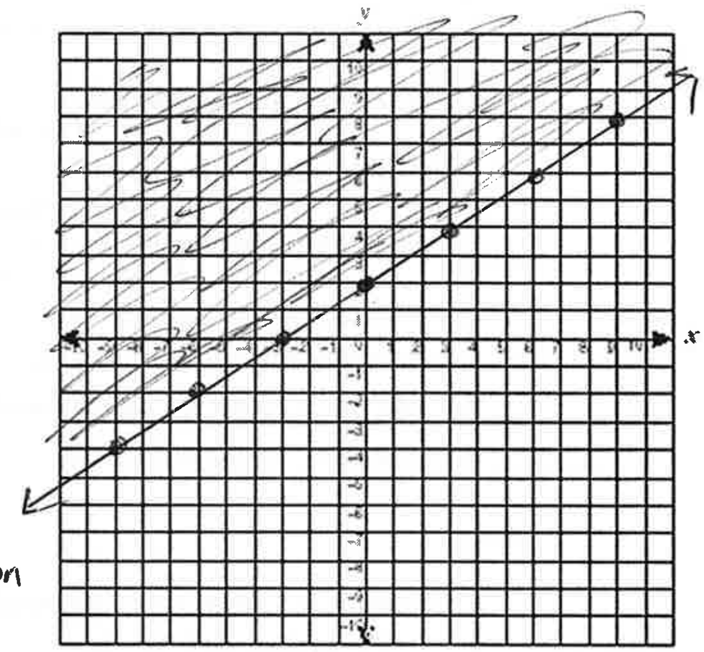
$$y \geq \frac{2}{3}x + 2$$

solid line

$$y \geq \frac{2}{3}x + 2$$

greater than, so shade ABOVE

All coordinates in the shaded region are solutions to the inequality



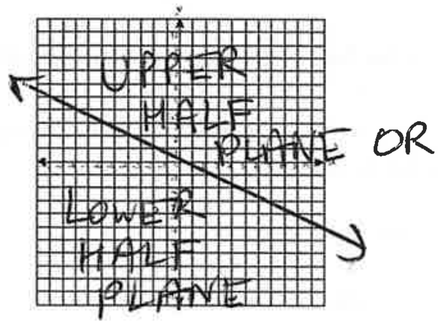
CHECK: Pick a point in the shaded region and see if inequality is satisfied

ex $(0,5)$: $3y - 2x \geq 6$

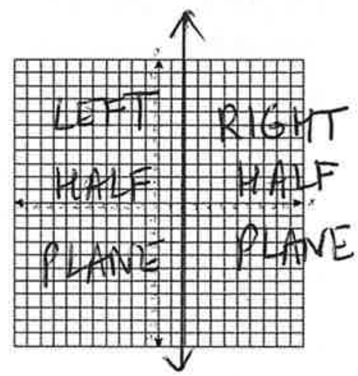
$3(5) - 2(0) \geq 6$

$15 \geq 6$ ✓

The graph of a linear equation is a line. The graph of a linear inequality is a half-plane with a boundary that is a straight line.

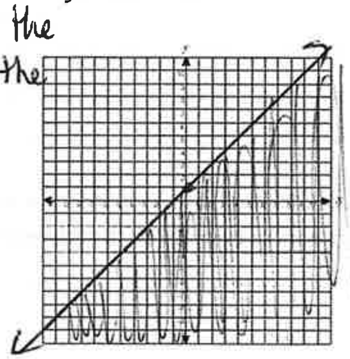


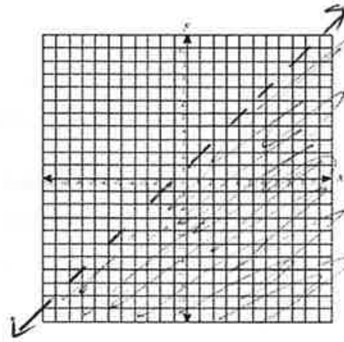
OR



The boundary line may or may not be part of the solution. How are each of these expressed? $\quad y \leq x + 1 \qquad\qquad y < x + 1$

Any point below the line, or on the line, are solutions





Only the points below the line are solutions

***When you read an inequality for shading purposes, it must be in *y = mx + b* form!**

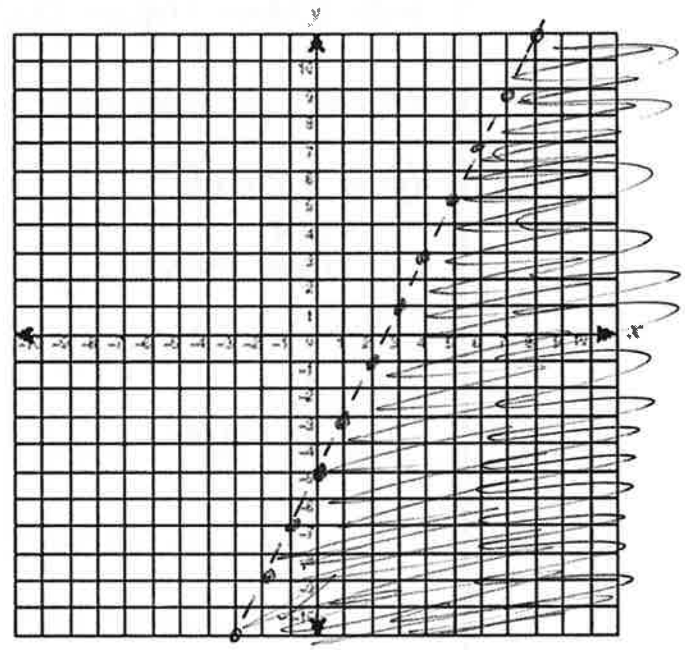
Example 2 – Solve $4x - 2y > 10$. Determine if (1, 3) is part of the solution.

$4x - 2y > 10$
$-4x \quad\quad -4x$

$\frac{-2y}{-2} > \frac{-4x}{-2} + \frac{10}{-2}$ flip!

$y < 2x - 5$

- dashed
- shade below



(1,3) not part of shaded region, so not part of solution

$4x - 2y > 10$
$4(1) - 2(3) > 10$
$4 - 6 > 10$
$-2 > 10$
not true

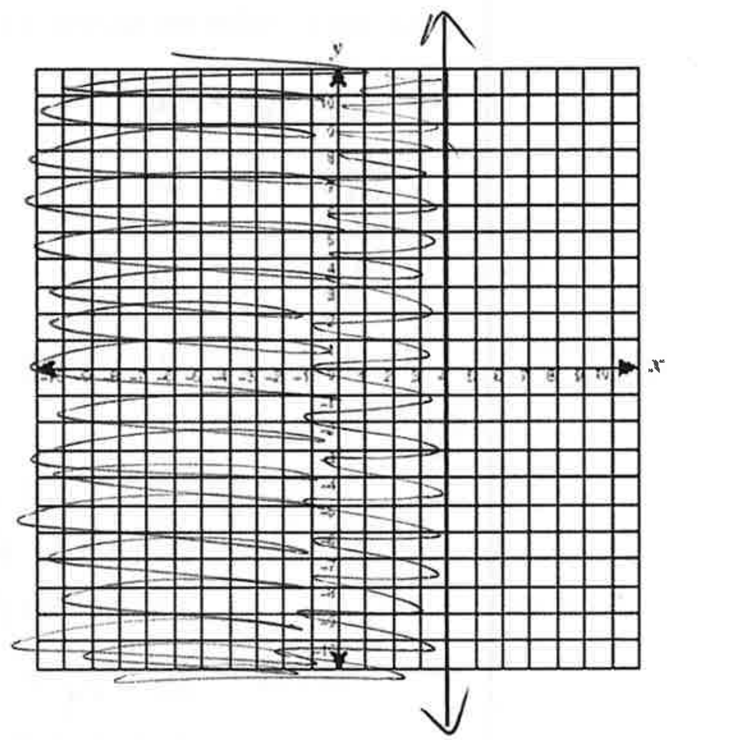
Example 3 – Solve $x \leq 4$

$x \leq 4$

vertical line where $x = 4$

$x \leq 4$

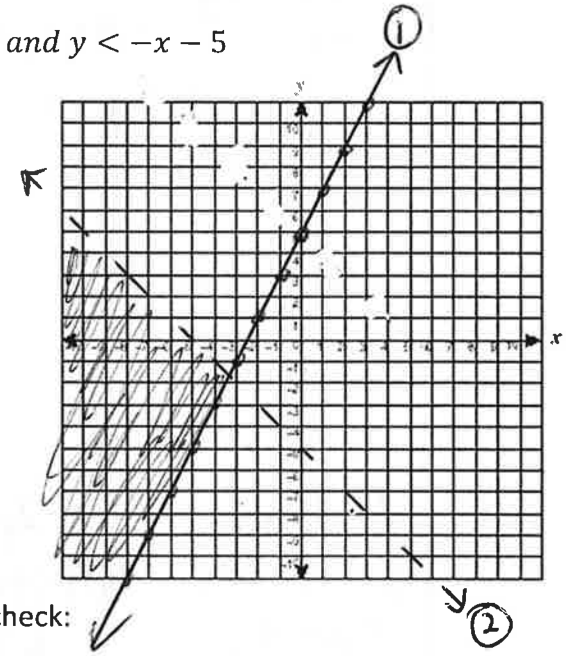
- solid
- less than, so shade LEFT



4.3B – Systems of Linear Inequalities in Two Variables

A **system** of linear inequalities is: Multiple boundary lines graphed together with a shaded region that satisfies <u>all</u> inequalities

Example 1 – Solve the system: $y \geq 2x + 5$ *and* $y < -x - 5$



① $y \geq 2x+5$
- solid
- shade ABOVE

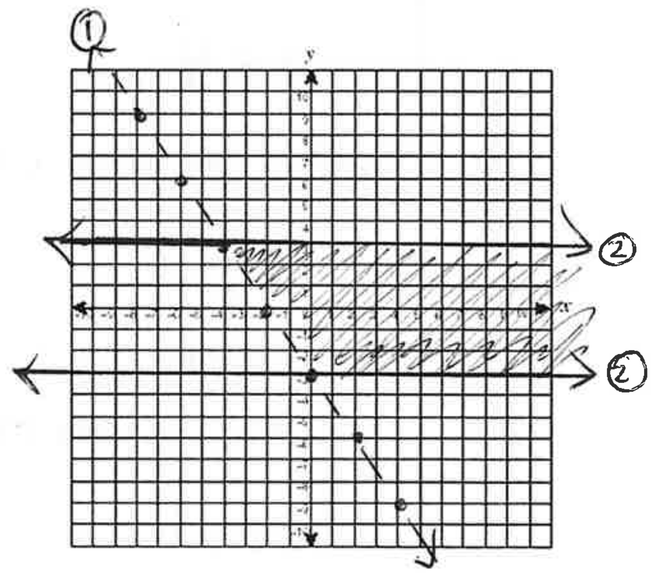
② $y < -x-5$
- dashed
- shade BELOW

Pick one possible solution and perform a check:

$(-9,0)$: ① $0 \geq 2(-9)+5$
$0 \geq -13$ ✓

② $y < -x-5$
$0 < -(-9)-5$
$0 < 4$ ✓

Example 2 – Solve the system: $3x + 2y > -6$ *and* $-3 \leq y \leq 3$



① $3x+2y > -6$

$2y > -3x-6$

$y > -\frac{3}{2}x - 3$

dashed ; shade ABOVE

② $-3 \leq y \leq 3$

y is between $y=-3$ and $y=3$
horiz line horiz line

Shade BETWEEN

Check: $(0,0)$: $3x+2y > -6$
$3(0)+2(0) > -6$
$0 > -6$ ✓

$-3 \leq y \leq 3$
$-3 \leq 0 \leq 3$ ✓

STEPS: 1. Rearrange each inequality into ***mx + b*** form.

2. Graph each line, using dashed (>, <) or solid (≥, ≤) lines.

3. To find the solution region (shaded region), look to see whether to shade above or below the first line, then above or below the second line (read the inequality in ***mx + b*** form).

4. Check your solution by picking a point in your solution and testing it in each of the two original inequalities. It must satisfy both inequalities. If it doesn't, an error was made at some point, so try to find out what it is, or redo the question.

Example 3 – Solve the system of linear inequalities

$x \geq -3,\ \ y > -4,\ \ y > 2x - 4,\ \ x + 6y \leq 15$

① $x \geq -3$

-solid

-vert line at $x=3$

-shade RIGHT

② $y > -4$

-dashed

-horiz line at $y=-4$

-shade ABOVE

③ $y > 2x - 4$

-dashed

- shade ABOVE

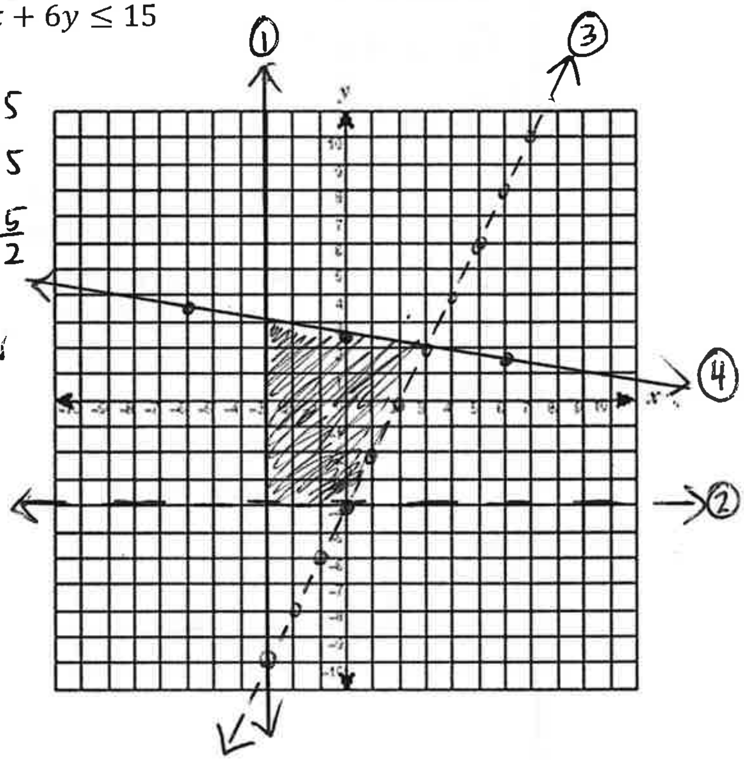
④ $x + 6y \leq 15$

$6y \leq -x + 15$

$y \leq -\frac{1}{6}x + \frac{5}{2}$

-solid

-shade BELOW



Example 4 – Write the system of inequalities for the following solution set.

① -horiz line at $y=3$

-solid line

-shaded below

so $y \leq 3$

② y-int $=0$

slope $= -1$

-solid line

- shaded above

$y \geq -x$

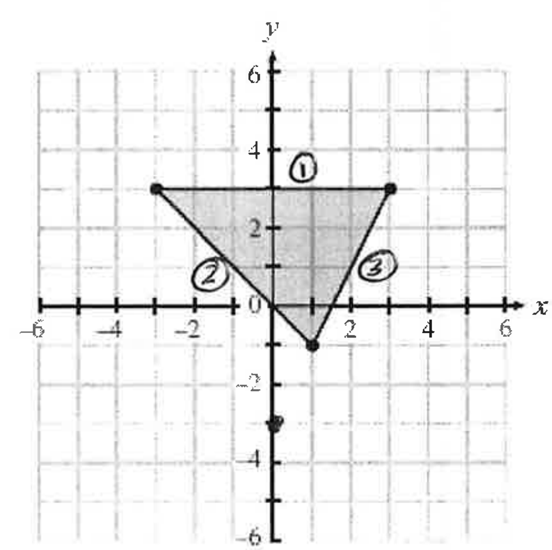
③ y-int $= -3$

slope $= \frac{2}{1} = 2$

-solid line

-shaded ABOVE

$y \geq 2x - 3$



word problem

Example 5 –The Canucks have 8 games left to play and need 10 points to make the playoffs. A win is worth 2 points and an overtime loss is worth 1 point. Write and graph a system of linear inequalities to see all the possible ways the Canucks can make the playoffs.

Let x = # of wins

Let y = # of overtime losses

inequalities:

① $x + y \leq 8$

② $2x + y \geq 10$

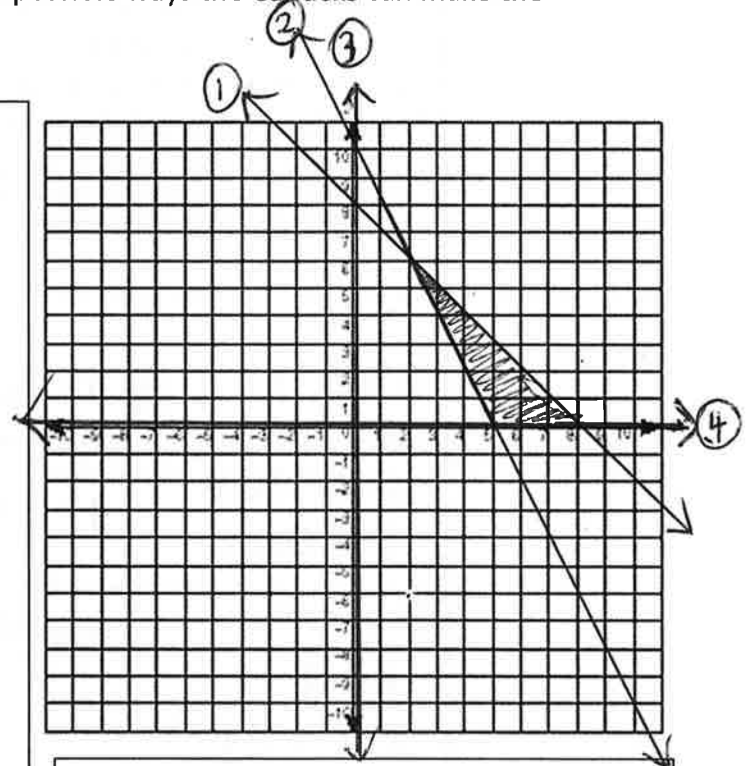
③ $x \geq 0$ vertical line, solid, shade RIGHT

④ $y \geq 0$ horiz line, solid, shade ABOVE

rearranged:

① $y \leq -x + 8$ solid, shade BELOW

② $y \geq -2x + 10$ solid, shade ABOVE



Ways to make the playoffs:

(2,6) ⇒ 2 wins, 6 OT losses = 10 pts

(4,3) ⇒ 4 wins, 3 OT losses = 11 pts.

all answers possible:

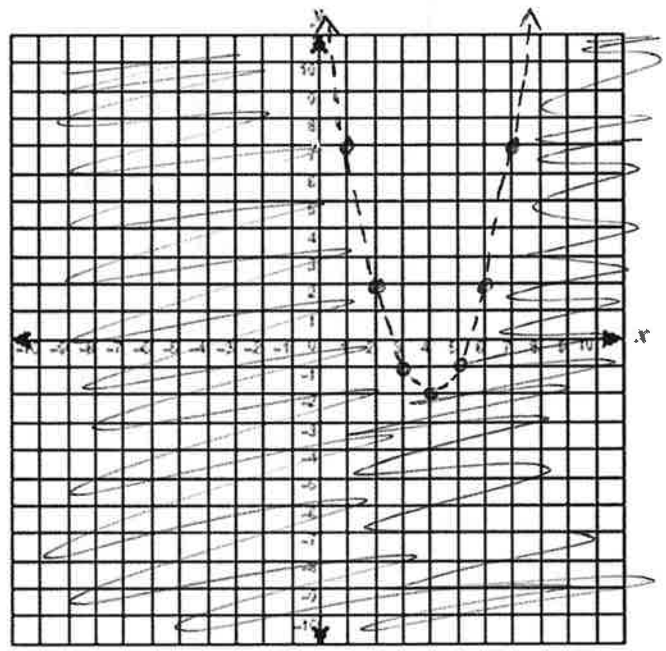
(2,6) (3,4) (3,5) (4,2) (4,3) (4,4)
(5,0) (5,1) (5,2) (5,3) (6,0) (6,1)
(6,2) (7,0) (7,1) (8,0)

Example 1 – Solve the inequality by graphing $y + 2 < (x-4)^2$.

steps

1. Rearrange the inequality so y is all by itself on one side.
2. Decide whether to use a solid curve or dotted curve:
3. Graph the parabola using ***standard form***. The line is called the **boundary**.
4. For y > $ax^2 \pm bx \pm c$ or y ≥ $ax^2 \pm bx \pm c$, solutions to the inequality are all of the points **above** the parabola, so shade above. For y < $ax^2 \pm bx \pm c$ or y ≤ $ax^2 \pm bx \pm c$, shade **below** the parabola. The shading represents the **solution region**: all of the points that satisfy the inequality.
5. CHECK: Pick a **test point** in the shaded region and test its coordinates in the inequality. If it satisfies the inequality, you've been successful. If it doesn't satisfy the inequality, you either shaded the incorrect region, or the boundary curve has been graphed incorrectly.

1)	$y < (x-4)^2 - 2$
2)	dotted
3)	$y < (x-4)^2 - 2$ vertex $(4,-2)$ $a = 1$
4)	$y < (x-4)^2 - 2$ less than = shade below
5)	$(0,0)$: $0 < (0-4)^2 - 2$ $0 < 16 - 2$ $0 < 14$ ✓



Example 2 – Solve by graphing: $y \leq -x^2 + 2x + 4$. Is (-1, 1) a solution? Is (2, 5)?

$y \leq (-x^2 + 2x) + 4$

$y \leq -(x^2 - 2x) + 4$ $b = -2, -1, 1$

$y \leq -(x^2 - 2x + 1 - 1) + 4$

$y \leq -(x^2 - 2x + 1) + 1 + 4$

$y \leq -(x-1)^2 + 5$

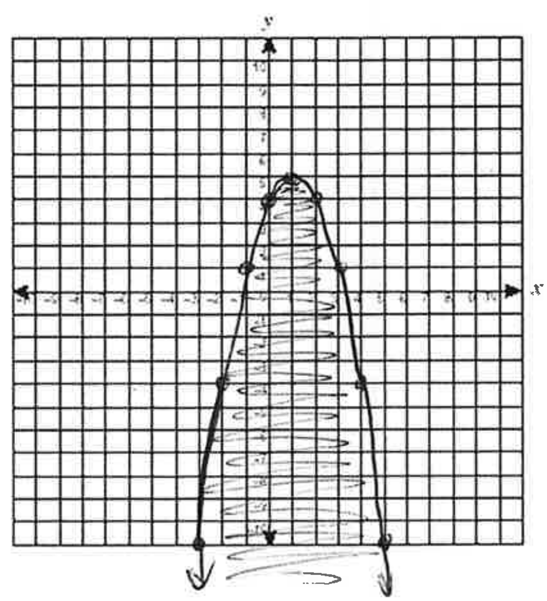
vertex $(1,5)$

$a = -1$

- solid line
- shade below

on graph:

- $(-1,1)$ is a solution
- $(2,5)$ is NOT



Example 2 – Solve by graphing: $y + 5 \geq x^2 - 4x$ and $y < \frac{1}{3}(x-2)^2$

① $y \geq x^2 - 4x - 5$ $-4, -2, 4$

$y \geq x^2 - 4x + 4 - 4 - 5$

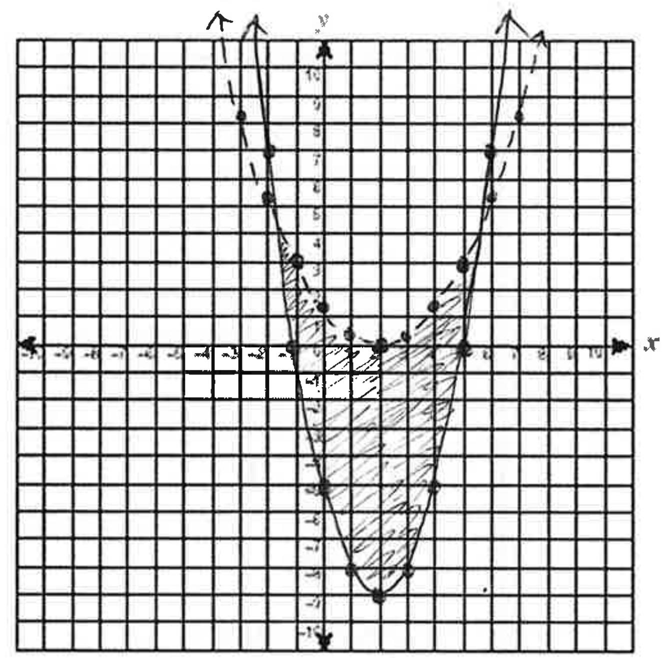
$y \geq (x-2)^2 - 9$

vertex $(2, -9)$

$a = 1$

solid line

shade ABOVE



② $y < \frac{1}{3}(x-2)^2$

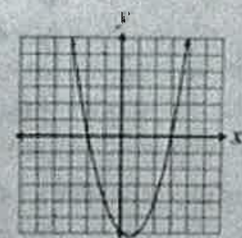
vertex $(2, 0)$

$a = \frac{1}{3}$

dashed

shade BELOW

For the quadratic inequality $x^2 - x - 6 > 0$, there are two approaches to determine the solution. The first is to consider the quadratic function $f(x) = x^2 - x - 6$ and its graph.



The parabola is above the x-axis when $x < -2$ or $x > 3$, and below the x-axis when $-2 < x < 3$. Therefore the quadratic inequality $x^2 - x - 6 > 0$ has a solution $x < -2$ or $x > 3$ and the quadratic inequality $x^2 - x - 6 < 0$ has a solution $-2 < x < 3$.

Example 1 – Solve $x^2 + 2x > 8$ **by graphing, and then using test intervals**. Graph the solution on a number line.

Graphing Steps

1. Get everything to the left side so that zero is on the right.
2. Find the roots (x-intercepts).
3. Sketch a graph and use the visual to solve the inequality.

→ if the quadratic is > 0, find the domain where the graph is above the x-axis
→ if the quadratic is < 0, find the domain where the graph is below the x-axis

1) $x^2+2x-8>0$

2) $(x+4)(x-2)$

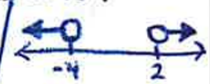
$x=-4,\ x=2$

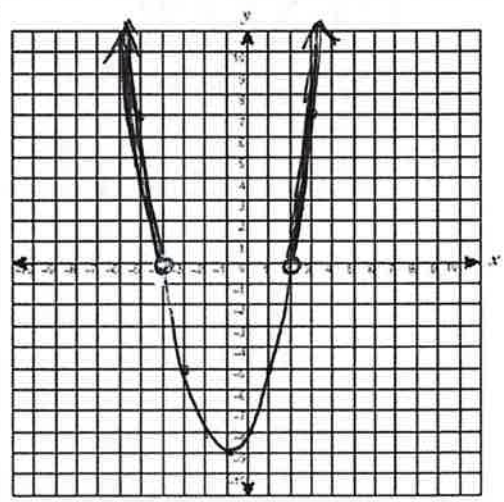
3) x-ints at $-4, 2$ / parabola opens up

$x^2+2x-8>0$

Where is the parabola ABOVE the x-axis?

$x<-4$ and $x>2$





Test Interval Steps

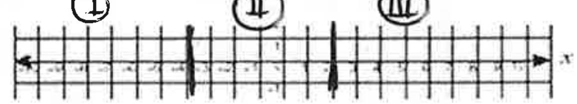
1. Find the critical numbers (the zeros) of the inequality.
2. Make an x-axis diagram of the resulting test intervals.
3. Test a value from each interval using the original inequality.

1) $x^2+2x-8>0$

$(x+4)(x-2)$

crit pts are $-4, 2$

2) I II III



3) I Test -5

$(-5)^2+2(-5)-8>0$

$25-10-8>0$

$7>0$ ✓

II Test 0

$(0)^2+2(0)-8>0$

$-8>0$ ✗

III test 3

$(3)^2+2(3)-8>0$

$9+6-8>0$

$7>0$ ✓

$x<-4$,

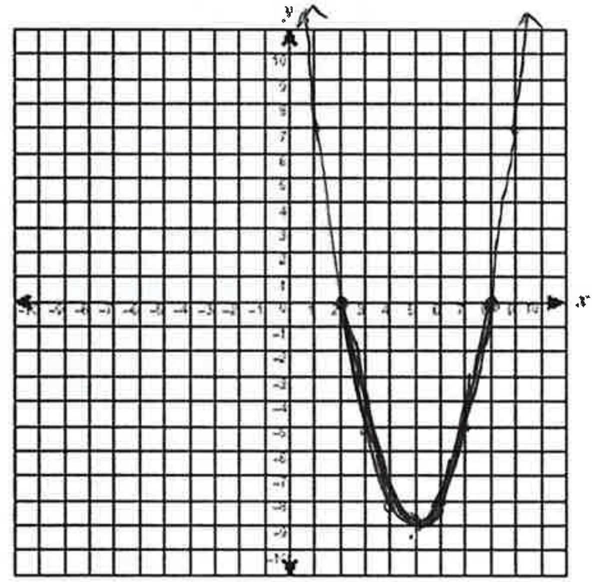
$x>2$

Example 2 – Solve $x^2 - 10x + 16 \leq 0$ using both methods and graph the solution on a number line

*if the quadratic is ≥ 0, find the domain where the graph is **above or on** the x-axis

*if the quadratic is ≤ 0, find the domain where the graph is **below or on** the x-axis

Graphing:



$x^2 - 10x + 16 \leq 0$

$(x-8)(x-2)$

$x = 8, 2$

$x^2 - 10x + 16 \leq 0$

Where is the parabola below the x axis
or
on

a>0 so opens up!

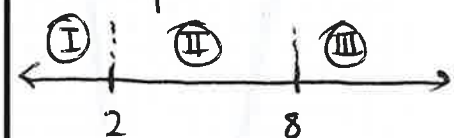
Solution: $2 \leq x \leq 8$

Test Intervals:

$x^2 - 10x + 16 \leq 0$

$(x-8)(x-2)$

critical pts are 8,2



I Test 0:

$0^2 - 10(0) + 16 \leq 0$

$16 \leq 0$

X

II Test 4:

$4^2 - 10(4) + 16 \leq 0$

$16 - 40 + 16 \leq 0$

$-8 \leq 0$

✓

between 2 and 8

$2 \leq x \leq 8$

III Test 9:

$9^2 - 10(9) + 16 \leq 0$

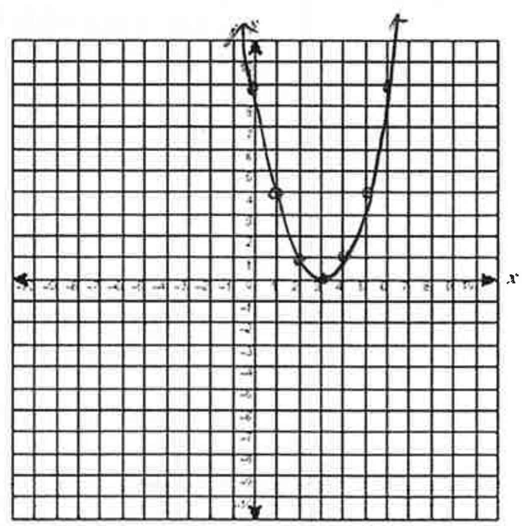
$81 - 90 + 16 \leq 0$

$7 \leq 0$

X

Example 3 – Graph the quadratic function $f(x) = x^2 - 6x + 9$. What is the solution to:

a) $x^2 - 6x + 9 \geq 0$ b) $x^2 - 6x + 9 > 0$ c) $x^2 - 6x + 9 \leq 0$ d) $x^2 - 6x + 9 < 0$



$f(x) = x^2 - 6x + 9$

$0 = (x-3)(x-3)$

$x = 3$

a) $x^2 - 6x + 9 \geq 0$

Above or on

Solution: $x \in \mathbb{R}$

b) $x^2 - 6x + 9 > 0$

Above x axis

- everywhere except 3

Solution: $x \neq 3$

c) $x^2 - 6x + 9 \leq 0$

below or on x axis

only at 3

Solution: $x = 3$

d) $x^2 - 6x + 9 < 0$

below x axis

- no where

Solution: $\varnothing$

Example 4 – Solve $x^2 - 2x > 2$. Then graph the solution on a number line.

$x^2 - 2x - 2 > 0$

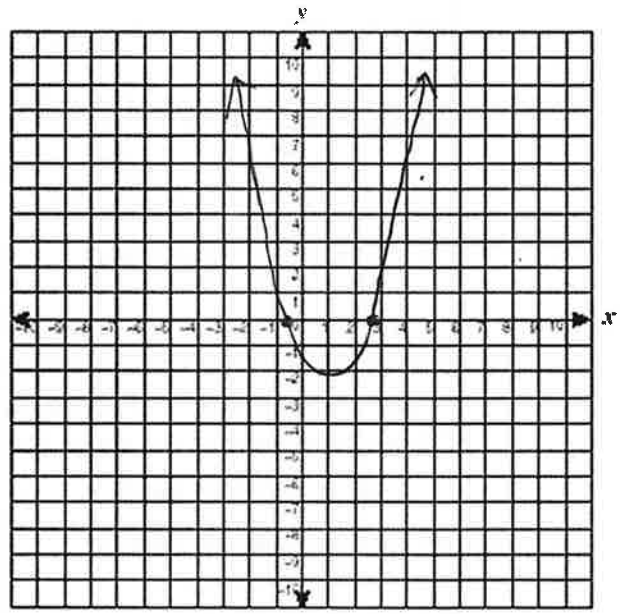
cannot factor so use quadratic formula:

$a=1, b=-2, c=-2$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2-4(1)(-2)}}{2(1)} = \frac{2 \pm \sqrt{4+8}}{2}$

$= \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3} = 2.73, -0.73$

$a > 0$ so opens up!



$x^2 - 2x - 2 > 0$

Where is the parabola ABOVE the x axis?

$x < 1 - \sqrt{3}, \ x > 1 + \sqrt{3}$

Example 1 – A certain website offers online interactive puzzles, but the puzzle-makers present the following problem for entry to their site. "Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93. By determining the smaller and larger numbers, use it as a password to gain access to the site.

Let x = smaller number
Let y = larger number

① $(x + 2y = 46) \times 3$
② $(x^2 - 3y = 93) \times 2$

② $2x^2 \quad - 6y = 186$
① + $\quad 3x + 6y = 138$

$2x^2 + 3x = 324$

$2x^2 + 3x - 324 = 0$

$2x^2 + 3x - 324 = 0$
$2x^2 - 24x + 27x - 324 = 0$
$2x(x-12) + 27(x-12) = 0$
$(x-12)(2x+27) = 0$
$x = 12, \cancel{\frac{-27}{2}}$ not an integer

$x = 12$
$x + 2y = 46$
$12 + 2y = 46$ $\quad y = 17$
$2y = 34$

The two integers are 12 and 17

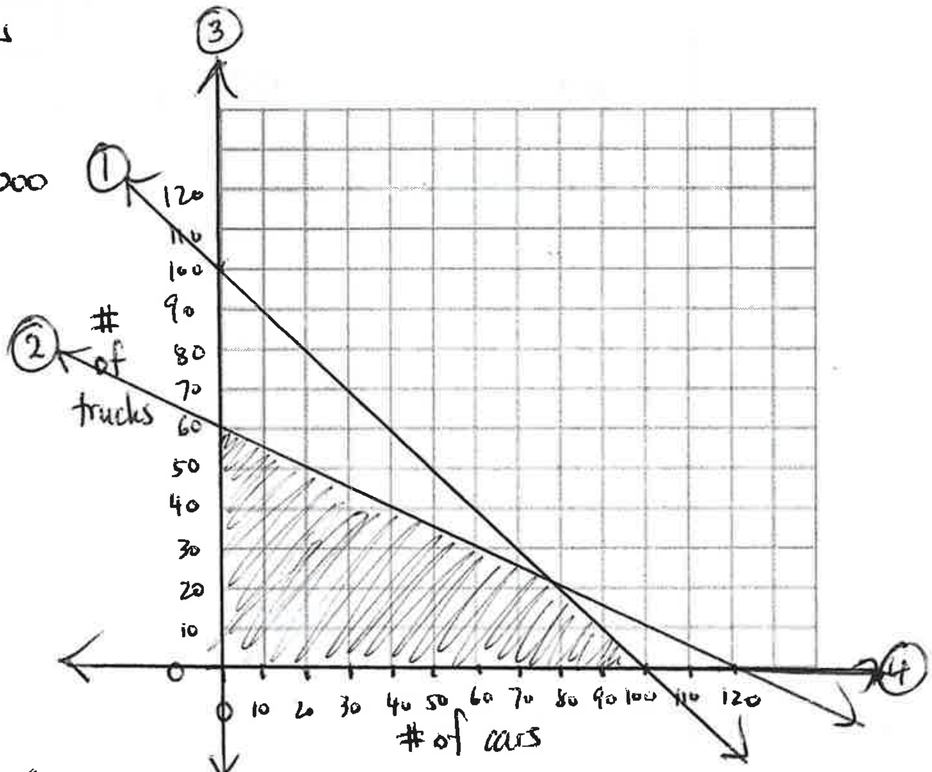
Example 2 – A parkade can fit at most 100 cars & trucks on its lot. A car covers 100 sq feet and a truck 200sq ft of space on a lot that is 12 000 sq ft. What are all the possibilities of cars & trucks that can be on the lot at any one time?

Let x = # of cars
Let y = # of trucks

① $x + y \leq 100$
② $100x + 200y \leq 12\,000$
③ $x \geq 0$
④ $y \geq 0$

① $y \leq -x + 100$
② $y \leq -\frac{1}{2}x + 60$

① shade below
② shade below
③ shade right
④ shade above



One possible solution: (70, 20)
70 cars, 20 trucks etc.

Example 3 - The height in metres of a projectile shot from the top of a building is given by $h(t) = -16t^2 + 60t + 25$, where t represents the time in seconds the projectile is in the air.

a) Find the time the projectile is in the air before hitting the ground, to the nearest thousandth.

b) Find the time interval that the projectile is above 25m, to the nearest hundredth.

ⓐ $h(t) = -16t^2 + 60t + 25$

$0 = -16t^2 + 60t + 25$

$a = -16,\ b = 60,\ c = 25$

$x = \dfrac{-60 \pm \sqrt{60^2 - 4(-16)(25)}}{2(-16)}$

$x = \dfrac{-60 \pm \sqrt{5200}}{-32}$

$x = \cancel{-0.378}, 4.128$
reject

The projectile is in the air for 4.128s.

ⓑ $h(t) = -16t^2 + 60t + 25$

$-16t^2 + 60t + 25 > 25$

$-16t^2 + 60t > 0$

$\dfrac{-4(4t^2 - 15t)}{-4} > \dfrac{0}{-4}$

flip!

$4t^2 - 15t < 0$

$t(4t - 15) < 0$

$t = 0, \frac{15}{4}$

$4t^2 - 15t < 0$
where is the parabola BELOW the x axis?

Test Intervals:

Ⅰ | 0 | Ⅱ | $\frac{15}{4}$ | Ⅲ

Ⅰ test -1:
$4(-1)^2 - 15(-1) < 0$
$4 + 15 < 0$
✗

Ⅱ test 1:
$4(1)^2 - 15(1) < 0$
$4 - 15 < 0$
✓

Ⅲ test 4:
$4(4)^2 - 15(4) < 0$
$64 - 60 < 0$ ✗

The ball is above 25m between 0 and $\frac{15}{4}$ s

$0 < t < \frac{15}{4}$

Example 4 – The price a stereo will be sold for is given by $S(x) = 200 - 0.1x$, $0 \le x \le 2000$, where x is the number of stereos produced each day. It costs \$18 000 per day to operate the factory and \$15 for material to produce each stereo.

a) Find the daily revenue. (b) Find the daily cost of producing stereos. (c) Find the interval that produces a profit.

ⓐ Revenue = (# sold)(price)

$R(x) = x(200 - 0.1x)$

$R(x) = 200x - 0.1x^2$

ⓑ $C(x) = 15x + 18000$

ⓒ $P(x) = R(x) - C(x)$

$P(x) = 200x - 0.1x^2 - (15x + 18000)$

$P(x) = -0.1x^2 + 185x - 18000$

$-0.1x^2 + 185x - 18000 > 0$

$a = -0.1,\ b = 185,\ c = -18000$

$x = \dfrac{-185 \pm \sqrt{(185)^2 - 4(-0.1)(-18000)}}{2(-0.1)}$

$x = 103.04,\ 1746.96$

Test Interval:

Ⅰ | 103.04 | Ⅱ | 1746.96 | Ⅲ

Ⅰ test 1:
$-0.1(1)^2 + 185(1) - 18000(1) > 0$
$-0.1 + 185 - 18000 > 0$
✗

Ⅱ test 1000:
$-0.1(1000)^2 + 185(1000) - 18000 > 0$
$-10000 + 185000 - 18000 > 0$
✓

Ⅲ ✗

Solution:
$104 < x < 1746$
will produce a profit