Principles of Mathematics 12

August 2005 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers	
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	C	K	1.5	2	D6	23.	C	H	1.5	2	B1; G7; A2
2.	A	U	1.5	2	C4	24.	A	K	1.5	2	D2
3.	A	U	1.5	2	C8	25.	D	U	1.5	2	D1
4.	A	U	1.5	2	D6; A1	26.	D	U	1.5	2	D2; F3; A2
5.	D	U	1.5	2	D5	27.	D	K	1.5	3	F1
6.	D	H	1.5	2	C6, C7	28.	A	U	1.5	3	F5
7.	A	U	1.5	2	D4	29.	B	U	1.5	3	F3; A1
8.	B	K	1.5	2	D3; A2	30.	D	H	1.5	3	F6
9.	A	H	1.5	2	D4; A1	31.	D	U	1.5	3	F6
10.	C	H	1.5	2	D1; A2; F2	32.	C	U	1.5	4	G5
11.	C	K	1.5	3	E2	33.	D	U	1.5	4	G5
12.	D	K	1.5	3	E2	34.	C	U	1.5	4	G8
13.	C	U	1.5	3	E2; A1	35.	B	U	1.5	4	G6
14.	D	U	1.5	3	E2	36.	B	U	1.5	4	G7
15.	B	U	1.5	3	E3	37.	B	U	1.5	4	G13
16.	A	H	1.5	3	E2	38.	A	U	1.5	4	G11
17.	B	U	1.5	2	C3	39.	D	H	1.5	4	G12
18.	B	U	1.5	2	C6	40.	A	U	1.5	4	G1; A1
19.	C	U	1.5	2	B1	41.	C	U	1.5	4	G1
20.	A	U	1.5	2	B1	42.	A	U	1.5	4	G3
21.	B	U	1.5	2	B1	43.	B	U	1.5	4	G2
22.	C	H	1.5	2	B3; A1	44.	D	H	1.5	4	G2

Multiple Choice = 66 marks

© 2005 Province of British Columbia. All rights reserved.

Part B: Written Response

Q	C	S	CO	PLO
1.	U	2	3	F6
2.	U	2	3	F4
3.	U	5	2	C2
4.	U	1	4	G13, G8; A1
5.	H	2	4	G13, G8; A1
6.	U	2	4	G13, G8; A1
7.	U	4	2	C5
8.	U	1	2	C6
9.	H	5	2	C7, C8

Written Response = 24 marks

Multiple Choice = 66 (44 questions)
Written Response = 24 (9 questions)
EXAMINATION TOTAL = 90 marks

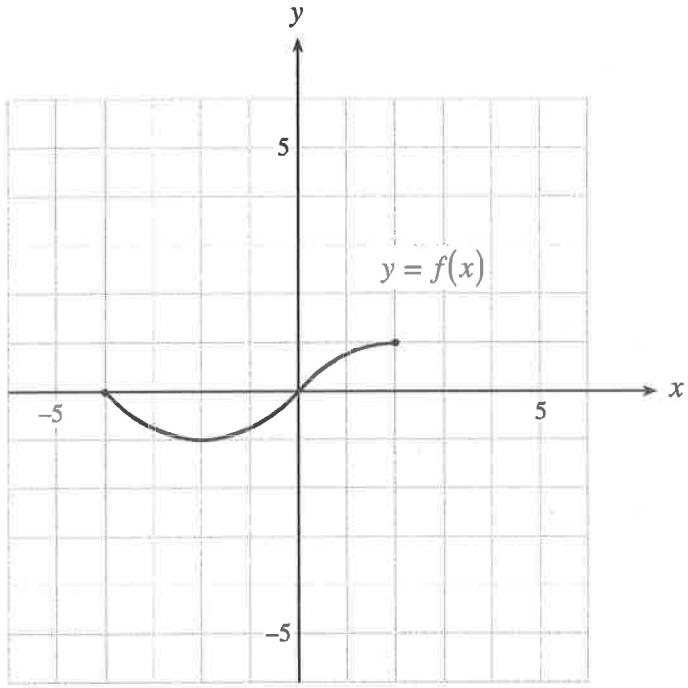
LEGEND:

Q = Question Number **K** = Keyed Response **C** = Cognitive Level
S = Score **CO** = Curriculum Organizer **PLO** = Prescribed Learning Outcome

© 2005 Province of British Columbia. All rights reserved.

Use the following graph to answer questions 1 and 2.

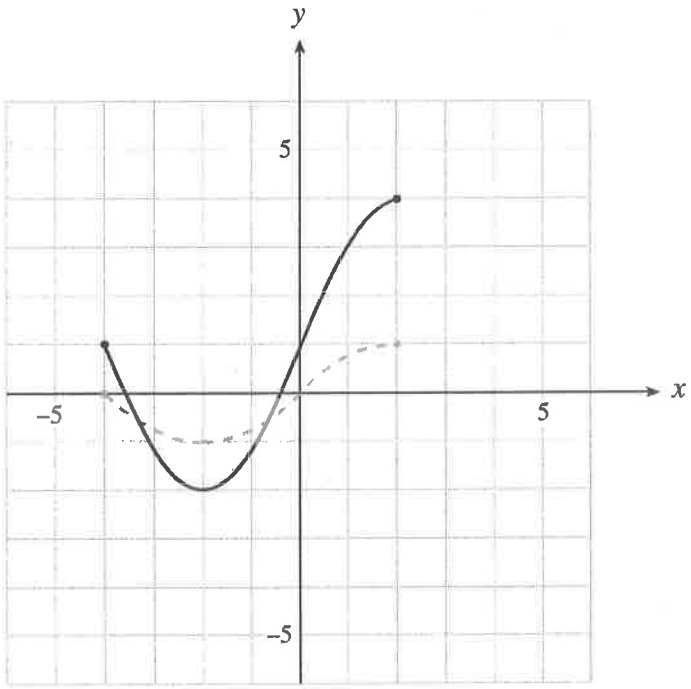
The graph of $y = f(x)$ is shown below.



© 2005 Province of British Columbia. All rights reserved.

1. On the grid provided, sketch the graph of $y = 3f(x) + 1$. **(2 marks)**

solution

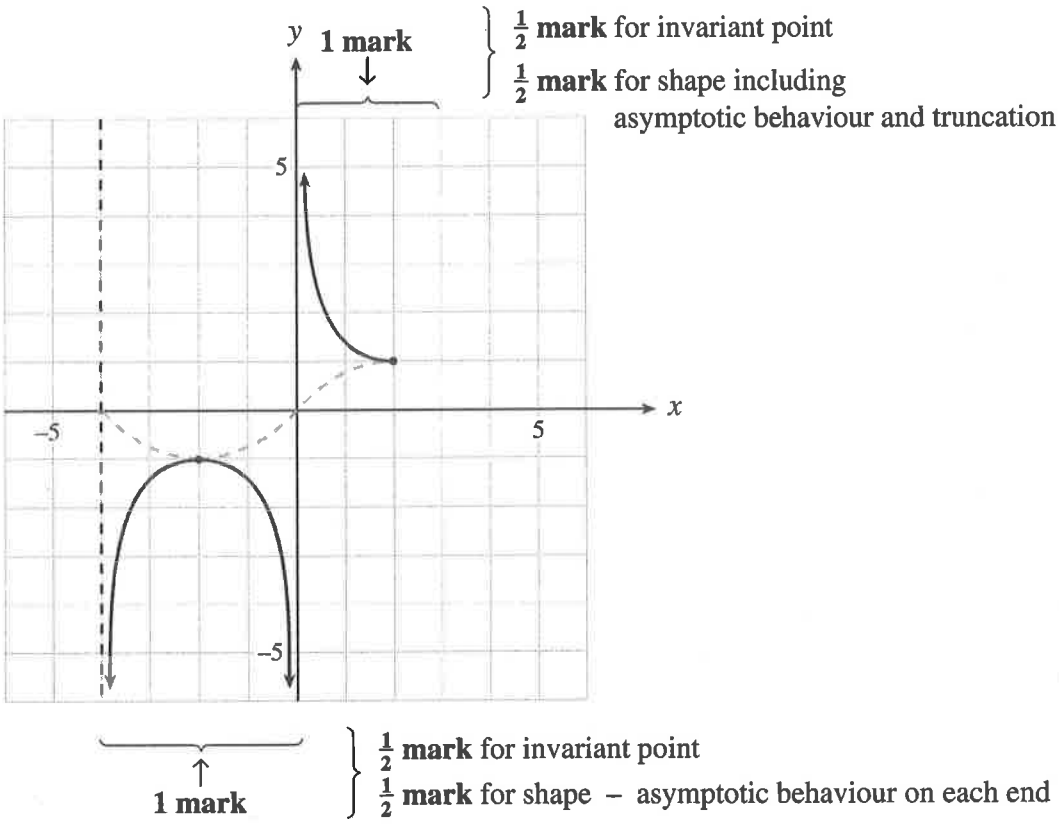


1 mark for vertical expansion
1 mark for vertical translation

© 2005 Province of British Columbia. All rights reserved.

2. On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(2 marks)**

solution



© 2005 Province of British Columbia. All rights reserved.

3. Solve algebraically: $2\log_3(x+4)-\log_3(-x)=2$ **(5 marks)**

solution

$$\log_3\left(x^2+8x+16\right)-\log_3(-x)=2 \qquad \leftarrow \textbf{1 mark}$$

$$\log_3\frac{x^2+8x+16}{-x}=2 \qquad \leftarrow \textbf{1 mark}$$

$$\frac{x^2+8x+16}{-x}=3^2 \qquad \leftarrow \textbf{1 mark}$$

$$x^2+8x+16=-9x$$

$$x^2+17x+16=0$$

$$(x+1)(x+16)=0 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\downarrow \qquad \downarrow$$

$$x=-1 \qquad x=-16 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\uparrow \qquad \Downarrow$$

$$\tfrac{1}{2}\textbf{ mk} \qquad \text{reject} \leftarrow \tfrac{1}{2}\textbf{ mark}$$

alternate solution

$$\log_3\left(x^2+8x+16\right)-\log_3(-x)=\log_3 9 \qquad \leftarrow \textbf{1}\tfrac{1}{2}\textbf{ marks}$$

$$\log_3\frac{x^2+8x+16}{-x}=\log_3 9 \qquad \leftarrow \textbf{1 mark}$$

$$\frac{x^2+8x+16}{-x}=9 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x^2+8x+16=-9x$$

$$x^2+17x+16=0$$

$$(x+1)(x+16)=0 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\downarrow \qquad \downarrow$$

$$x=-1 \qquad x=-16 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\uparrow \qquad \Downarrow$$

$$\tfrac{1}{2}\textbf{ mk} \qquad \text{reject} \leftarrow \tfrac{1}{2}\textbf{ mark}$$

© 2005 Province of British Columbia. All rights reserved.

Use the following information to answer questions 4 to 6.

A biased (weighted) coin is designed so that the probability of a head on each flip is $\frac{3}{5}$.

4. If this biased coin is flipped 3 times, what is the probability that the first 2 flips are tails and the third flip is a head? **(1 mark)**

solution

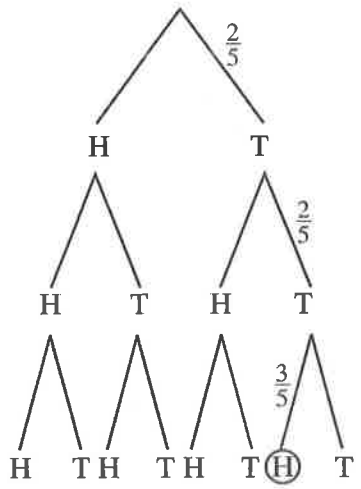
T, T, H

$\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{3}{5}$ ← $\frac{1}{2}$ **mark**

$= \frac{12}{125}$

≈ 0.10 ← $\frac{1}{2}$ **mark**

alternate solution



← $\frac{1}{2}$ **mark** tree diagram

$\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{3}{5}$

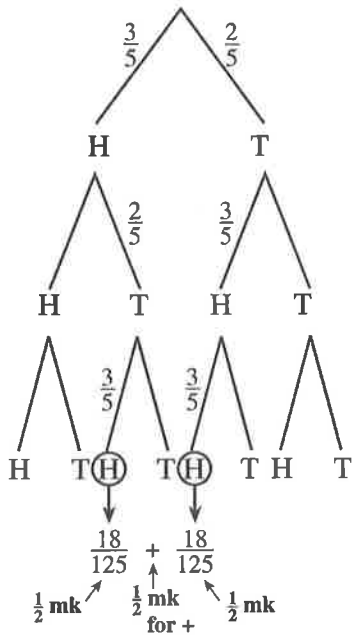
$= \frac{12}{125}$

≈ 0.10 ← $\frac{1}{2}$ **mark**

© 2005 Province of British Columbia. All rights reserved.

5. If this biased coin is flipped until exactly 2 heads appear, what is the probability that it takes exactly 3 flips until the second head appears? **(2 marks)**

solution



$$P(\text{2H in 3 flips}) = \frac{36}{125}$$

$$\approx 0.29 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

alternate solution

$$P(\text{H}) = \frac{3}{5}$$

$$\underbrace{P(\text{1H})}\ \downarrow$$

$$\underline{\ \ }\ \underline{\ \ }\ \underline{\text{H}}$$

$$n = 2,\ p = \frac{3}{5},\ x = 1$$

$$P(\text{2H in 3 flips}) = \overbrace{\text{binompdf}\left(2,\ \frac{3}{5},\ 1\right)}^{\textbf{1 mark}} \times \overset{\tfrac{1}{2}\ \textbf{mark}}{\frac{3}{5}}$$

$$\approx 0.29 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

© 2005 Province of British Columbia. All rights reserved.

6. If this biased coin is flipped 7 times, what is the probability that exactly 3 or 4 heads appear? **(2 marks)**

solution

$n = 7$

$p = \frac{1}{2}$

$x = 3,\ 4$

$$P(3\text{H}) + P(4\text{H}) = \overbrace{\text{binompdf}\left(7,\ \frac{3}{5},\ 3\right)}^{\frac{1}{2}\text{ mark}} + \overbrace{\text{binompdf}\left(7,\ \frac{3}{5},\ 4\right)}^{\frac{1}{2}\text{ mark}}$$

$\frac{1}{2}$ **mark**

$$= 0.193536 + 0.290304$$

$$\approx 0.48 \quad \leftarrow \tfrac{1}{2}\text{ mark}$$

alternate solution 1

$\frac{1}{2}$ **mark** $\quad$ $\frac{1}{2}$ **mark** $\quad$ $\frac{1}{2}$ **mark**

$$\underbrace{{}_7C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^4}_{\frac{1}{2}\text{ mark}} + \underbrace{{}_7C_3\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^3}_{\frac{1}{2}\text{ mark}}$$

$$= 0.193536 + 0.290304$$

$$\approx 0.48 \quad \leftarrow \tfrac{1}{2}\text{ mark}$$

alternate solution 2

$$\overbrace{\text{binomcdf}\left(7,\ \frac{3}{5},\ 4\right)}^{\frac{1}{2}\text{ mark}} - \overbrace{\text{binomcdf}\left(7,\ \frac{3}{5},\ 2\right)}^{\frac{1}{2}\text{ mark}}$$

$\frac{1}{2}$ **mark**

$$\approx 0.48 \quad \leftarrow \tfrac{1}{2}\text{ mark}$$

© 2005 Province of British Columbia. All rights reserved.

Use the following equation to answer questions 7 and 8.

$$2\cos^2 x + 3\cos x + 1 = 0$$

7. Solve the equation algebraically, giving exact values for x, where $0 \le x < 2\pi$. **(4 marks)**

solution

$$(2\cos x + 1)(\cos x + 1) = 0$$

$\frac{1}{2}$ **mark** $\rightarrow$ $\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = -1$ $\leftarrow$ $\frac{1}{2}$ **mark**

2 marks $\rightarrow$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\qquad$ $x = \pi$ $\leftarrow$ **1 mark**

8. Give the general solution for this equation.
(Solve over the set of real numbers giving exact value solutions.) **(1 mark)**

solution

$$\underbrace{x = \pi + 2n\pi,\ x = \frac{2\pi}{3} + 2n\pi,\ x = \frac{4\pi}{3} + 2n\pi}_{\uparrow}$$

1 mark for $+2n\pi$

n is an integer

Note: Do not deduct if n is an integer is missing.

© 2005 Province of British Columbia. All rights reserved.

9. Prove the identity. **(5 marks)**

$$\cos 2x = \frac{\cot x - \sin 2x}{\cot x}$$

solution

LEFT SIDE	RIGHT SIDE
$\cos 2x$	$\dfrac{\cot x - \sin 2x}{\cot x}$
	$\frac{1}{2}$ **mark** $\downarrow$ $\frac{1}{2}$ **mark** $\downarrow$ $= \dfrac{\frac{\cos x}{\sin x} - 2\sin x\cos x}{\frac{\cos x}{\sin x}}$ $\leftarrow \frac{1}{2}$ **mark**
	$\frac{1}{2}$ **mark** $\downarrow$ $= \dfrac{\frac{\cos x}{\sin x} - 2\sin x\cos x}{\frac{\cos x}{\sin x}}\left(\dfrac{\sin x}{\sin x}\right)$
	$= \dfrac{\cos x - 2\sin^2 x\cos x}{\cos x}$ $\leftarrow$ **1 mark**
	$= \dfrac{\cos x\left(1 - 2\sin^2 x\right)}{\cos x}$ $\leftarrow$ **1 mark**
	$= 1 - 2\sin^2 x$ $\leftarrow \frac{1}{2}$ **mark**
	$= \cos 2x$ $\leftarrow \frac{1}{2}$ **mark**

LS = RS

© 2005 Province of British Columbia. All rights reserved.

9. Prove the identity. **(5 marks)**

$$\cos 2x = \frac{\cot x - \sin 2x}{\cot x}$$

alternate solution

LEFT SIDE	RIGHT SIDE
$\cos 2x$	$\dfrac{\cot x - \sin 2x}{\cot x}$
	$= \dfrac{\cot x}{\cot x} - \dfrac{\sin 2x}{\cot x}$ ← **1 mark**
	$= 1 - \dfrac{2\sin x \cos x}{\dfrac{\cos x}{\sin x}}$ ← $\frac{1}{2}$ **mark** ← $\frac{1}{2}$ **mark** ↑ $\frac{1}{2}$ **mk**
	$\frac{1}{2}$ **mark** ↓ $= 1 - \dfrac{2\sin x \cos x}{\dfrac{\cos x}{\sin x}} \left(\dfrac{\sin x}{\sin x}\right)$
	$= 1 - \dfrac{2\sin^2 x \cos x}{\cos x}$ ← **1 mark**
	$= 1 - 2\sin^2 x$ ← $\frac{1}{2}$ **mark**
	$= \cos 2x$ ← $\frac{1}{2}$ **mark**

LS = RS

END OF KEY

© 2005 Province of British Columbia. All rights reserved.