✗ No POLYNOMIALS (Unit 2)

✗ minimal RATIONALS (Unit 3)

BRITISH COLUMBIA

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✗ no RADICALS (Unit 3)

Principles of Mathematics 12

August 2004

Course Code = MA

Student Instructions

1. Place the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**

2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.

3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.

4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION .

5. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle on the Response Form that has the letter corresponding to your answer.

1. An arc of length x cm subtends an angle of $\dfrac{2\pi}{5}$ radians at the centre of a circle with radius 10 cm. Determine the value of x.

    A. 7.20

    B. $\dfrac{25}{\pi}$

    C. 10

    D. 4π

2. Determine $\sec\theta$ using the information in the diagram below.
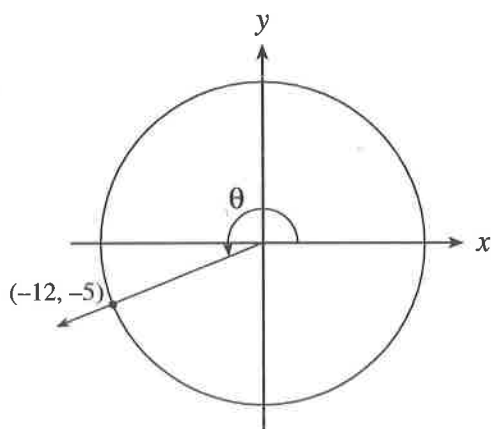
    A. $-\dfrac{13}{12}$

    B. $-\dfrac{13}{5}$

    C. $\dfrac{13}{5}$

    D. $\dfrac{13}{12}$

3. Determine the amplitude of the function $y = -4\sin 3x - 2$.

    A. −4

    B. −2

    C. 2

    D. 4

*4. Solve: $\log x = 2\cos x$, $0 < x < 2\pi$

graphing calc only

    A. 0.17, 0.71

    B. 1.38

    C. 1.48, 5.07

    D. 1.57, 5.11

5. Determine the exact value of $\cot\left(-\dfrac{5\pi}{6}\right)$.

    A. $-\sqrt{3}$

    B. $-\dfrac{1}{\sqrt{3}}$

    C. $\dfrac{1}{\sqrt{3}}$

    D. $\sqrt{3}$

6. Determine the phase shift of the function: $f(x) = \cos\left(\dfrac{1}{3}x - \pi\right)$

    A. $\dfrac{\pi}{3}$ to the right

    B. π to the right

    C. 3π to the right

    D. 6π to the right

7. Solve: $\csc x + 2 = 0$, $\quad 0 \le x < 2\pi$

    A. $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

    B. $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

    C. $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$

    D. $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

8. If $A = B + 90°$, simplify $\cos A \cos B + \sin A \sin B$.

    A. 0

    B. 1

    C. $\cos 2B$

    D. $\cos(2B + 90°)$

9. Determine the general solution for $\sin 2x = 1$.

    A. $x = \dfrac{\pi}{2} + n\pi$, n is an integer

    B. $x = \dfrac{\pi}{4} + n\pi$, n is an integer

    C. $x = \dfrac{\pi}{6} + 2n\pi$, $\dfrac{5}{6}\pi + 2n\pi$, n is an integer

    D. $x = \dfrac{\pi}{3} + 2n\pi$, $\dfrac{2}{3}\pi + 2n\pi$, n is an integer

10. Determine the number of terms in the geometric sequence: $\dfrac{1}{81}, \dfrac{1}{27}, \dfrac{1}{9}, \ldots, 531441$

    A. 14

    B. 15

    C. 16

    D. 17

OVER

11. Evaluate: $\displaystyle\sum_{k=3}^{13} 5(-2)^{k-1}$

    A. 13660
    B. 13655
    C. −13651.67
    D. −13646.67

12. If $600 is invested at 7% compounded quarterly, which expression will give the value of the investment in 9 years?

    A. $600(1+0.07)^9$

    B. $600\left(\dfrac{1+0.07}{4}\right)^{36}$

    C. $600\left(1+\dfrac{0.07}{4}\right)^{9}$

    D. $600\left(1+\dfrac{0.07}{4}\right)^{36}$

13. Determine the sum of the infinite geometric series: $800 + 300 + \dfrac{225}{2} + \ldots$

    A. 1280
    B. 1212.5
    C. 1254.69
    D. no finite sum

14. The third term of a geometric sequence is 3 and the sixth term is $\dfrac{64}{9}$. Find the fifth term of this sequence.

    A. $\dfrac{27}{16}$

    B. $\dfrac{16}{3}$

    C. $\dfrac{32}{3}$

    D. $\dfrac{256}{27}$

15. Determine an equation of the asymptote of $y = 4\log_2(x+3)+4$.

    A. $x = -3$
    B. $x = 3$
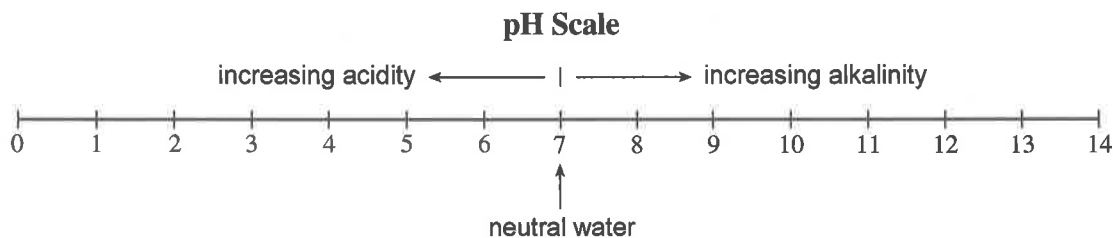   C. $y = -4$
    D. $y = 4$

16. Evaluate: $5\log_4 3$

    A. 1.95
    B. 2.39
    C. 3.96
    D. 6.31

17. Express as a single logarithm:
$$\log a - \log b - 5\log c$$

    A. $\log\dfrac{ac^5}{b}$

    B. $\log\dfrac{5ac}{b}$

    C. $\log\dfrac{a}{5bc}$

    D. $\log\dfrac{a}{bc^5}$

18. In chemistry, the pH scale measures the acidity (0–7) or alkalinity (7–14) of a solution. It is a logarithmic scale in base 10. Thus a pH of 9 is 10 times more alkaline than a pH of 8. An unknown solution is 40 times more alkaline than neutral water which has a pH of 7. Determine the pH of the unknown solution.

pH Scale

increasing acidity ◄——— | ———► increasing alkalinity

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

neutral water

    A. 7.4
    B. 8.4
    C. 8.6
    D. 11

OVER

19. The population of rabbits is increasing by 70% every 6 months. Presently there are 200 rabbits. Determine an expression for the number of months it will take for the population to reach 800.

A. $6\log_{1.7} 4$

B. $\dfrac{1}{6}\log_{1.7} 4$

C. $4\log_{1.7} 6$

D. $\dfrac{1}{4}\log_{1.7} 6$

20. If $f(x) = 10^{x-3} + 4$, find $f^{-1}(x)$, the inverse of $f(x)$.

A. $f^{-1}(x) = \log(x-3) + 4$

B. $f^{-1}(x) = \log(x-4) + 3$

C. $f^{-1}(x) = 10^{3-x} + 4$

D. $f^{-1}(x) = \dfrac{1}{10^{x-3} + 4}$

21. Determine the value of $\log_n ab^2$ if $\log_n a = 5$ and $\log_n b = 3$.

A. 11
B. 14
C. 16
D. 45

22. Which equation below represents a hyperbola with asymptotes $y = \pm\dfrac{3}{2}x$?

A. $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$

B. $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$

C. $\dfrac{x^2}{3} - \dfrac{y^2}{2} = 1$

D. $\dfrac{x^2}{2} - \dfrac{y^2}{3} = 1$

25. Which equation represents the graph of $y = g(x)$ after it is reflected in the line $y = x$?

    A. $y = g(-x)$

    B. $y = -g(x)$

    C. $y = \dfrac{1}{g(x)}$

    D. $x = g(y)$

26. How is the graph of $y = \dfrac{1}{2} f(x)$ related to the graph of $y = f(x)$?

    A. $y = f(x)$ has been expanded vertically by a factor of 2.

    B. $y = f(x)$ has been expanded horizontally by a factor of 2.

    C. $y = f(x)$ has been compressed vertically by a factor of $\dfrac{1}{2}$.

    D. $y = f(x)$ has been compressed horizontally by a factor of $\dfrac{1}{2}$.

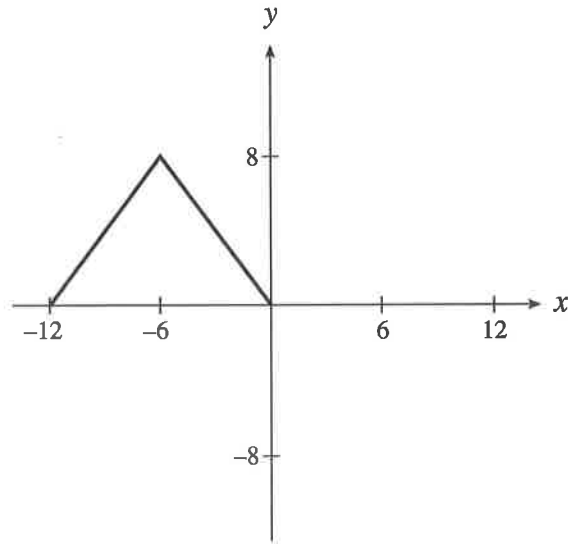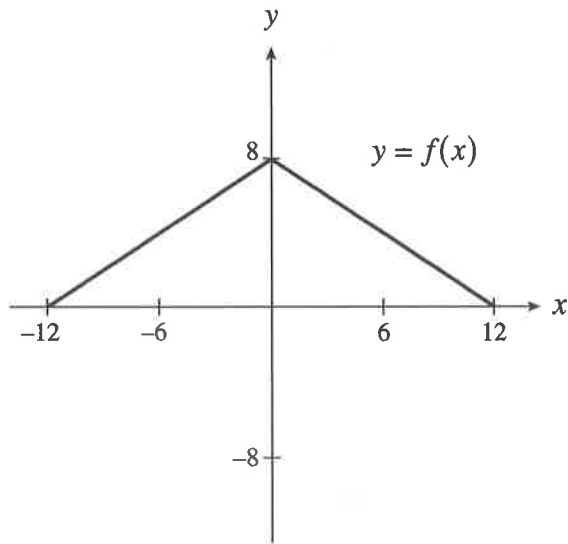27. If the range of $y = f(x)$ is $-1 \le y \le 2$, what is the range of $y = \dfrac{1}{f(x)}$?

    A. $-1 \le y \le \dfrac{1}{2}$

    B. $-1 \le y \le \dfrac{1}{2},\ y \ne 0$

    C. $y \ge \dfrac{1}{2}$ or $y \le -1$

    D. $y \ge 2$ or $y \le -1$

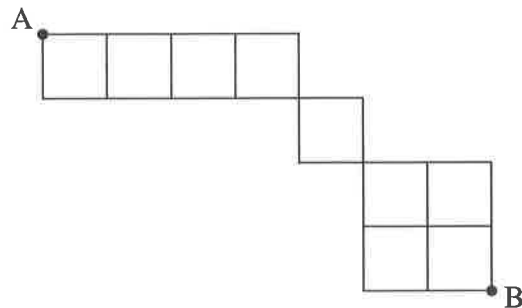28. The function $y = f(x)$ is graphed to the left below. Determine an equation of the function shown on the right.

A. $y = f(2x)$
B. $y = f(2x + 6)$
C. $y = f(2x - 6)$
D. $y = f(2x + 12)$

29. Determine the number of terms in the expansion of $(a + b)^7$.

A. 6
B. 7
C. 8
D. 9

30. Moving only to the right or down, determine the number of different pathways from A to B.

A
B

A. 13
B. 24
C. 60
D. 80

31. Codes with 5 digits are made from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. If repetitions are not permitted and each code must contain 2 odd digits followed by 3 even digits, determine the number of different codes that can be made.

A.   126
B.   480
C.   1 600
D. 15 120

Use the following information to answer questions 32 and 33.

A class of 14 students is made up of 6 girls and 8 boys. From this class, a group of 5 students is chosen to represent the class at a competition.

32. Determine the number of different groups of 5 that can be formed if there must be 2 girls and 3 boys in each group.

    A.   71
    B.   560
    C.   840
    D. 10 080

33. Determine the number of different groups of 5 that can be formed if there must be at most 1 boy in each group.
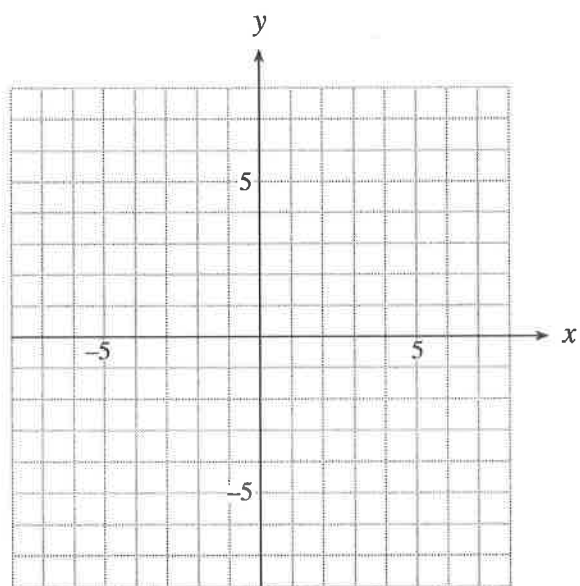
    A.   23
    B.   30
    C. 120
    D. 126

34. Two fair six-sided dice are rolled and the face values are added. What is the probability of obtaining a sum that is an even number less than 8?

    A. $\dfrac{1}{4}$

    B. $\dfrac{7}{18}$

    C. $\dfrac{1}{2}$

    D. $\dfrac{7}{12}$

OVER

a) On the grid provided, sketch the graph of $y = -f(x+3)$. **(2 marks)**

b) On the grid provided, sketch the graph of $y = 2|f(x)| - 3$. **(3 marks)**
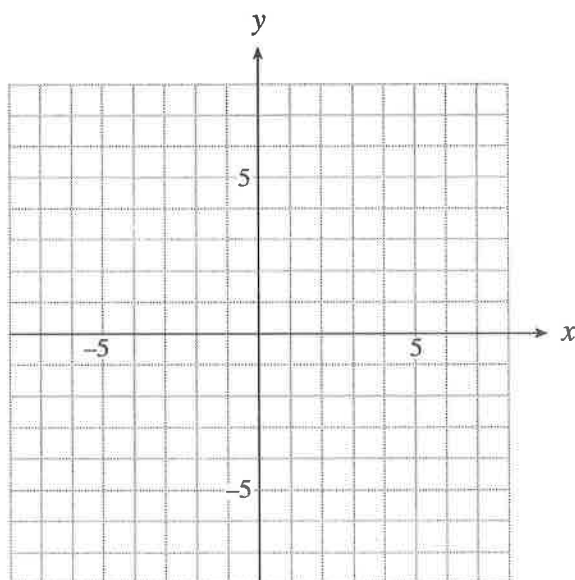
OVER

3. Solve algebraically: $\log(3-x)+\log(4-3x)-\log x = \log 7$ **(5 marks)**

6. A sinusoidal curve has a minimum point at $\left(-\dfrac{\pi}{3},\ -5\right)$ and the closest maximum point to the right is $\left(\dfrac{\pi}{6},\ 3\right)$. Determine an equation of this curve. **(4 marks)**

A grid is provided for rough work only.
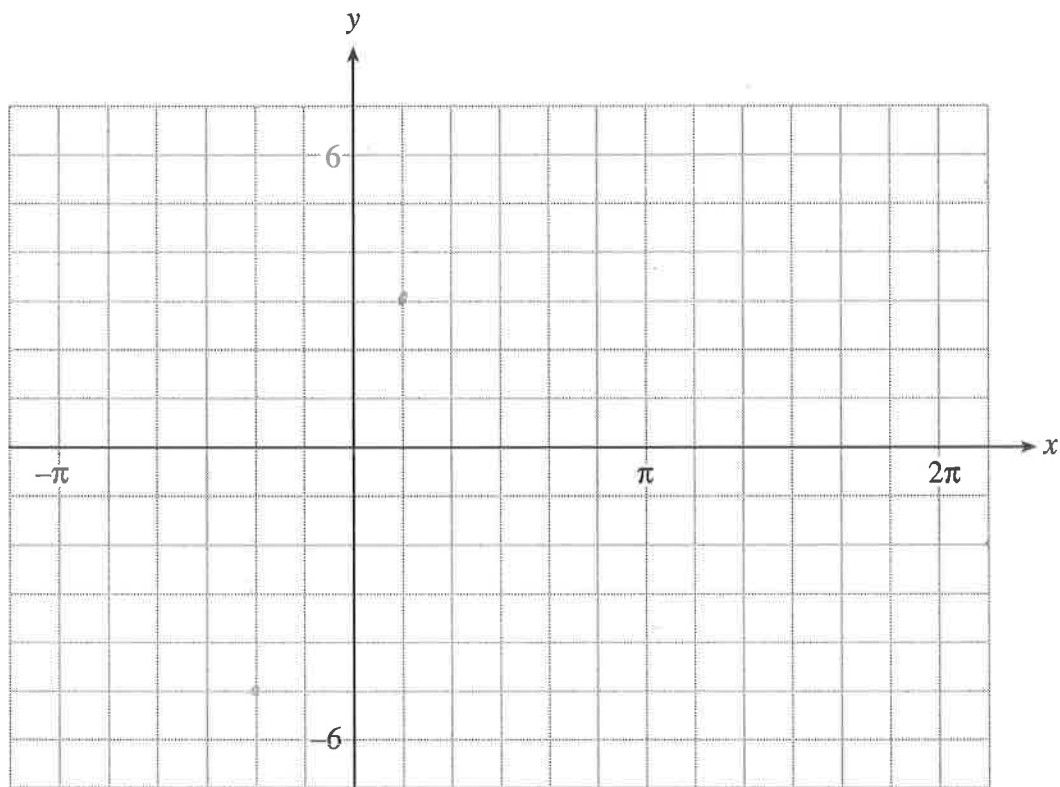
7. Prove the identity: **(4 marks)**

$$\frac{1 - \cos 2x}{\sin 2x} = \frac{1 + \tan x}{1 + \cot x}$$

LEFT SIDE	RIGHT SIDE

Principles of Mathematics 12
August 2004 Provincial Examination
ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers	
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	U	1.5	2	C3	21.	A	U	1.5	2	D4
2.	A	U	1.5	2	D5	22.	B	U	1.5	3	E3
3.	D	K	1.5	2	D6	23.	B	U	1.5	3	E2
4.	C	U	1.5	2	A9	24.	D	H	1.5	3	E2
5.	D	U	1.5	2	C4	25.	D	K	1.5	3	F3
6.	C	U	1.5	2	D6	26.	C	K	1.5	3	F2
7.	B	U	1.5	2	C4, C5	27.	C	H	1.5	3	F4
8.	A	H	1.5	2	C4, C8	28.	D	H	1.5	3	F1, F2
9.	B	H	1.5	2	C6, C8	29.	C	K	1.5	4	G8
10.	D	U	1.5	2	B1	30.	C	U	1.5	4	G4
11.	A	U	1.5	2	B1	31.	B	U	1.5	4	G5
12.	D	K	1.5	2	B2	32.	C	U	1.5	4	G7
13.	A	U	1.5	2	B3	33.	D	U	1.5	4	G7
14.	B	U	1.5	2	B1	34.	A	U	1.5	4	G11
15.	A	K	1.5	2	D3	35.	B	U	1.5	4	G11
16.	C	U	1.5	2	D4	36.	A	U	1.5	4	G13
17.	D	U	1.5	2	D4	37.	C	H	1.5	4	G8
18.	C	U	1.5	2	D1	38.	B	U	1.5	4	G1
19.	A	H	1.5	2	D1	39.	B	U	1.5	4	G2
20.	B	U	1.5	2	F3; D2	40.	D	U	1.5	4	G3

Multiple Choice = 60 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	3	F1, F5
1b.	2	U	3	3	F2, F1
2.	3	U	4	3	E3
3.	4	U	5	2	C2
4a.	5	U	2	4	G11
4b.	6	H	2	4	G12
5a.	7	U	2	4	G2
5b.	8	U	2	4	G1
6.	9	H	4	2	D6
7.	10	H	4	2	C8

Written Response = 30 marks

Multiple Choice = 60 (40 questions)
Written Response = 30 (7 questions)
EXAMINATION TOTAL = 90 marks

LEGEND:

Q = Question Number   **K** = Keyed Response   **C** = Cognitive Level
B = Score Box Number   **S** = Score   **CO** = Curriculum Organizer
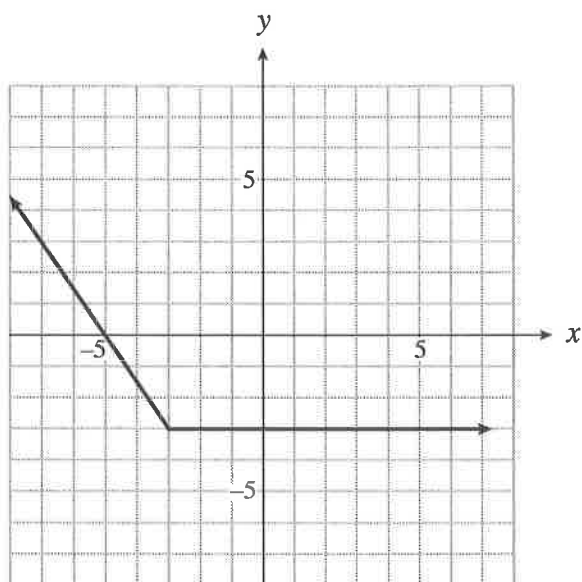**PLO** = Prescribed Learning Outcome

1. The function $y = f(x)$ is graphed below.
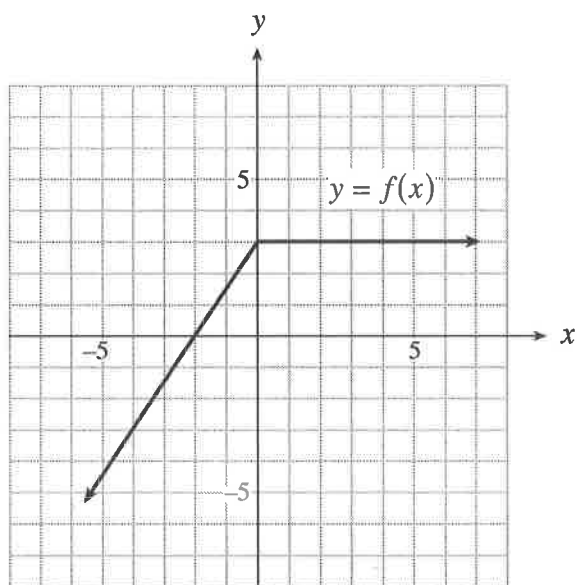
a) On the grid provided, sketch the graph of $y = -f(x+3)$. **(2 marks)**
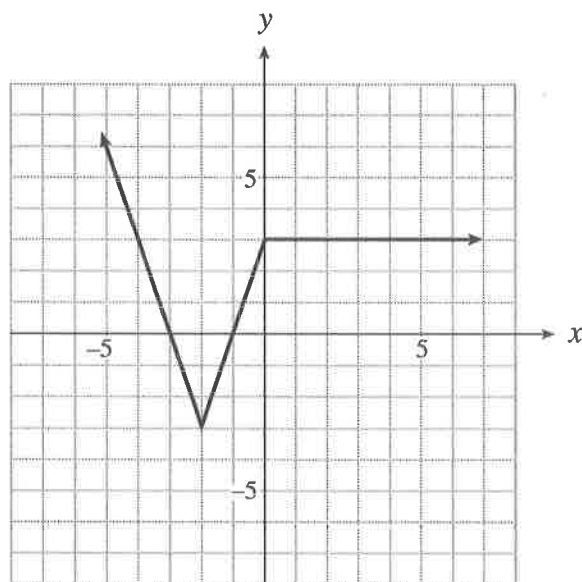
solution

1 mark for horizontal translation
1 mark for reflection in x-axis

b) On the grid provided, sketch the graph of $y = 2|f(x)| - 3$. **(3 marks)**

solution

1 mark for absolute value
1 mark for vertical expansion
1 mark for vertical translation

3. Solve algebraically: $\log(3-x) + \log(4-3x) - \log x = \log 7$ **(5 marks)**

solution

$$\log(3-x) + \log(4-3x) = \log 7 + \log x$$

$$\log(3-x)(4-3x) = \log 7x \qquad \leftarrow \textbf{1 mark}$$

$$(3-x)(4-3x) = 7x \qquad \leftarrow \textbf{1 mark}$$

$$12 - 13x + 3x^2 = 7x$$

$$3x^2 - 20x + 12 = 0 \qquad \leftarrow \textbf{1 mark}$$

$$(3x-2)(x-6) = 0$$

$$x = \frac{2}{3}, \ 6 \qquad \leftarrow \textbf{1 mark} \text{ (for both solutions)}$$

$$\uparrow$$

$$\text{reject} \qquad \leftarrow \textbf{1 mark}$$

$$\therefore \ x = \frac{2}{3}$$

alternate solution

$$\log(3-x) + \log(4-3x) - \log x = \log 7$$

$$\log \frac{(3-x)(4-3x)}{x} = \log 7 \qquad \leftarrow \textbf{1 mark}$$

$$\frac{12 - 13x + 3x^2}{x} = 7 \qquad \leftarrow \textbf{1 mark}$$

$$3x^2 - 13x + 12 = 7x$$

$$3x^2 - 20x + 12 = 0 \qquad \leftarrow \textbf{1 mark}$$

$$(3x-2)(x-6) = 0$$

$$x = \frac{2}{3}, \ x = 6 \qquad \leftarrow \textbf{1 mark} \text{ (for both solutions)}$$

$$\uparrow$$

$$\text{reject} \qquad \leftarrow \textbf{1 mark}$$

6. A sinusoidal curve has a minimum point at $\left(-\frac{\pi}{3}, -5\right)$ and the closest maximum point to the right is $\left(\frac{\pi}{6}, 3\right)$. Determine an equation of this curve. **(4 marks)**

solution

$$\overset{\underset{\displaystyle\downarrow}{\textbf{1mk}}\ \ \overset{\underset{\displaystyle\downarrow}{\textbf{1mk}}}{}\ \ \overset{\underset{\displaystyle\downarrow}{\textbf{1mk}}}{}\ \ \overset{\underset{\displaystyle\downarrow}{\textbf{1mk}}}{}}{}$$

$$y = 4\cos 2\left(x - \frac{\pi}{6}\right) - 1$$

or

$$y = -4\cos 2\left(x + \frac{\pi}{3}\right) - 1$$

or

$$y = 4\sin 2\left(x + \frac{\pi}{12}\right) - 1$$

or

$$y = -4\sin 2\left(x - \frac{5\pi}{12}\right) - 1$$

7. Prove the identity: **(4 marks)**

$$\frac{1-\cos 2x}{\sin 2x} = \frac{1+\tan x}{1+\cot x}$$

![solution]

LEFT SIDE	RIGHT SIDE

LEFT SIDE:

$$\frac{1-\cos 2x}{\sin 2x}$$

$\frac{1}{2}$ **mark** $\rightarrow$
$\frac{1}{2}$ **mark** $\rightarrow$
$$= \frac{1-\left(1-2\sin^2 x\right)}{2\sin x \cos x}$$

$\frac{1}{2}$ **mark** $\rightarrow$
$$= \frac{2\sin^2 x}{2\sin x \cos x}$$

$\frac{1}{2}$ **mark** $\rightarrow$
$$= \frac{\sin x}{\cos x}$$

RIGHT SIDE:

$$\frac{1+\tan x}{1+\cot x} \quad \tfrac{1}{2}\ \textbf{mark}$$

$$= \frac{1+\tan x}{1+\dfrac{1}{\tan x}} \times \frac{\tan x}{\tan x} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$= \frac{\left(1+\tan x\right)\tan x}{\tan x + 1} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$= \tan x$$

$$= \frac{\sin x}{\cos x} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

LS = RS

7. Prove the identity: **(4 marks)**

$$\frac{1-\cos 2x}{\sin 2x} = \frac{1+\tan x}{1+\cot x}$$

solution

	LEFT SIDE	RIGHT SIDE

$$\frac{1-\cos 2x}{\sin 2x} \qquad\qquad \frac{1+\tan x}{1+\cot x}$$

$\frac{1}{2}$ mark $\rightarrow$
$\frac{1}{2}$ mark $\rightarrow$
$$= \frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x}$$

$\frac{1}{2}$ mark $\rightarrow$
$$= \frac{2\sin^2 x}{2\sin x\cos x}$$

$\frac{1}{2}$ mark $\rightarrow$
$$= \frac{\sin x}{\cos x}$$

Right side:

$\frac{1}{2}$ mark

$\frac{1}{2}$ mark

$$= \frac{1+\dfrac{\sin x}{\cos x}}{1+\dfrac{\cos x}{\sin x}} \times \frac{\sin x\cos x}{\sin x\cos x}$$

$$= \frac{\sin x\cos x + \sin^2 x}{\sin x\cos x + \cos^2 x} \qquad \leftarrow \tfrac{1}{2}\ \text{mark}$$

$$= \frac{\sin x(\cos x + \sin x)}{\cos x(\sin x + \cos x)}$$

$$= \frac{\sin x}{\cos x} \qquad \leftarrow \tfrac{1}{2}\ \text{mark}$$

$$\text{LS} = \text{RS}$$

END OF KEY