MINISTRY USE ONLY

MT. DOUGLAS SECONDARY SCHOOL
000069972

Place Personal Education Number (PEN) here.

Principles of Mathematics 12

AUGUST 2005

Course Code = MA

Principles of Mathematics 12

AUGUST 2005

Course Code = MA

BRITISH COLUMBIA

Student Instructions

1. Place the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have **two Examination Response Forms**, one blue and one white. Follow the directions on the front of each Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION .
5. At the end of the examination, place your white Response Form inside the front cover of this booklet and return the booklet and your white Response Form to the supervisor.

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Place Personal Education Number (PEN) here.

Principles of Mathematics 12

AUGUST 2005

Course Code = MA

Question 1								
0 ☐	1 ☐	2 ☐					(.5) ☐	NR ☐
Question 2								
0 ☐	1 ☐	2 ☐					(.5) ☐	NR ☐
Question 3								
0 ☐	1 ☐	2 ☐	3 ☐	4 ☐	5 ☐		(.5) ☐	NR ☐
Question 4								
0 ☐	1 ☐						(.5) ☐	NR ☐
Question 5								
0 ☐	1 ☐	2 ☐					(.5) ☐	NR ☐
Question 6								
0 ☐	1 ☐	2 ☐					(.5) ☐	NR ☐
Question 7								
0 ☐	1 ☐	2 ☐	3 ☐	4 ☐			(.5) ☐	NR ☐
Question 8								
0 ☐	1 ☐						(.5) ☐	NR ☐
Question 9								
0 ☐	1 ☐	2 ☐	3 ☐	4 ☐	5 ☐		(.5) ☐	NR ☐

General Instructions

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.

2. All multiple-choice answers must be entered on the Response Forms using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.

3. For each of the written-response questions, write your answer in the space provided in this booklet. Rough-work space has been incorporated into the space allowed for answering each written-response question. You may not need all of the space provided to answer each question.

4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.

5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time	Allowable Time
1. This examination consists of **two** parts:			
PART A:			
Section I: 16 multiple-choice questions **Note:** No calculator may be used for the first 45 minutes of the examination.	24 marks	35 minutes	45 minutes
Section II: 28 multiple-choice questions (some of which require the use of a calculator)	42 marks	55 minutes	105 minutes
PART B:			
5 written-response topics covered by 9 questions	24 marks	30 minutes	
Total:	**90 marks**	**120 minutes**	**150 minutes**

2. **After 45 minutes, the blue Response Form (Section I) will be collected. When all blue Response Forms are handed in, you will be permitted to use your calculator.**
   During the first 45 minutes, you may proceed to other questions on the examination, many of which do not require the use of a calculator. Once the blue Response Forms have been handed in, you will **not** be able to go back to any of the first 16 questions; therefore, ensure you have checked your answers before proceeding to the rest of the examination.

3. The last **four** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **The Standard Normal Distribution Table**, **Rough Work for Graphing**, and **Rough Work for Multiple Choice**.

4. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

PART A: MULTIPLE CHOICE (non-calculator)
SECTION I

Value: 24 marks

Suggested Time: 35 minutes
Allowable Time: 45 minutes

INSTRUCTIONS: **No calculator may be used for this section of the examination.** For each question, select the **best** answer and record your choice on the blue Response Form provided. Using an HB pencil, completely fill in the circle on the blue Response Form that has the letter corresponding to your answer.

1. Determine the amplitude of $y = -2\cos x - 3$.

   A. -3
   B. -2
   C. 2
   D. 3

2. Evaluate: $\sec \frac{4\pi}{3}$

   A. -2

   B. $-\frac{2}{\sqrt{3}}$

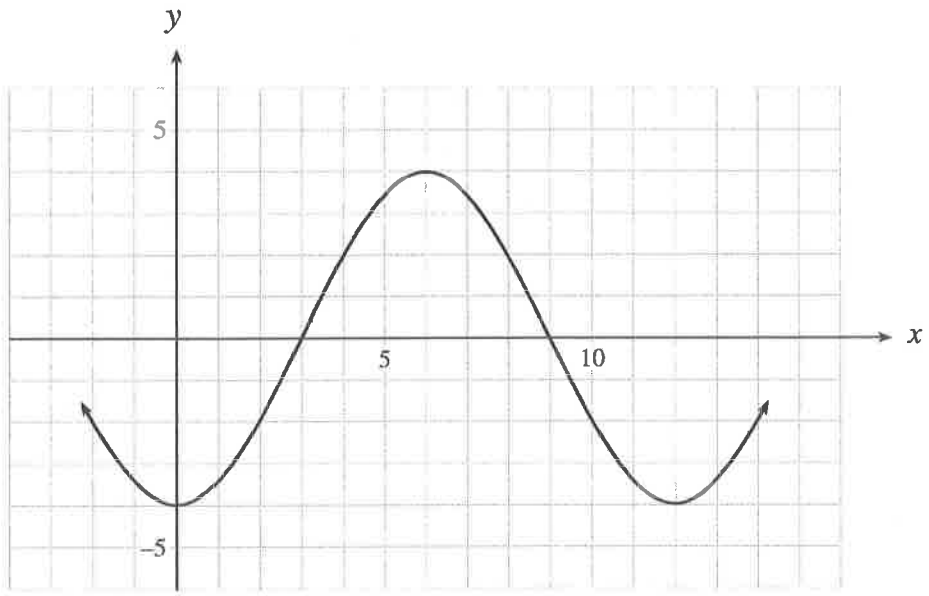
   C. $\frac{2}{\sqrt{3}}$

   D. 2

3. Simplify: $\cos 2x \cos x + \sin 2x \sin x$

   A. $\cos x$
   B. $\sin x$
   C. $\cos 3x$
   D. $\sin 3x$

4. Given the graph below, determine an equation of this function.



A. $y = -4\cos\frac{\pi}{6}x$

B. $y = 4\cos\frac{\pi}{6}x$

C. $y = -4\cos\frac{\pi}{12}x$

D. $y = 4\cos\frac{\pi}{12}x$

5. If $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$ and $\tan\theta = -\frac{4}{3}$, determine the exact value of $\sin\theta$.

A. $-\frac{4}{5}$

B. $-\frac{3}{5}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

6. Determine the restriction(s) for the expression $\frac{\tan\theta}{2\cos\theta - 1}$.

A. $\cos\theta \neq \frac{1}{2}$

B. $\sin\theta \neq 0$

C. $\sin\theta \neq 0,\ \cos\theta \neq \frac{1}{2}$

D. $\cos\theta \neq 0,\ \cos\theta \neq \frac{1}{2}$

7. Express as a single logarithm:

$$\log m - \log n - 3\log k$$

A. $\log \frac{m}{nk^3}$

B. $\log \frac{m}{3nk}$

C. $\log \frac{mk^3}{n}$

D. $\log \frac{3mk}{n}$

8. Determine the domain of the function $y = \log(x - 5)$.

A. $x \geq 5$
B. $x > 5$
C. $x \leq 5$
D. $x < 5$

9. Simplify: $9\log_{27} x - 4\log_9 x$

A. $\log_3 x$

B. $\log_9 x$

C. $\log_{27} x$

D. $\frac{3}{4}\log_3 x$

10. If the function $y = 3^x$ is expanded vertically by a factor of 9 to produce a new function, which of the following is an equation of the new function?

A. $y = 3^{2x}$

B. $y = 3^{3x}$

C. $y = 3^{x+2}$

D. $y = 3^{x-2}$

11. Determine an equation of the circle with centre $(2, -3)$ and radius 4.

A. $(x-2)^2 + (y+3)^2 = 4$

B. $(x+2)^2 + (y-3)^2 = 4$

C. $(x-2)^2 + (y+3)^2 = 16$

D. $(x+2)^2 + (y-3)^2 = 16$

12. Determine the vertex of the parabola $x = (y+1)^2 - 3$.

A. $(-1, -3)$

B. $(-1, 3)$

C. $(3, -1)$

D. $(-3, -1)$

13. An ellipse has vertices at $(1, 4)$ and $(1, -4)$. If the ellipse passes through the point $(4, 0)$, determine the length of the minor axis.

A. 3

B. 4

C. 6

D. 8

14. Which hyperbola has asymptotes $y = \pm\frac{2}{3}x$?

A. $\frac{x^2}{3} - \frac{y^2}{2} = 1$

B. $\frac{x^2}{2} - \frac{y^2}{3} = 1$

C. $\frac{x^2}{16} - \frac{y^2}{36} = 1$

D. $\frac{x^2}{36} - \frac{y^2}{16} = 1$

15. Change to standard form $2x^2 - 3y^2 + 12y = 0$.

A. $\frac{x^2}{6} - \frac{(y+2)^2}{4} = -1$

B. $\frac{x^2}{6} - \frac{(y-2)^2}{4} = -1$

C. $\frac{x^2}{2} - \frac{(y+2)^2}{\frac{4}{3}} = 1$

D. $\frac{x^2}{2} - \frac{(y-2)^2}{\frac{4}{3}} = 1$

16. Determine the restriction on the constants A and C such that the following equation represents an ellipse (not a circle) with major axis on the y-axis. $(\text{where } A > 0,\ C > 0)$

$$Ax^2 + Cy^2 = AC$$

A. $A > C$
B. $A < C$
C. $A = C$
D. $AC = -1$

This is the end of Part A, Section I.

You may proceed to the rest of the examination *without* the use of a calculator until directed by the supervisor to access your calculator. At the end of 45 minutes, you will not be able to go back to Part A, Section I; therefore, ensure you have checked this section.

PART A: MULTIPLE CHOICE
SECTION II

Value: 42 marks **Suggested Time: 55 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the white Response Form provided. Using an HB pencil, completely fill in the circle on the white Response Form that has the letter corresponding to your answer.

17. Determine the reference angle for $\frac{7\pi}{5}$.

A. $\frac{\pi}{5}$

B. $\frac{2\pi}{5}$

C. $\frac{3\pi}{5}$

D. $\frac{4\pi}{5}$

18. Solve: $\sin^2 x = 3 - x$

A. 2.18
B. 2.97
C. 3.02
D. 3.09

19. Determine the common ratio of the geometric sequence $3\sqrt{3},\ 3,\ \sqrt{3},\ \ldots$

A. $\sqrt{3}$

B. $3\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{3}{\sqrt{3}}$

20. In the geometric sequence $162, -108, 72, \ldots,$ determine the 10^{th} term.

A. -4.21
B. -1.87
C. 2.81
D. 6.32

21. The sum of the first 8 terms of a geometric series is 16 400. If the common ratio of this series is 3, determine the first term.

A. -128.13
B. 5
C. 7.5
D. 15

22. An infinite geometric series has all positive terms. If $t_1 = 64$ and $t_3 = 1$, determine the sum of this series.

A. $\frac{256}{5}$
B. $\frac{512}{9}$
C. $\frac{512}{7}$
D. $\frac{256}{3}$

23. Evaluate: $\sum_{k=1}^{4} {}_4C_k$

A. 1
B. 4
C. 15
D. 16

24. Change to exponential form $a = \log_b c$.

A. $b^a = c$

B. $b^c = a$

C. $a^b = c$

D. $a^c = b$

25. A particular type of bacteria multiplies 5-fold every 30 minutes. Initially there are 100 bacteria. Determine an expression for the number of bacteria after k minutes.

A. $\frac{100(5)^k}{30}$

B. $100(5)^{30k}$

C. $100(5)^{\frac{30}{k}}$

D. $100(5)^{\frac{k}{30}}$

26. Given $f(x) = 2^x + 5$, determine $f^{-1}(x)$, the inverse of $f(x)$.

A. $f^{-1}(x) = 5 + \log_2 x$

B. $f^{-1}(x) = -5 + \log_2 x$

C. $f^{-1}(x) = \log_2(x + 5)$

D. $f^{-1}(x) = \log_2(x - 5)$

27. Which equation represents the graph of $y = g(x)$ after it is translated 3 units to the right?

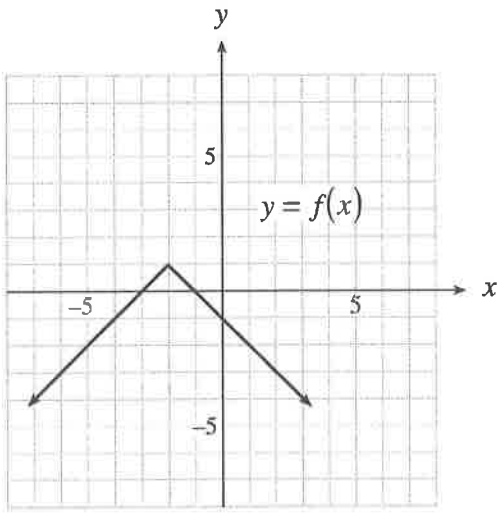
A. $y = g(x) + 3$

B. $y = g(x) - 3$

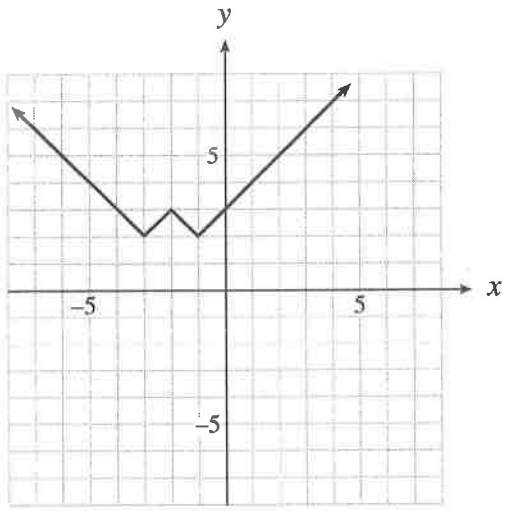
C. $y = g(x + 3)$

D. $y = g(x - 3)$

28. The graph of $y = f(x)$ is shown below. Which graph represents $y = |f(x)| + 2$?



A.



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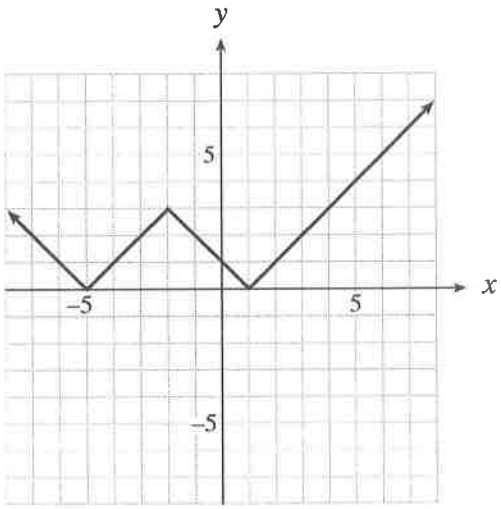
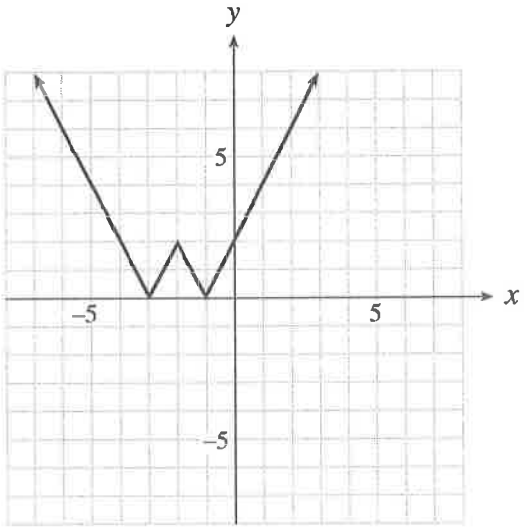
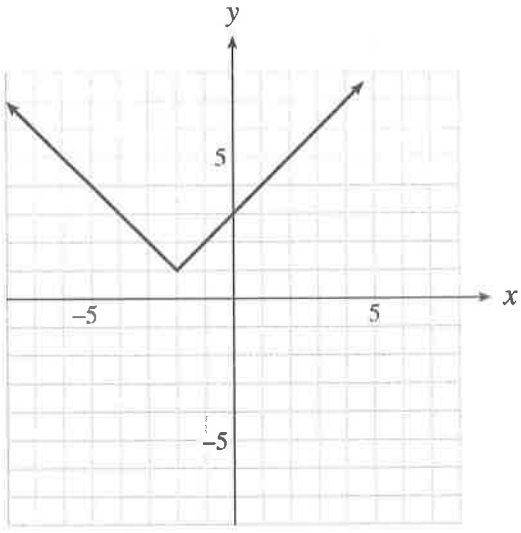


C.

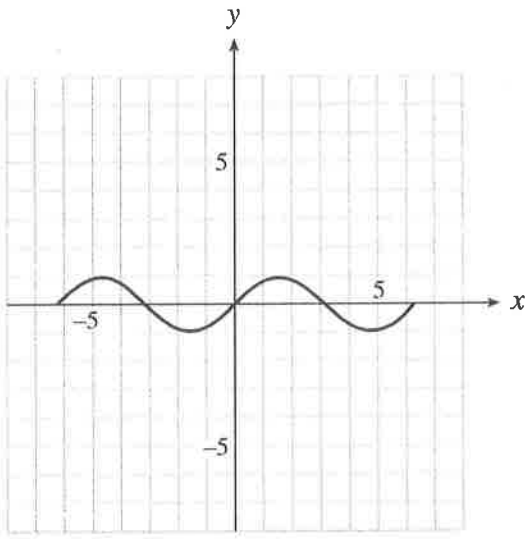


D.

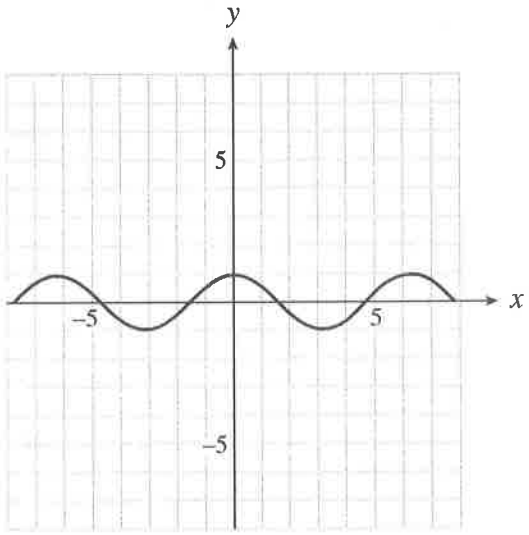


29. For which of the following functions is $f(-x) = f(x)$?

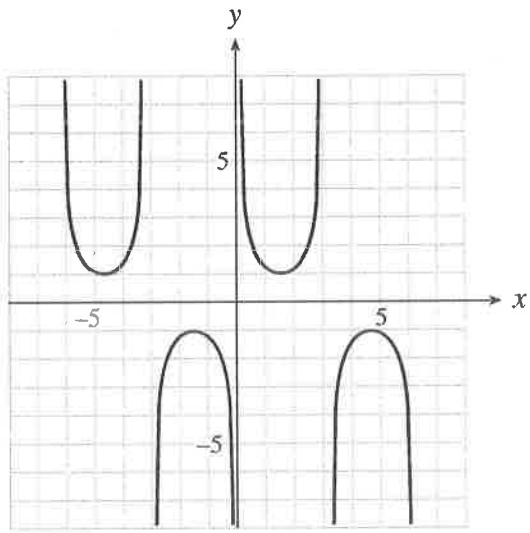
A.



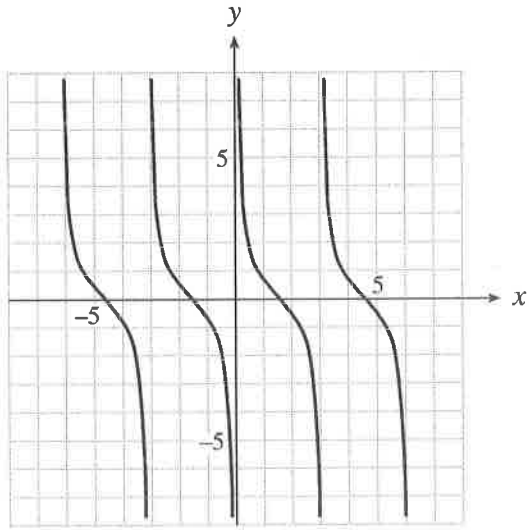
B.



C.



D.



30. If the point $(6, 10)$ is on the graph of $y = f(x)$, which point must be on the graph of $y = \frac{1}{2f(x)}$?

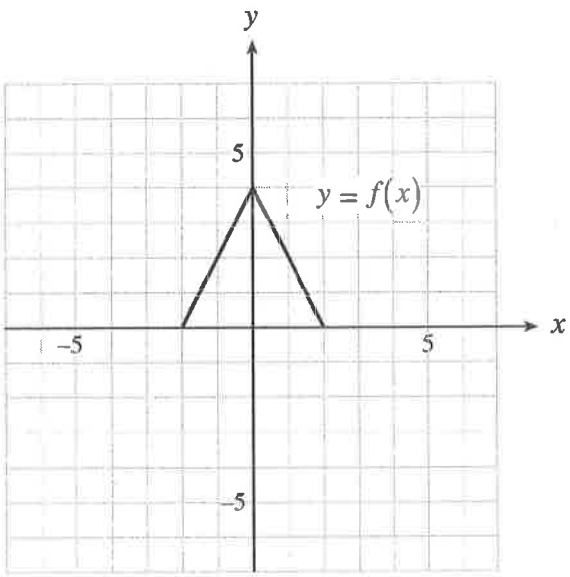
A. $\left(3, \frac{1}{10}\right)$

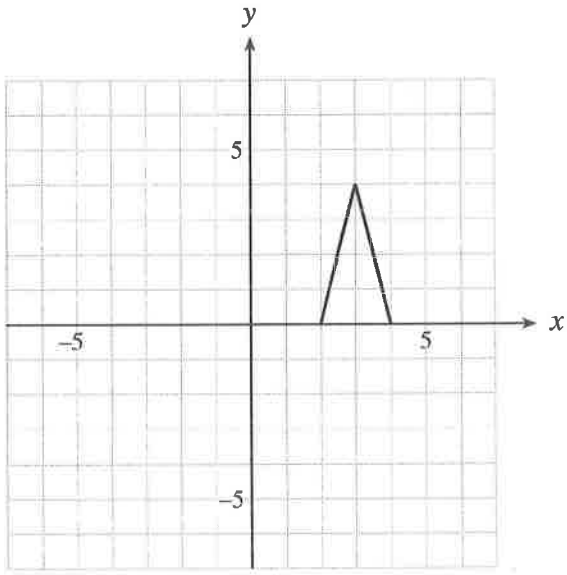
B. $\left(6, \frac{1}{5}\right)$

C. $\left(6, \frac{1}{10}\right)$

D. $\left(6, \frac{1}{20}\right)$

31. Given the graph of the function $y = f(x)$ on the left, determine the equation of the function on the right.





A. $y = f\left(\frac{x}{2} - 3\right)$

B. $y = f\left(\frac{x-3}{2}\right)$

C. $y = f(2x - 3)$

D. $y = f(2x - 6)$

32. Evaluate: $\frac{200!}{198!}$

A. 2
B. 200
C. 39 800
D. infinity

33. A postal code consists of three letters and three digits arranged with a letter first, then a digit, a letter, then a digit, and a letter and a digit. If the first letter must be V, W or X and there are no other restrictions on the other letters or digits, determine how many different postal codes are possible. (An example of a postal code is V0N 5Y2.)

A. 1 259 712
B. 1 478 412
C. 1 728 000
D. 2 028 000

34. Determine the coefficient of the 3rd term in the expansion of $(x+2y)^7$.

A. 21
B. 35
C. 84
D. 140

35. Determine the number of different arrangements of all the letters in the word BALLOON.

A. 210
B. 1260
C. 2520
D. 5040

36. A class of 34 students consists of 20 girls and 14 boys. How many different committees of 5 girls and 3 boys can be formed from this class?

A. 2 282 280
B. 5 643 456
C. 18 643 456
D. 40 632 288 320

37. Two cards are drawn without replacement from a standard deck of 52 cards.
What is the probability that the first card is a heart and the second card is a black face card?

A. $\frac{3}{104}$

B. $\frac{1}{34}$

C. $\frac{1}{17}$

D. $\frac{13}{102}$

Use the following information and table to answer questions 38 and 39.

A survey of 750 students conducted in two schools showed the following results:

	Students who have a graphing calculator	Students who do not have a graphing calculator	Total
School A	250	50	300
School B	300	150	450
Total	550	200	750

38. Calculate the probability that a randomly chosen student from this group of 750 students has a graphing calculator and is from school A.

A. $\frac{1}{3}$

B. $\frac{2}{5}$

C. $\frac{5}{11}$

D. $\frac{4}{5}$

39. Calculate the probability that a randomly chosen student from this group of 750 students has a graphing calculator, given that the student is from school A.

A. $\frac{2}{5}$

B. $\frac{5}{11}$

C. $\frac{11}{15}$

D. $\frac{5}{6}$

40. When 32 students wrote a test, they achieved a mean score of 53 and a standard deviation of 10.4. If the teacher added 10 marks to each student's score, determine the standard deviation of the new scores for these 32 students.

A. 10.4
B. 11.4
C. 13.4
D. 20.4

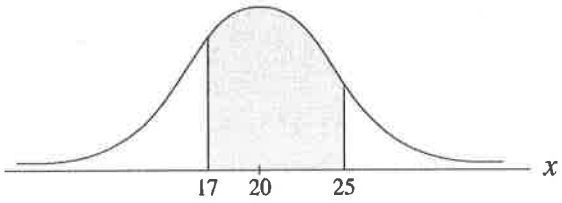
41. Determine a simplified expression for the mean of the population $a-3,\ a,\ a+6$.

A. $3a$
B. $3a+1$
C. $a+1$
D. $a+3$

42. The probability distribution for the number of times a four appears when a fair, six-sided die is rolled 720 times is a binomial distribution. What is the standard deviation of this distribution?

A. 10
B. $\sqrt{135}$
C. 100
D. 135

43. The diagram below shows a normal distribution with mean 20 and standard deviation 5. Determine $P(17 < X < 25)$.



A. 0.51
B. 0.57
C. 0.59
D. 0.62

44. Eggs chosen at random are normally distributed with a mean length of 6 cm and a standard deviation of 0.3 cm. If the longest 9% of the eggs are classified as premium eggs, what is the length of the shortest premium egg?

A. 5.6 cm
B. 5.7 cm
C. 6.3 cm
D. 6.4 cm

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

PART B: WRITTEN RESPONSE

Value: 24 marks **Suggested Time: 30 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

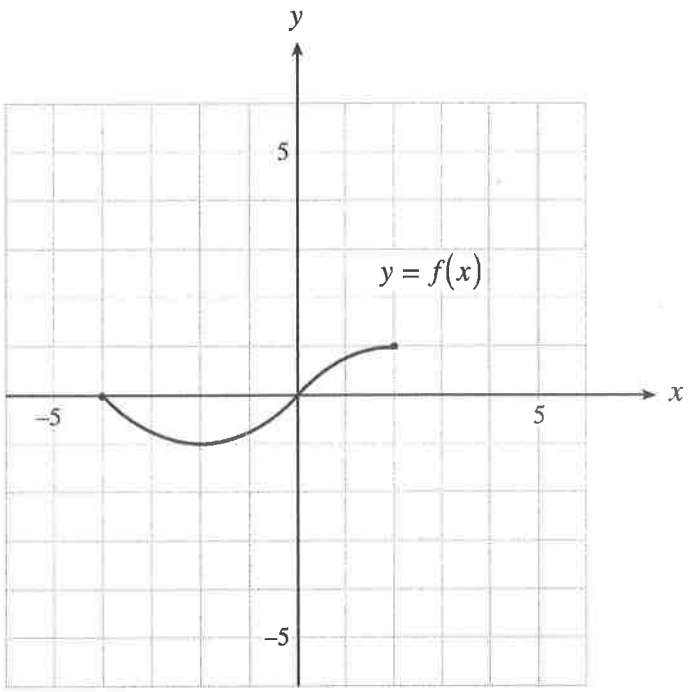
If, in a justification, you refer to information produced by the graphing calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem it is important to sketch the graph, showing its general shape and indicating the appropriate values. If the statistical features of the calculator are used, it is important to show the function with the substitution of the relevant numbers. For example: in part of the solution it is acceptable to show normalcdf (10, 40, 50, 20) or the equivalent syntax for the calculator used.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

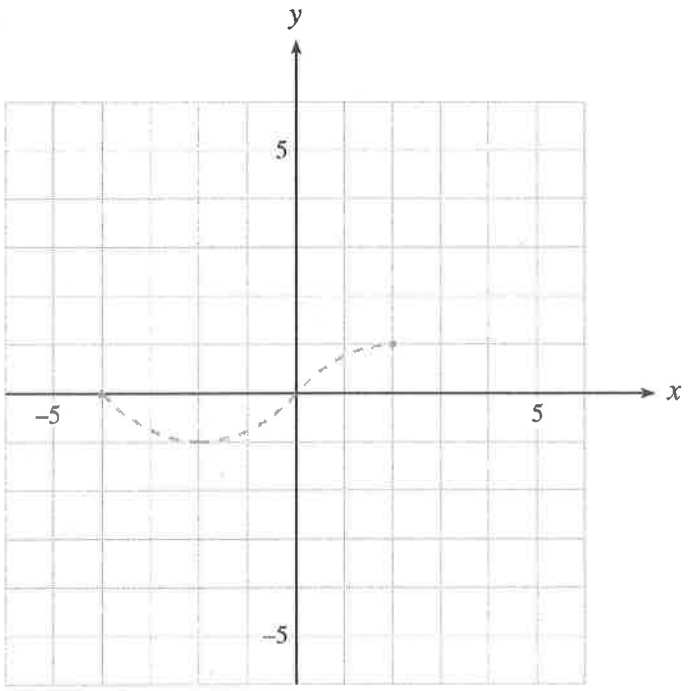
Full marks will NOT be given for the final answer only.

Use the following graph to answer questions 1 and 2.

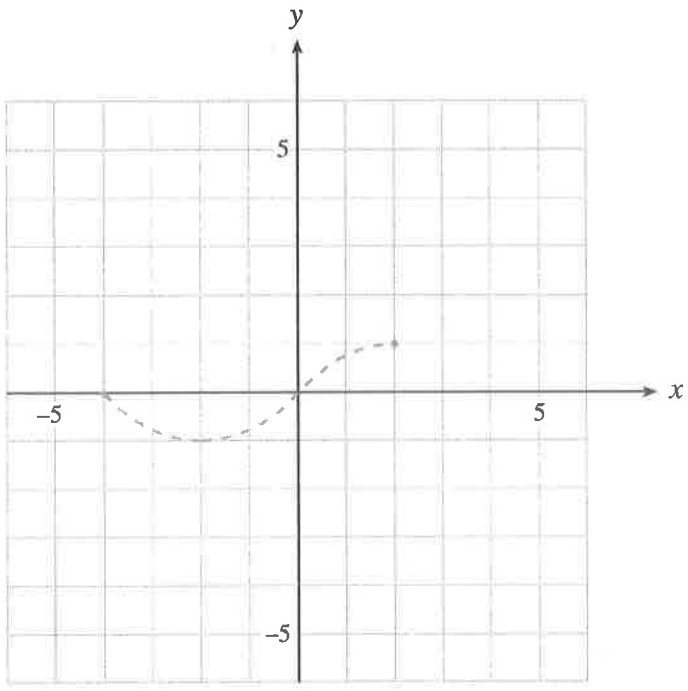
The graph of $y = f(x)$ is shown below.



1. On the grid provided, sketch the graph of $y = 3f(x) + 1$. **(2 marks)**



2. On the grid provided, sketch the graph of $y = \frac{1}{f(x)}$. **(2 marks)**



3. Solve algebraically: $2\log_3(x+4) - \log_3(-x) = 2$ **(5 marks)**

ANSWER:

Use the following information to answer questions 4 to 6.

A biased (weighted) coin is designed so that the probability of a head on each flip is $\frac{3}{5}$.

4. If this biased coin is flipped 3 times, what is the probability that the first 2 flips are tails and the third flip is a head? **(1 mark)**

ANSWER:

5. If this biased coin is flipped until exactly 2 heads appear, what is the probability that it takes exactly 3 flips until the second head appears? **(2 marks)**

ANSWER:

6. If this biased coin is flipped 7 times, what is the probability that exactly 3 or 4 heads appear? **(2 marks)**

ANSWER:

Use the following equation to answer questions 7 and 8.

$$2\cos^2 x + 3\cos x + 1 = 0$$

7. Solve the equation algebraically, giving exact values for x, where $0 \leq x < 2\pi$. **(4 marks)**

ANSWER:

8. Give the general solution for this equation.
(Solve over the set of real numbers giving exact value solutions.) **(1 mark)**

ANSWER:

9. Prove the identity. **(5 marks)**

$$\cos 2x = \frac{\cot x - \sin 2x}{\cot x}$$

LEFT SIDE	RIGHT SIDE