BRITISH
COLUMBIA
Ministry of Education

Principles of Mathematics 12
Examination Booklet
August 2006
Form A

* No POLYNOMIALS (Unit 2)

* minimal RATIONALS (Unit 3)

* No RADICALS (Unit 3)

DO NOT OPEN ANY EXAMINATION MATERIALS UNTIL INSTRUCTED TO DO SO.

FOR FURTHER INSTRUCTIONS REFER TO THE RESPONSE BOOKLET.

Suggested Time: 35 minutes
Value: 24 marks   **Allowable Time: 45 minutes**

INSTRUCTIONS: **No calculator may be used for this section of the examination.** For each question, select the **best** answer and record your choice on the **blue Answer Sheet** provided. Using an HB pencil, completely fill in the bubble that has the letter corresponding to your answer.

You have **Examination Booklet Form A**. In the box above #1 on your **Answer Sheet**, fill in the bubble as follows.

Exam Booklet Form/ Cahier d'examen   A ● B ○ C ○ D ○ E ○ F ○ G ○ H ○

1. Determine the amplitude of the function $y = -4\cos(x-2)$.

   A. -4
   B. -2
   C. 2
   D. 4

2. A circle has a radius of 12 cm. If the central angle is $45°$, as shown in the diagram, determine the length of arc AB.
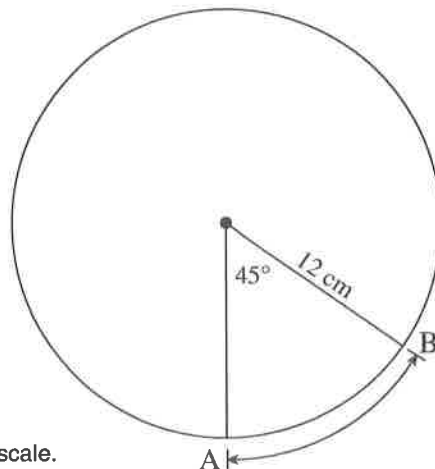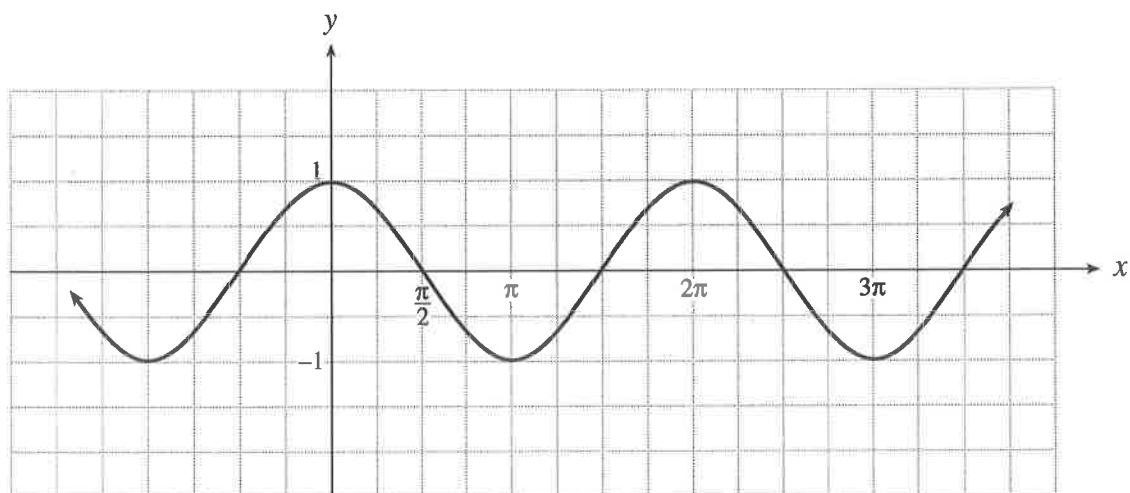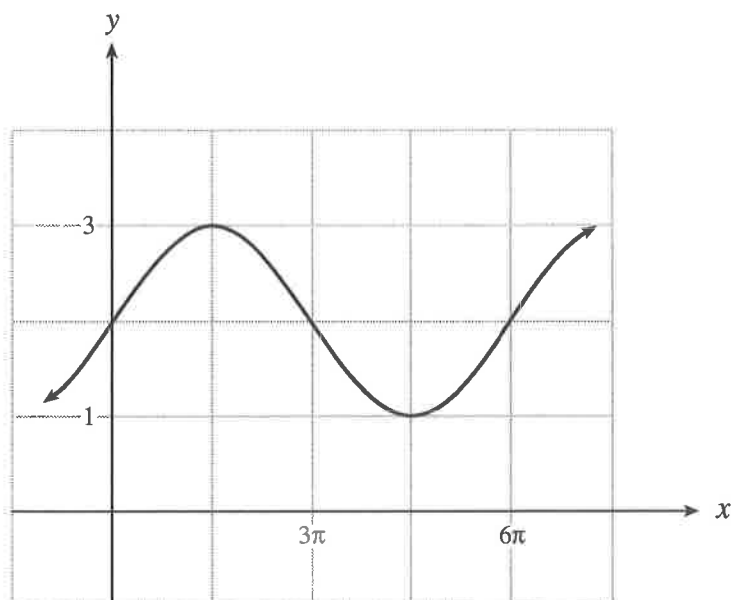

   Diagram not drawn to scale.

   A. 2π cm
   B. 3π cm
   C. 4π cm
   D. 6π cm

3. Which equation represents the function graphed below?

A. $y = \cos\left(x + \dfrac{\pi}{2}\right)$

B. $y = \sin\left(x - \dfrac{\pi}{2}\right)$

C. $y = -\cos\left(x - \dfrac{\pi}{2}\right)$

D. $y = -\sin\left(x - \dfrac{\pi}{2}\right)$

4. If the graph of the function shown below has the equation $y = a \sin bx + d$, determine the value of b $(b > 0)$.

A. $\dfrac{1}{3}$

B. 3

C. 2π

D. 6π

5. Determine an equivalent expression to $\sin(2x - \pi)$.

A. $2 \sin x \cos x$
B. $-2 \sin x \cos x$
C. $\cos^2 x - \sin^2 x$
D. $\sin^2 x - \cos^2 x$

6. Determine the number of solutions for $(a \sin x - b)(a \cos x - a)(b \sin x + a) = 0$ where $0 \le x < 2\pi$, if $0 < a < b$.

A. 3
B. 4
C. 5
D. 6

7. Change $\log_a p = t$ to exponential form.

    A. $p^t = a$

    B. $a^t = p$

    C. $a^p = t$

    D. $t^p = a$

8. Determine an equivalent expression for $\log a + 2\log b - 3\log c$.

    A. $\log \dfrac{ab^2}{c^3}$

    B. $\log \dfrac{a}{b^2 c^3}$

    C. $\log \dfrac{a}{6bc}$

    D. $\log \dfrac{2ab}{3c}$

9. Solve: $\log_5(3x) - \log_5(x-3) = 2$

    A. -6

    B. $-\dfrac{1}{2}$

    C. $\dfrac{75}{22}$

    D. 11

10. Solve: $9^{x+2} = \left(3^{4x-3}\right)\left(3^5\right)$

    A. 0

    B. 1

    C. $\dfrac{17}{19}$

    D. $\dfrac{19}{18}$

11. Determine the radius of the circle $16x^2 + 16y^2 = 25$.

    A. $\dfrac{5}{4}$

    B. $\dfrac{25}{16}$

    C. 5

    D. 25

14. Determine a possible value for D such that $x^2 + y^2 + Dx - 6y - 4 = 0$ represents a circle with radius 7.

   A. 6
   B. 12
   C. 18
   D. 36

15. Which equation represents the graph of $\dfrac{(x-2)^2}{4} + \dfrac{(y-3)^2}{9} = 1$ after it is translated 5 units to the right and 1 unit up?

   A. $\dfrac{(x-7)^2}{4} + \dfrac{(y-4)^2}{9} = 1$

   B. $\dfrac{(x-7)^2}{4} + \dfrac{(y-2)^2}{9} = 1$

   C. $\dfrac{(x+3)^2}{4} + \dfrac{(y-4)^2}{9} = 1$

   D. $\dfrac{(x+3)^2}{4} + \dfrac{(y-2)^2}{9} = 1$

16. Which equation represents the graph of $y = 2^x$ after it is reflected in the x-axis?

   A. $y = 2^{-x}$
   B. $y = -2^x$
   C. $y = \log_2 x$
   D. $y = -\log_2 x$

This is the end of Part A, Section I.
You may proceed to the rest of the examination *without* the use of a calculator until directed by the supervisor to access your calculator. At the end of 45 minutes, you will not be able to go back to Part A, Section I; therefore, ensure you have checked this section.

Value: 42 marks   **Suggested Time: 55 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the **white Answer Sheet** provided. Using an HB pencil, completely fill in the bubble that has the letter corresponding to your answer.

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17. How is the graph of $y = f(4x)$ related to the graph of $y = f(x)$?

   A. $y = f(x)$ has been compressed vertically by a factor of $\frac{1}{4}$.

   B. $y = f(x)$ has been compressed horizontally by a factor of $\frac{1}{4}$.

   C. $y = f(x)$ has been expanded vertically by a factor of 4.

   D. $y = f(x)$ has been expanded horizontally by a factor of 4.

18. If the maximum value of the function $y = f(x)$ is 6, determine the maximum value of $y = \frac{1}{3} f\left(\frac{1}{2}x\right)$.

   A. 2
   B. 3
   C. 12
   D. 18

19. If the point $(-2, -5)$ is on the graph of $y = f(x)$, which point must be on the graph of $y = |f(x-1)| - 3$?

   A. $(-3, 2)$
   B. $(-1, 2)$
   C. $(1, -8)$
   D. $(3, -8)$

20. Which conic is formed by the intersection of a plane and a double-napped cone when the intersecting plane is parallel to the generator of the cone?

   A. circle
   B. ellipse
   C. parabola
   D. hyperbola

22. Let θ be an angle in standard position such that $\cot\theta = -\dfrac{4}{3}$ and $\sin\theta < 0$.

Determine the exact value of $\sec\theta$.

A. $-\dfrac{5}{3}$

B. $-\dfrac{5}{4}$

C. $\dfrac{5}{4}$

D. $\dfrac{5}{3}$

23. A wheel rolling along the ground has a diameter of 16 cm and rotates every 12 seconds. At time $t = 0$ s, a point P on the outside edge of the wheel is at its highest point. Determine a cosine function that gives the height, h, of point P above the ground at any time, t, where h is in cm and t is in seconds.

A. $h(t) = -8\cos\dfrac{\pi}{6}t + 8$

B. $h(t) = 8\cos\dfrac{\pi}{12}t + 8$

C. $h(t) = 8\cos\dfrac{\pi}{6}t + 8$

D. $h(t) = -8\cos\dfrac{\pi}{12}t + 8$

24. Determine the sum of the infinite geometric series $3 + \dfrac{6}{5} + \dfrac{12}{25} + \ldots$

A. 5

B. 7.5

C. 12.5

D. 15

25. Determine the sum of the first 10 terms of the geometric series $-36 + 24 - 16 + \ldots$

A. −21.23

B. −21.60

C. −21.97

D. −22.16

26. What is the first term of the geometric series defined by $\sum_{k=3}^{7} 5\left(-\dfrac{1}{2}\right)^{k-1}$?

A. $-\dfrac{5}{2}$

B. $-\dfrac{5}{8}$

C. $\dfrac{5}{4}$

D. 5

27. A ball is dropped from a height of 4 m. After each bounce, the ball rises to 70% of its previous height. What is the maximum height the ball will reach after it hits the ground for the 5th time?

A. 0.47 m
B. 0.67 m
C. 0.96 m
D. 11.09 m

28. If x, 6, $3x$ are three consecutive terms in a geometric sequence, determine the values of x.

A. ± 1
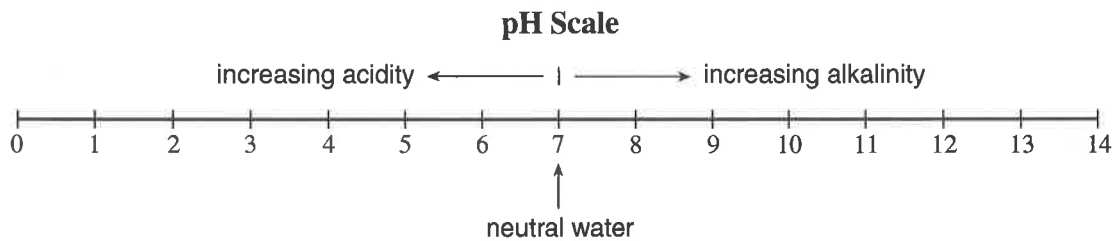B. $\pm\sqrt{3}$
C. ± 2
D. $\pm 2\sqrt{3}$

29. A population grows continuously according to the formula $P = P_0 e^{kt}$, where P is the final population in t years, P_0 is the initial population and k is the continuous growth rate. What will be the population in 7 years if the initial population is 25 000 and the continuous growth rate is 1.2%?

A. 27 191
B. 57 909
C. 177 113
D. 197 312

30. In chemistry, the pH scale measures the acidity (0–7) or alkalinity (7–14) of a solution. It is a logarithmic scale in base 10. Thus a pH of 5 is 10 times more **acidic** than a pH of 6.

Solution A has a pH of 5.7. Solution B is 1260 times more **acidic** than Solution A. Find the pH of solution B.

pH Scale

increasing acidity ← | → increasing alkalinity

0   1   2   3   4   5   6   7   8   9   10   11   12   13   14

neutral water

A. 2.6
B. 4.4
C. 7.0
D. 8.8

31. How many permutations are there using all of the letters in the word P E P P E R ?

A. 60
B. 120
C. 360
D. 720

32. In a particular city, all of the streets run continuously north-south or east-west. The mayor lives 4 blocks east and 5 blocks north of city hall. Determine the number of different routes, 9 blocks in length, that the mayor can take to get to city hall.
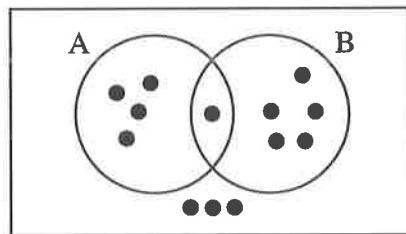
A. 20
B. 126
C. 3 024
D. 15 120

33. In the expansion of $(x + y)^{10}$, determine the coefficient of the term containing $x^8 y^2$.

A. 9
B. 10
C. 36
D. 45

34. In a standard deck of 52 cards, how many different 4-card hands are there that contain at most one heart?

    A.   91 403
    B.   118 807
    C.   188 474
    D.   201 058

35. In a library, 4 different English books, 2 different Chemistry books and 3 different Mathematics books are arranged on a shelf. Determine the number of different arrangements for the 9 books if the books on each subject must be kept together.

    A.   24
    B.   288
    C. 1260
    D. 1728

36. The diagram shows a sample space of 13 equally likely outcomes.

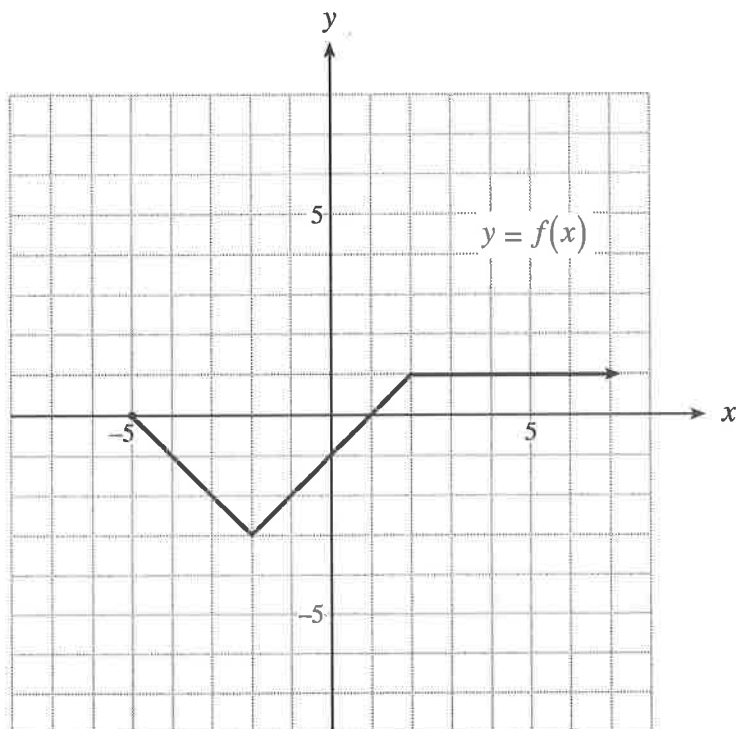Determine $P(\text{B})$.

    A. $\dfrac{5}{13}$

    B. $\dfrac{6}{13}$

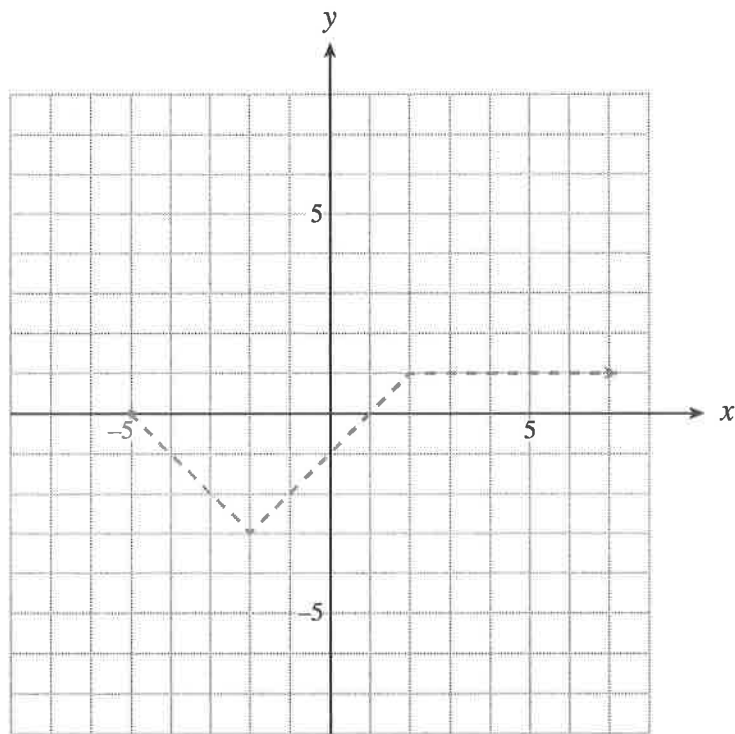   C. $\dfrac{5}{10}$

    D. $\dfrac{6}{10}$

Use the following graph to answer questions 1 and 2.

The graph of $y = f(x)$ is shown below.

1. On the grid provided, sketch the graph of $y = 2|f(x)| + 1$. **(3 marks)**

2. On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(2 marks)**
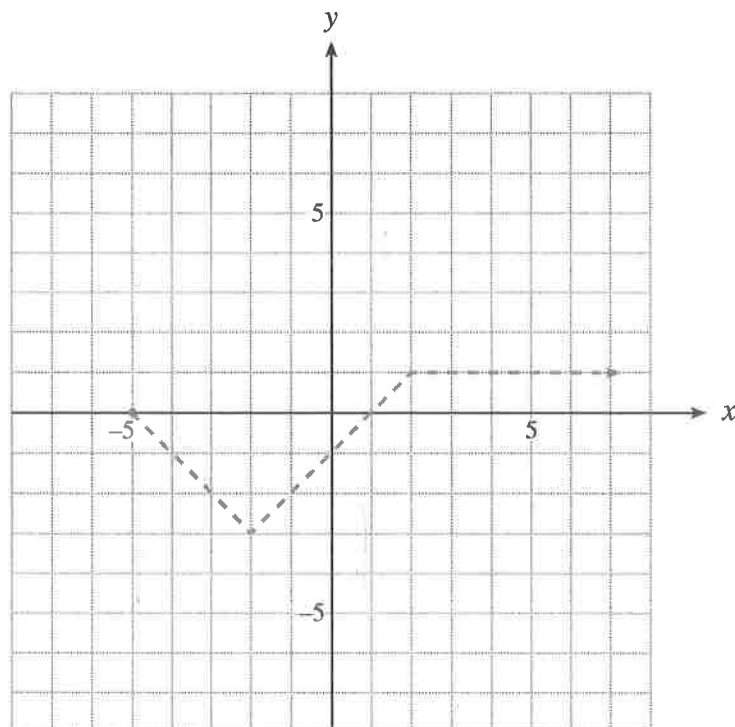
3. A radioactive substance has a half-life of 17 d. How long will it take for 300 g of this substance to decay to 95 g?

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

(5 marks)

5. Determine the general solution algebraically. (Solve over the set of real numbers.)

$$3\cos^2 x - 8\cos x + 4 \doteq 0$$

(Answer accurate to at least 2 decimal places.) **(4 marks)**

6. Prove the identity: **(5 marks)**

$$\frac{\tan x(\cos x + \cot x)}{\sec x + \tan x} = \frac{\sin x \sin 2x}{2 - 2\cos^2 x}$$

LEFT SIDE	RIGHT SIDE

Principles of Mathematics 12
August 2006 — Form A
Provincial Examination — Answer Key

Cognitive Processes	Weightings	Question Type
K = Knowledge	10%	**44** = Multiple Choice (MC)
U = Understanding	70%	**6** = Written Response (WR)
H = Higher Mental Processes	20%	

Topics	Prescribed Learning Outcomes (PLOs)		
1. Problem Solving	A	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B, C, D	B	Geometric Sequences and Series
3. Shape and Space	E, F	C/D	Logarithms and Exponents
4. Statistics and Probability	G	C/D	Trigonometry
		E	Conics
		F	Transformations
		G	Combinatorics
		G	Probability
		G	Statistics

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	D	K	1.5	2	D6	MC
2.	B	U	1.5	2	C3	MC
3.	D	U	1.5	2	D6	MC
4.	A	U	1.5	2	D6	MC
5.	B	U	1.5	2	C8	MC
6.	A	H	1.5	2	C5	MC
7.	B	K	1.5	2	D2	MC
8.	A	U	1.5	2	D4	MC
9.	C	U	1.5	2	C2	MC
10.	B	H	1.5	2	C1	MC
11.	A	U	1.5	3	E2	MC
12.	A	U	1.5	3	E2	MC
13.	D	H	1.5	3	E2	MC
14.	B	H	1.5	3	E3	MC
15.	A	U	1.5	3	F1	MC
16.	B	U	1.5	3	F3	MC
17.	B	K	1.5	3	F2	MC
18.	A	U	1.5	3	F2	MC
19.	B	H	1.5	3	F6	MC
20.	C	K	1.5	3	E1	MC
21.	D	U	1.5	3, 1	E2; F3; A2	MC
22.	C	U	1.5	2	D5	MC
23.	C	H	1.5	1, 2	A9, D7	MC

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
24.	A	U	1.5	2	B3	MC
25.	A	U	1.5	2	B1	MC
26.	C	U	1.5	2	B1	MC
27.	B	U	1.5	2	B1	MC
28.	D	H	1.5	2	B1	MC
29.	A	U	1.5	2	D3	MC
30.	A	H	1.5	2	D3	MC
31.	A	U	1.5	4	G6	MC
32.	B	U	1.5	4	G4	MC
33.	D	U	1.5	4	G8	MC
34.	D	U	1.5	4	G7	MC
35.	D	H	1.5	4	G5, G6	MC
36.	B	U	1.5	4	G10	MC
37.	C	U	1.5	4	G13	MC
38.	D	H	1.5	4	G8	MC
39.	C	U	1.5	4	G2	MC
40.	C	U	1.5	4	G1	MC
41.	B	H	1.5	4	G2	MC
42.	D	U	1.5	4	G2	MC
43.	C	U	1.5	4	G2	MC
44.	B	U	1.5	4	G3	MC

Principles of Mathematics 12
August 2006
Provincial Examination — Written-Response Key / Scoring Guide

Cognitive Processes	Weightings	Question Types
K = Knowledge	10%	**44** = Multiple Choice (MC)
U = Understanding	70%	**6** = Written Response (WR)
H = Higher Mental Processes	20%	

Topics	Prescribed Learning Outcomes (PLOs)		
1. Problem Solving	A	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B, C, D	B	Geometric Sequences and Series
3. Shape and Space	E, F	C/D	Logarithms and Exponents
4. Statistics and Probability	G	C/D	Trigonometry
		E	Conics
		F	Transformations
		G	Combinatorics
		G	Probability
		G	Statistics

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	—	U	3	3	F5, F6	WR
2.	—	U	2	3	F4	WR
3.	—	U	5	2	D1	WR
4.	—	U	5	4	G12	WR
5.	—	U	4	2	C5, C6	WR
6.	—	H	5	2	C7, C8	WR

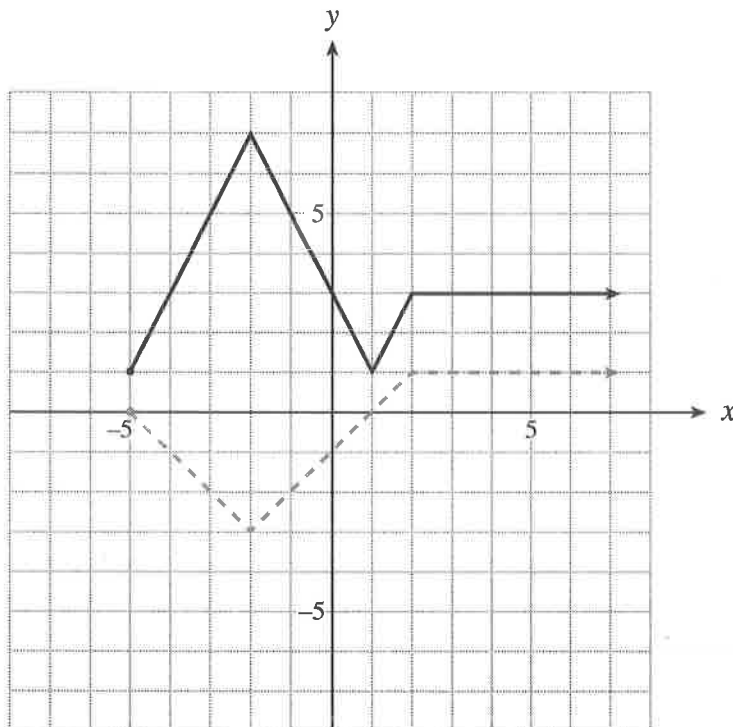**Use the following graph to answer questions 1 and 2.**

The graph of $y = f(x)$ is shown below.

$y = f(x)$
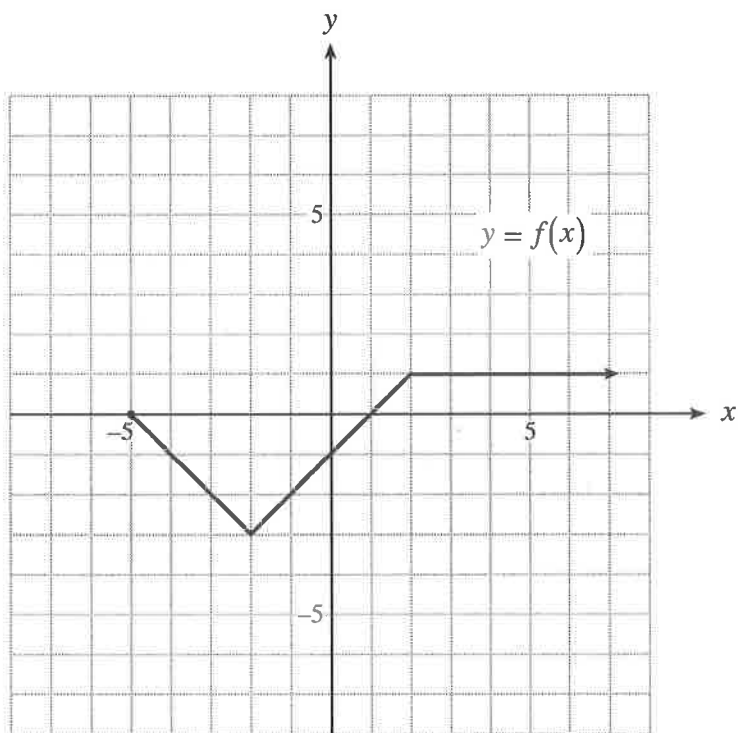
1. On the grid provided, sketch the graph of $y = 2|f(x)| + 1$. (3 marks)

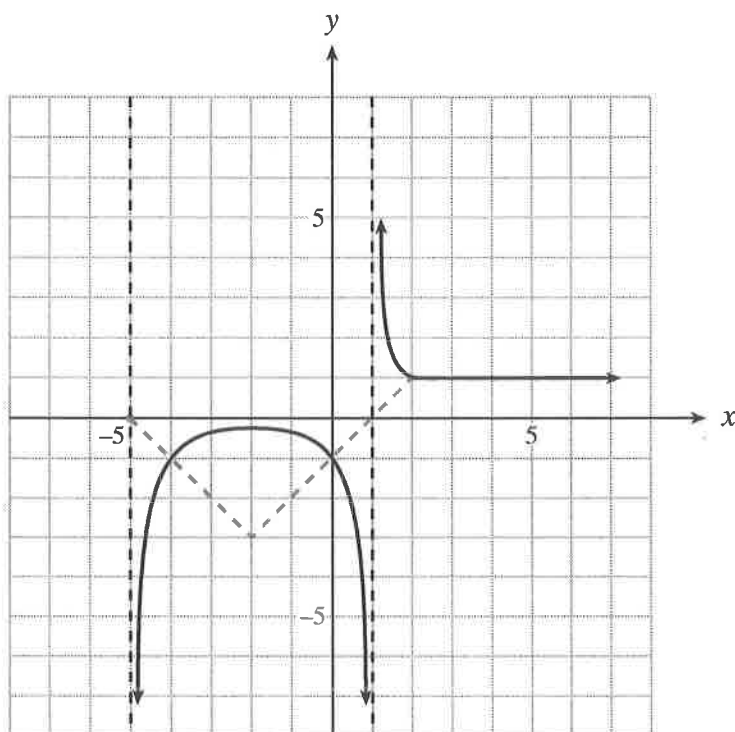**1 mark: absolute value**
1 mark: vertical expansion
1 mark: vertical translation

Note: deduct $\frac{1}{2}$ mark if graph does not stop on the left side or does not continue on the right side

2. On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(2 marks)**

$\frac{1}{2}$ **mark: invariant points – each half**

$\frac{1}{2}$ **mark: shape – each half**

3. A radioactive substance has a half-life of 17 d. How long will it take for 300 g of this substance to decay to 95 g?

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)   **(5 marks)**

solution

$$95 = 300\left(\frac{1}{2}\right)^{\frac{d}{17}} \qquad \leftarrow \textbf{2 marks}$$

$$\frac{95}{300} = \left(\frac{1}{2}\right)^{\frac{d}{17}} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\log\left(\frac{95}{300}\right) = \log\left(\frac{1}{2}\right)^{\frac{d}{17}} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\log\left(\frac{95}{300}\right) = \frac{d}{17}\log\left(\frac{1}{2}\right) \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$d = \frac{17\log\left(\frac{95}{300}\right)}{\log\left(\frac{1}{2}\right)} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$d = 28.20 \text{ days} \qquad \leftarrow \textbf{1 mark}$$

5. Determine the general solution algebraically. (Solve over the set of real numbers.)

$$3\cos^2 x - 8\cos x + 4 = 0$$

(Answer accurate to at least 2 decimal places.)   **(4 marks)**

solution

$$(3\cos x - 2)(\cos x - 2) = 0$$

$\frac{1}{2}$ **mark** $\rightarrow$ $\cos x = \dfrac{2}{3}$ $\cos x = 2$ $\leftarrow \frac{1}{2}$ **mark**

$$\Downarrow$$

$x = 0.84,\ x = 5.44$ reject $\leftarrow \frac{1}{2}$ **mark**

    $\uparrow$ $\uparrow$

 $\frac{1}{2}$ **mk**   **1 mk**

$x = 0.84 + 2n\pi,\quad x = 5.44 + 2n\pi,\quad n$ is an integer

    $\uparrow$ $\uparrow$

 $\frac{1}{2}$ **mk** $\frac{1}{2}$ **mk**

6. Prove the identity: **(5 marks)**

$$\frac{\tan x(\cos x + \cot x)}{\sec x + \tan x} = \frac{\sin x \sin 2x}{2 - 2\cos^2 x}$$

solution

LEFT SIDE	RIGHT SIDE

$$\frac{\tan x(\cos x + \cot x)}{\sec x + \tan x}$$

$$\frac{\sin x \sin 2x}{2 - 2\cos^2 x}$$

$\frac{1}{2}$ **mark** →
$$\frac{\dfrac{\sin x}{\cos x}\left(\cos x + \dfrac{\cos x}{\sin x}\right)}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$$

$\frac{1}{2}$ **mark** →

$\frac{1}{2}$ **mark** →
$$\frac{\dfrac{\sin x + 1}{1 + \sin x}}{\cos x}$$

1 **mark** → $(\sin x + 1)\left(\dfrac{\cos x}{1 + \sin x}\right)$

$\frac{1}{2}$ **mark** → $\cos x$

$$\frac{\sin x(2\sin x \cos x)}{2(1 - \cos^2 x)} \quad \leftarrow \frac{1}{2}\text{ mark}$$
$$\leftarrow \frac{1}{2}\text{ mark}$$

$$\frac{2\sin^2 x \cos x}{2\sin^2 x} \quad \leftarrow \frac{1}{2}\text{ mark}$$

$$\cos x \qquad \leftarrow \frac{1}{2}\text{ mark}$$

LS = RS

6. Prove the identity: **(5 marks)**

$$\frac{\tan x (\cos x + \cot x)}{\sec x + \tan x} = \frac{\sin x \sin 2x}{2 - 2\cos^2 x}$$

alternate solution

LEFT SIDE	RIGHT SIDE

LEFT SIDE:

$$\frac{\tan x (\cos x + \cot x)}{\sec x + \tan x}$$

$\frac{1}{2}$ **mark** → $\dfrac{\dfrac{\sin x}{\cos x}\left(\cos x + \dfrac{\cos x}{\sin x}\right)}{}$

$\frac{1}{2}$ **mark** → $\dfrac{}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$

↙ $\frac{1}{2}$ **mark**

$$\frac{\dfrac{\cos x}{\cos x}(\sin x + 1)}{\left(\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}\right)}$$

1 mark

clearing the complex fraction → $\dfrac{\cos x (\sin x + 1)}{1 + \sin x}$

$\frac{1}{2}$ **mark** → $\cos x$

RIGHT SIDE:

$$\frac{\sin x \sin 2x}{2 - 2\cos^2 x}$$

$\dfrac{\sin x (2 \sin x \cos x)}{2(1 - \cos^2 x)}$ ← $\frac{1}{2}$ **mark**
                                              ← $\frac{1}{2}$ **mark**

$\dfrac{2 \sin^2 x \cos x}{2 \sin^2 x}$ ← $\frac{1}{2}$ **mark**

$\cos x$ ← $\frac{1}{2}$ **mark**

$$LS = RS$$

END OF KEY