

Some solutions to Ch. 2.4 p 76-77

2s) $\frac{\frac{x}{x+1} + 3}{\frac{x}{x+1} + \frac{3}{x+1}} = \frac{\frac{x}{x+1} + \frac{3(x+1)}{x+1}}{\frac{x}{x+1} + \frac{3(x+1)}{x}}$

$= \frac{4x+3}{x+1}$

$\frac{x^2 + 3(x+1)^2}{(x+1)(x)} \rightarrow x^2 + 2x + 1$

$= \left(\frac{4x+3}{x+1} \right) \cdot \left(\frac{x(x+1)}{4x^2+6x+3} \right)$

$= \frac{(4x+3)(x)}{4x^2+6x+3}$

$\frac{x}{12} + \frac{1}{6}$
cannot be factored

t) $3 - \frac{x}{3 - \frac{x}{3-x}} = 3 - \frac{x}{\frac{3(3-x)}{3-x} - \frac{x}{3-x}}$

$= 3 - \frac{x}{\frac{-4x+9}{3-x}} = 3 - x \frac{(3-x)}{(-4x+9)}$

$= 3 - \left(\frac{(-x^2+3x)}{(-4x+9)} \right)$

$= \frac{3(-4x+9) + x^2 - 3x}{-4x+9}$

$= \frac{-12x+27+x^2-3x}{-4x+9}$

$= \frac{x^2-15x+27}{9-4x}$

$$3a) \frac{x^{-2} + x}{x} = \frac{\frac{1}{x^2} + x}{x} = \frac{\frac{1 + x^3}{x^2}}{x} = \left(\frac{1 + x^3}{x^2}\right) \cdot \frac{1}{x} = \boxed{\frac{1 + x^3}{x^3}}$$

$$b) \frac{x^{-2} - 3x^{-3}}{3x^{-2} - 9x^{-3}} = \frac{\frac{1}{x^2} - \frac{3}{x^3}}{\frac{3}{x^2} - \frac{9}{x^3}} = \frac{\frac{x-3}{x^3}}{\frac{3x-9}{x^3}} \rightarrow = \left(\frac{x-3}{x^3}\right) \left(\frac{x^3}{3x-9}\right) = \frac{x-3}{3(x-3)} = \boxed{\frac{1}{3}}$$

$$c) (x^{-1} + y^{-1})^{-1} = \left(\frac{1}{x} + \frac{1}{y}\right)^{-1} = \left(\frac{y+x}{xy}\right)^{-1} \rightarrow = \frac{1}{\frac{y+x}{xy}} = 1 \left(\frac{xy}{y+x}\right) = \boxed{\frac{xy}{y+x}}$$

$$d) (x^{-1} - y^{-1})^{-2} = \left(\frac{1}{x} - \frac{1}{y}\right)^{-2} = \left(\frac{y-x}{xy}\right)^{-2} \rightarrow = \frac{1}{\left(\frac{y-x}{xy}\right)^2} = \boxed{\frac{x^2 y^2}{x^2 - 2xy + y^2}}$$

$$4. T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} = \boxed{\frac{R_1 R_2}{R_2 + R_1}}$$

$$5. i = \frac{E}{R + \frac{r}{2}} \Rightarrow i = \frac{E}{\frac{2R + r}{2}} \Rightarrow i = \frac{2E}{2R + r}$$

$$\hookrightarrow (2R + r)i = 2E \Rightarrow 2R + r = \frac{2E}{i} \Rightarrow 2R = \frac{2E}{i} - r$$

$$\hookrightarrow 2R = \frac{2E - ri}{i} \Rightarrow \boxed{R = \frac{2E - ri}{2i}}$$

$$r = \frac{2E}{i} - 2R$$

$$\boxed{r = \frac{2E - 2Ri}{i}}$$

$$6. f^{-1} = d_i^{-1} + d_o^{-1}$$

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$1 = \frac{f}{d_i} + \frac{f}{d_o}$$

$$1 = \frac{fd_o + fd_i}{d_i d_o}$$

$$1 = \frac{f(d_o + d_i)}{d_i d_o}$$

$$\rightarrow d_i d_o = f(d_o + d_i)$$

$$\boxed{f = \frac{d_i d_o}{d_o + d_i}}$$

$$7) \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} \Rightarrow \frac{1}{\frac{x+1}{x}} = \frac{x}{x+1}$$

$$\text{Rewrite: } = \frac{1}{1 + \frac{1}{1 + \frac{x}{x+1}}} \Rightarrow \frac{1}{\frac{x+1+x}{x+1}} = \frac{1}{\frac{2x+1}{x+1}} = \frac{x+1}{2x+1}$$

$$\text{Rewrite: } = \frac{1}{1 + \frac{x+1}{2x+1}} \Rightarrow \frac{1}{\frac{2x+1+x+1}{2x+1}} = \frac{1}{\frac{3x+2}{2x+1}} = \frac{2x+1}{3x+2}$$