Pre-Calculus 12
Logarithm Review

KEY

November 1, 2012

PLEASE READ

This chapter is challenging because it requires you to solve questions in a method you are not used to using. The ***understanding*** of exponent laws and logarithm laws means that you have to be able to *use* them, not merely *recite* them. This means you will have to practice, practice, practice and even after that you will have practice some more. Unless otherwise stated, all of these questions are to be done **without** the use of a calculator. If you are using a calculator to answer the question, don't bother doing the review (IT WILL BE OF NO BENEFIT ⊗)

Have your laws of exponents/ logarithms handy and let's begin.

PART A: Simplify the following. (If you understand logs, these should take about 10 seconds each.)

1. $log_3 9 = \frac{\log 3^2}{\log 3} = \frac{2\log 3}{\log 3} \boxed{=2}$

2. $log_2 32 = \frac{5\log 2}{\log 2} \boxed{=5}$

3. $log_5 \frac{1}{125} = \frac{-3\log 5}{\log 5} \boxed{=-3}$

4. $log_2 1 = \frac{\log 2^0}{\log 2} = \boxed{0}$

5. $log_2 \sqrt[3]{4} = \frac{\log 2^{\frac{2}{3}}}{\log 2} = \boxed{\frac{2}{3}}$

6. $log_3 81 = \frac{4\log 3}{\log 3} \boxed{4}$

7. $log_8 2 = \frac{\log 2}{\log 2^3} = \boxed{\frac{1}{3}}$

8. $log_9 \frac{1}{3} = \frac{\log 3^{-1}}{\log 3^2} = \boxed{\frac{-1}{2}}$

9. $log_{\frac{1}{2}}\left(\frac{\sqrt{2}}{2}\right)$ ← Ok ... this should take you more than 10 seconds... 1 or 2 minutes maybe ???

$= \frac{\log \frac{\sqrt{2}}{2}}{\log \frac{1}{2}} = \frac{\log\sqrt{2} - \log 2}{\log 1 - \log 2} = \frac{\frac{1}{2}\log 2 - \log 2}{-\log 2} = \frac{-\frac{1}{2}\log 2}{-\log 2} = \boxed{\frac{1}{2}}$

PART B: Determine the properties of the following exponential and logarithmic functions.

10. Find the domain, range, equation of asymptote, x-intercepts and y-intercepts.

a. $y = 2^{x+1} - 3$
b. $y - 2 = -3^x$
c. $y = log_2(x-3)$
d. $y = log_3(2-x)$
e. $y = -log_5(2x-1)$
f. $y = 5 + log_2(-x)$

	DOMAIN	RANGE	EQ. ASYPTOTE	X-INTERCEPT	Y-INTERCEPT
a)	$x \in \mathbb{R}$	$y > -3$	$y = 3$	$x = \log_2 3 - 1$	$y = -1$ $(0,-1)$
b)	$x \in \mathbb{R}$	$y < 2$	$y = 2$	$\frac{\log 2}{\log 3} = (\log_3 2, 0)$	$y = 1$ $(0,1)$
c)	$x > 3$	$y \in \mathbb{R}$	$x = 3$	$x = 4$ $(4,0)$	NONE
d)	$x < 2$	$y \in \mathbb{R}$	$x = 2$	$x = 1$ $(1,0)$	$\log_3 2 = \frac{\log 2}{\log 3}$
e)	$x > \frac{1}{2}$	$y \in \mathbb{R}$	$x = \frac{1}{2}$	$x = 1$ $(1,0)$	NONE
f)	$x < 0$	$y \in \mathbb{R}$	$x = 0$	$x = \frac{-1}{32}$ $(\frac{-1}{32}, 0)$	NONE

11. Determine the restriction for $f(x) = log_x(x+3)$. $x \neq 1$

$x > -3$ but $x > 0$ so $\boxed{x > 0,\ x \neq 1}$

12. Determine the restriction for $g(x) = log_{x-1}(5-x)$ $x - 1 \neq 1;\ x \neq 2$

$x < 5$ but $x - 1 > 0 \Rightarrow x > 1$ $\boxed{1 < x < 5\ (x \neq 2)}$

13. What two integers will $\log 666$ be between? (Remember... no Kalkulader.. Use Grey Matter)

2 and 3

$\log 100 = 2$ $\log 1000 = 3$

PART C: Use the laws of logs to simplify the following expressions.

14. $log_2 4^3$ $= \frac{\log 4^3}{\log 2}$ $= \frac{6 \log 2}{\log 2} = \boxed{6}$

15. $log_3 \sqrt[3]{9}$ $= \frac{\log 3^{\frac{2}{3}}}{\log 3}$ $\boxed{= \frac{2}{3}}$

16. $log_2 20 - log_2 5$ $= \log_2 (4) = \frac{2 \log 2}{\log 2} = \boxed{2}$

17. $log_3 6 + log_3 12 - log_3 8$

$= \log_3 \left(\frac{6 \cdot 12}{8}\right) = \log_3 9 = \boxed{2}$

18. $2log_2 10 - 3log_2 5$

$= \log_2 100 - \log_2 125 = \log_2 4 - \log_2 5$

$= \log_2 \left(\frac{100}{125}\right)$

$= \log_2 \left(\frac{4}{5}\right)$

$\boxed{= 2 - \log_2 5}$

19. $log5 + log6 + log100 - log3$

$\log \left(\frac{5 \cdot 6 \cdot 100}{3}\right) = \log 1000$

$\boxed{= 3}$

20. $log_2 7 - log_2 18 - log_2 14 + 2log_2 3$

Don't forget.... No Calculator... Use your brain !!!

$\log_2 \left(\frac{\frac{7}{18}}{14}\right)(9) = \log_2 \left(\frac{7 \cdot 9}{252}\right) = \log_2 \frac{1}{4}$

$= \frac{\log 2^{-2}}{\log 2} = \boxed{-2}$

21. Simplify the following as a single quantity: $log_5(x^2 - x - 2) - [log_5(x - 2) - log_5(x + 1)]$.

$\log_5 \left(\frac{x^2 - x - 2}{\left(\frac{x-2}{x+1}\right)}\right) = \log_5 \frac{\cancel{(x-2)}(x+1)(x+1)}{\cancel{(x-2)}} = \log_5 (x+1)^2$

$\boxed{= 2 \log_5 (x+1)}$

PART D: Solving exponential equations

Use the Laws of Logs to answer and then use your calculator to round the answer to 2 decimal places.

22. Solve for x in $2 = 5^{x+1}$

$\log 2 = (x+1) \log 5$

$x + 1 = \frac{\log 2}{\log 5}$

$x = \frac{\log 2}{\log 5} - 1 = \boxed{-0.57}$

23. Solve for x in $3^x = 7^x$

$x \log 3 = x \log 7$

$x \log 3 - x \log 7 = 0$

$x (\log 3 - \log 7) = 0$

$\boxed{x = 0}$

24. Solve for x in $2^{x+1} = 5^{x+2}$

$$(x+1)\log 2 = (x+2)\log 5$$

$$x+1 = (x+2)\left(\frac{\log 5}{\log 2}\right)$$

$$x+1 = (x+2)(2.3219)$$

$$-3.6439 = 1.3219x$$

$$\boxed{x = -2.76}$$

25. Solve for x in $3 \times 2^{x+1} = 7^{x-2}$

$$\log(3 \cdot 2^{x+1}) = \log 7^{x-2}$$

$$\log 3 + (x+1)\log 2 = (x-2)\log 7$$

$$\frac{\log 3}{\log 7} + (x+1)\left(\frac{\log 2}{\log 7}\right) = x-2$$

$$0.5646 + (x+1)(0.3562) = x-2$$

$$2.5646 + 0.3562x + 0.3562 = x$$

$$2.9208 = 0.6438x$$

$$\boxed{x = 4.54}$$