

Enrichment Assignment 1

$$\begin{aligned}1. \quad A'(w) &= \frac{-2(w+h)^2 + 100(w+h) - (-2w^2 + 100w)}{h} \\&= \frac{-2(w^2 + 2wh + h^2) + 100w + 100h + 2w^2 - 100w}{h} \\&= \frac{-2w^2 - 4wh - 2h^2 + 100w + 100h + 2w^2 - 100w}{h} \\&= \frac{-4wh - 2h^2 + 100h}{h} \\&= \frac{h(-4w - 2h + 100)}{h} = \boxed{-4w - 2h + 100}\end{aligned}$$

$$2. \text{ When } h \rightarrow 0, \quad A'(w) = \boxed{-4w + 100}$$

$$3. \text{ set } A'(w) = 0$$

$$0 = -4w + 100$$

$$4w = 100$$

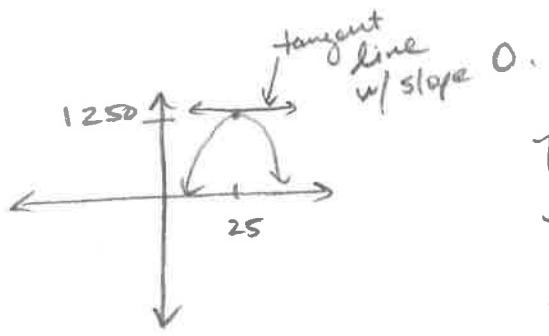
$$\boxed{w = 25}$$

4. The width of the rectangular yard is 25m.

The length, then, is 50m
 $(100 = l + 2w)$

The area (max.) is $25 \cdot 50 = 1250 \text{ m}^2$.

* with respect to calculus: over →



The w -value of 25 represents the w -value when the slope of the tangent to the curve (parabola) is zero (horizontal). The y -value here is 1250 (the max. area).

Yay, Calculus!

fyi, the short way:

$$A(w) = -2w^2 + 100w$$

$$A'w = -4w + 100$$

how?

take $-2w^2 \rightarrow$ ^{exponent}
 $2 \cdot -2$
^{coefficient} = -4
 then subtract 1
 from exponent.

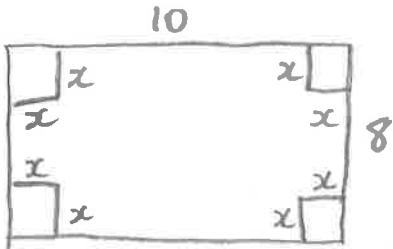
Thus $-4w$

$$\text{take } 100w \rightarrow 100w' \rightarrow 1 \cdot 100 = 100$$

$$1 - 0 = 0$$

$$100w^0 = \underline{\underline{100}}$$

$$5. \quad V(x) = x(10-2x)(8-2x)$$



$$48 = 4x^3 - 36x^2 + 80x$$

$$0 = 4x^3 - 36x^2 + 80x - 48$$

$$\begin{array}{r} | \\ 1 \quad 4 \quad -36 \quad 80 \quad -48 \\ \downarrow \quad 4 \quad -32 \quad 48 \\ \hline 4 \quad -32 \quad 48 \quad \boxed{0} \end{array}$$

* 1 works!

So, $(x-1)$ is a factor.

So is $4x^2 - 32x + 48$

$$(x-1)(4x^2 - 32x + 48) = 0$$

$$(x-1)(4)(x^2 - 8x + 12) = 0$$

$$(x-1)(x^2 - 8x + 12) = 0$$

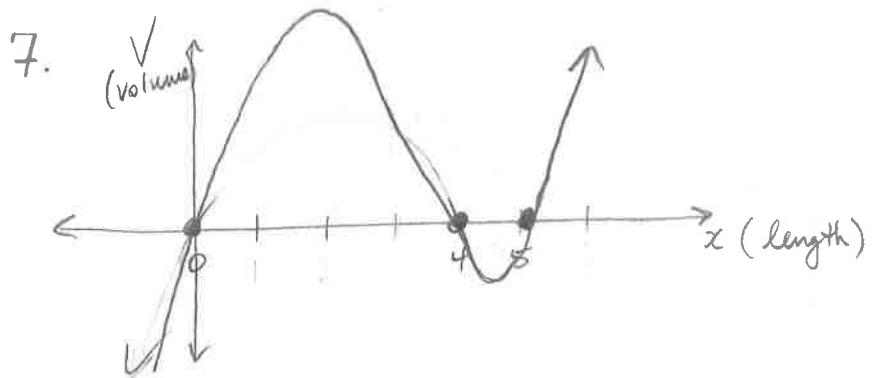
$$(x-1)(x-2)(x-6) = 0$$

$$x = 1, 2,$$

~~X~~
too large ($y: 8-2(6) = -4$)

Either $\frac{1}{\text{cm}} \times \frac{1}{\text{cm}}$ square or $\frac{2}{\text{cm}} \times \frac{2}{\text{cm}}$ square.

6. $V(x) = 4x^3 - 36x^2 + 80x$



$$\begin{aligned} V(x) &= 4x(x^2 - 9x + 20) \\ &= 4x(x-5)(x-4) \quad \text{ROOTS of } x=0, 4, 5 \end{aligned}$$

8. x -values between 0 and 4 make boxes of positive volume.

x -values between 4 and 5 make neg. volume
no solution.

x -values greater than 5 \rightarrow infinite volume.
makes no sense!

Plus... $10 - 2(5) = 0$ length.
 $8 - 2(5) = \text{neg. width.}$

$$\begin{aligned}
 9. \quad f'(x) &= \frac{f(x+h) - f(x)}{h} \\
 &= \frac{4(x+h)^3 - 36(x+h)^2 + 80(x+h) - (4x^3 - 36x^2 + 80x)}{h} \\
 &= \frac{4(x^3 + 3x^2h + 3xh^2 + h^3) - 36(x^2 + 2xh + h^2) + 80x + 80h}{h} \\
 &\quad - \frac{4x^3 + 36x^2 - 80x}{h} \\
 &= \frac{4x^3 + 12x^2h + 12xh^2 + 4h^3 - 36x^2 - 72xh - 36h^2 + 80x + 80h}{h} \\
 &\quad - \frac{4x^3 + 36x^2 - 80x}{h} \\
 &= \frac{12x^2h + 12xh^2 + 4h^3 - 72xh - 36h^2 + 80h}{h} \\
 a) \quad &= \frac{(12x^2 + 12xh + 4h^2 - 72x - 36h + 80)}{h}
 \end{aligned}$$

b) as $h \rightarrow 0 = 12x^2 - 72x + 80$

$V'(x) = 12x^2 - 72x + 80$

c) $0 = 12x^2 - 72x + 80$
 $0 = 3x^2 - 18x + 20$

$x = 1.47, 4.53$

10. 1.47 cm , not 4.53 cm
because

$$8 - 2(4.53) = \underline{\text{negative}}$$

$$\begin{aligned}\text{Max Volume} &= (1.47)(10 - 2(1.47))(8 - 2(1.47)) \\ &= (1.47)(7.06)(5.06) \\ &= \boxed{52.51 \text{ cm}^3}\end{aligned}$$