Ch. 1.4 – Defining a Power

Scenario: In a five-question T/F quiz, how many different answer sequences are possible?

$$\boxed{\underset{Q1}{2} \times \underset{Q2}{2} \times \underset{Q3}{2} \times \underset{Q4}{2} \times \underset{Q5}{2} = 32}$$

ie. multiplying a number by itself multiple times.

- powers/exponents can help!

$$\boxed{2 \times 2 \times 2 \times 2 \times 2 = 2^5 \quad (\text{"2 to the 5th power"})}$$

2^5 ← EXPONENT (POWER)

BASE → 2

$$a \cdot a \cdot a \cdot \ldots \text{(n times)} = \boxed{a^n}$$

Note: $a = \underline{a^1}$

The Product of Negative Numbers

- very important to know the <u>base</u>.

$(-x)^n \rightarrow$ base is $\underline{-x}$

$-x^n \rightarrow$ base is $\underline{x}$

BEDMAS says that 'E' (exponents) comes before 'M' (multiplication).

$(-2)^4 \rightarrow$ multiply 2 by -1 first, then raise to 4^{th} power.

$-2^4 \rightarrow$ raise 2 to 4^{th} power first, then multiply by -1.

- also, a negative number multiplied by itself an EVEN number of times results in a <u>POSITIVE</u> number.

- whereas, a negative number multiplied by itself an ODD number of times results in a <u>NEGATIVE</u> number.

eg: $(-2)\cdot(-2)\cdot(-2)\cdot(-2) = \underline{(-2)^4 = 16}$

$(-2)\cdot(-2)\cdot(-2) = \underline{(-2)^3 = -8}$

eg1: Determine if the following, when simplified, would be $\oplus$ or $\ominus$:

a) $(-1)^2$ $\underline{\oplus}$

d) -2^4 $\underline{\ominus}$

b) $(-3)^7$ $\underline{\ominus}$

e) $-(-5)^2$ $\underline{\ominus}$

c) 4^9 $\underline{\oplus}$

f) $-(-1)^9$ $\underline{\oplus}$

SUMMARY: Given $x > 0$,

$$(-x)^{\text{even}} = \oplus$$

$$(-x)^{\text{odd}} = \ominus$$

$$-x^{\text{odd or even}} = \ominus$$

One and Zero as Exponents

$$2 \times 2 \times 2 \times 2 = 2^4 = 16$$

$$2 \times 2 \times 2 = 2^3 = 8$$

$$2 \times 2 = 2^2 = 4$$

$$2 = 2^1 = 2$$

$$1 = 2^0 = 1$$

$$\frac{1}{2} = 2^{-1} = \frac{1}{2}$$

So, $a^1 = \underline{a}$, for any number a.

$a^0 = \underline{1}$, for any non-zero number a.

Note: 0^0 is <u>UNDEFINED</u>

eg 2: Evaluate: $a \neq 0$ $b \neq 0$

a) $5^0 = 1$

b) $3^1 = 3$

c) $\left(\frac{1}{4}\right)^0 = 1$

d) $(-5)^0 = 1$

e) $-5^0 = -1$

f) $0^0 =$ undefined

g) $(a+b)^0 = 1$

h) $a + b^0 = a + 1$

i) $a^0 + b^0 = 1 + 1 = 2$

j) $(a \cdot b)^0 = 1$

k) $a \cdot b^0 = a \cdot 1 = a$

* Note the following:

i) $\left(\frac{x}{y}\right)^n = \boxed{\frac{x^n}{y^n}}$

ii) $-\frac{x}{y} = \boxed{\frac{-x}{y}} = \boxed{\frac{x}{-y}}$

p. 23-26 #1-12

Ch. 1.5 – Order of Operations

Formal Rules:

1. Perform all calculations within brackets FIRST! If more than one set of brackets are 'layered' within one another, perform calculations from the innermost brackets to the outermost.

2. Evaluate all exponential expressions

3. Perform all multiplication and division IN ORDER from L to R.

4. Perform all addition and subtraction IN ORDER from L to R.

B RACKETS

E XPONENTS

D IVISION

M ULTIPLICATION

A DDITION

S UBTRACTION

eg 1: Simplify each of the following:

a) $5 + 4 \times 3 = 5 + 12 = \boxed{17}$

b) $6 - (2+3)^2 = 6 - (5)^2$

$= 6 - 25 = \boxed{-19}$

c) $(3 - 2 \times 4)^2 - \left(3 + \frac{6^2}{2}\right)$

$= (3-8)^2 - \left(3 + \frac{36}{2}\right)$

$= (-5)^2 - (3 + 18)$

$= 25 - 21 = \boxed{4}$

d) $\frac{42 - 18}{2} = \frac{24}{2} = \boxed{12}$

OR $= \frac{42}{2} - \frac{18}{2} = 21 - 9$

$= \boxed{12}$

p. 28 - 30 # 1-9

Ch.1.6 - Exponent Laws

Multiplying with Exponents

eg: $2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 2^7$

Check: $2^3 \cdot 2^4 = 2^7$

$8 \cdot 16 = 128$ ✓

Note: $3 + 4 = 7$

(the exponents of an identical base)

The Product Rule

If x is a REAL number and, m and n are INTEGERS, then:

$$x^m \cdot x^n = \underline{x^{m+n}}$$

Note: $x \neq \underline{0}$

Why? If $m \leq 0$ or $n \leq 0$, then x^m or x^n is UNDEFINED.

eg1: Simplify (but do not evaluate):

a) $3^5 \cdot 3^4 = 3^{5+4} = \boxed{3^9}$

b) $(2^2)(2^4)(2^3) = 2^{2+4+3} = \boxed{2^9}$

c) $2^5 \cdot 2^3 \cdot 3^2 = (2^{5+3})(3^2) = \boxed{2^8 \cdot 3^2}$

Common Errors while using Product Rule:

i) $2^2 \cdot 2^3 \neq 4^{2+3} = 4^5$

ii) $2^2 \cdot 2^3 \neq 2^{2 \cdot 3} = 2^6$

iii) $2^2 \cdot 2^3 \neq 4^{2 \cdot 3} = 4^6$

Dividing with Exponents

eg: $2^5 \div 2^2 = \dfrac{2^5}{2^2} = \dfrac{(\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2)}{(\cancel{2} \cdot \cancel{2})}$

$= \dfrac{2 \cdot 2 \cdot 2}{1} = 2^3$

Check: $2^5 \div 2^2 = 2^3$

$32 \div 4 = 8$ ✓

Note: $5 - 2 = 3$

The Quotient Rule

If x is a REAL number, and m and n are INTEGERS, then:

$$x^m \div x^n = \boxed{\frac{x^m}{x^n} = x^{m-n}}$$

Note: $x \neq \underline{0}$

eg2: Simplify (but do NOT evaluate):

a) $5^8 \div 5^4 = \frac{5^8}{5^4} = 5^{8-4} = \boxed{5^4}$

b) $\frac{2^7 \cdot 3^2}{2^4} = \frac{2^7}{2^4} \cdot 3^2 = 2^{7-4} \cdot 3^2$

$= \boxed{2^3 \cdot 3^2}$

c) $\frac{3^4 \cdot 3^5}{3^6} = \frac{3^{4+5}}{3^6} = \frac{3^9}{3^6} = 3^{9-6}$

$= \boxed{3^3}$

OR $= 3^{4+5-6}$

$= \boxed{3^3}$

Common Errors while using Quotient Rule:

i) $\dfrac{3^8}{3^2} \neq 1^{8-2} = 1^6$

ii) $\dfrac{3^8}{3^2} \neq 3^{8 \div 2} = 3^4$

iii) $\dfrac{3^8}{3^2} \neq 1^{8 \div 2} = 1^4$

Summary:

a) When multiplying, if the bases are the same, keep the base and ADD the exponents.

b) When dividing, if the bases are the same, keep the base and SUBTRACT the exponents.

Zero Exponent Proof → Why does $x^0 = 1$?

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

$$\frac{x^3}{x^3} = 1$$

✓

eg3: Simplify (do <u>not</u> evaluate):

a) $(-2)^5 \cdot 2^3$

$= -2^5 \cdot 2^3$

$= -1\left(2^{5+3}\right)$

$= \boxed{-2^8}$

b) $\dfrac{(-3)^8}{3^4} = \dfrac{3^8}{3^4} = 3^{8-4} = \boxed{3^4}$

c) $2^7 - 2^2 \cdot 2^3$

$= 2^7 - \left(2^{2+3}\right)$

$= \boxed{2^7 - 2^5}$

d) $3 \cdot 3^4 + \dfrac{3^5}{3^3}$

$= 3^{1+4} + 3^{5-3}$

$= \boxed{3^5 + 3^2}$

p. 34-37 #1-9

Ch. 1.7 - Power Rules

eg: $(5^3)^2$

$= (5^3)(5^3) = 5^{3+3} = 5^6$

* NOTICE: $3 \cdot 2 = \underline{\underline{6}}$

The Power Rule

For any REAL number x, and any INTEGERS m and n:

$$(x^m)^n = x^{m \cdot n} = x^{mn} \quad (x \neq 0)$$

eg1. Simplify (but do NOT evaluate):

a) $(2^4)^2 = 2^{4 \cdot 2} = \boxed{2^8}$

b) $(3^3)^5 = 3^{3 \cdot 5} = \boxed{3^{15}}$

c) $(x^2)^3 = x^{2 \cdot 3} = \boxed{x^6}$

Raising a Product to a Power

eg: $(3 \cdot 4)^2 = (3 \cdot 4) \cdot (3 \cdot 4)$

$= 3 \cdot 3 \cdot 4 \cdot 4$

$= 3^2 \cdot 4^2$

Check:

$(3 \cdot 4)^2 = 3^2 \cdot 4^2$

$(12)^2 = 9 \cdot 16$

$144 = 144$ ✓

Rule:

For any REAL number x and y, and any INTEGER n:

$$(xy)^n = \boxed{x^n y^n \quad \begin{matrix}(x \neq 0) \\ (y \neq 0)\end{matrix}}$$

eg 2: Simplify, then evaluate (where applicable):

a) $(2 \cdot 3)^4 = 2^4 \cdot 3^4$

$= 16 \cdot 81 = \boxed{1296}$

b) $(a^2 b^3)^2 = (a^2)^2 \cdot (b^3)^2$

$= a^{2 \cdot 2} \cdot b^{3 \cdot 2}$

$\boxed{= a^4 b^6}$

Raising a Quotient to a Power

eg: $\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$

$= \left(\frac{2\cdot 2\cdot 2}{3\cdot 3\cdot 3}\right)$

$= \boxed{\frac{2^3}{3^3}}$

Check:

$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$

$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$

$\frac{8}{27} = \frac{8}{27}$ ✓

Rule:

For any REAL number x and y, and any INTEGER n:

$$\left(\frac{x}{y}\right)^n = \boxed{\frac{x^n}{y^n} \quad \begin{matrix}(x \neq 0)\\(y \neq 0)\end{matrix}}$$

eg3: Simplify, and evaluate (when applicable):

a) $\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \boxed{\frac{9}{25}}$

b) $\left(\frac{a^3}{b^2}\right)^4 = \frac{(a^3)^4}{(b^2)^4} = \frac{a^{3\cdot4}}{b^{2\cdot4}} = \boxed{\frac{a^{12}}{b^8}}$

Common Errors:

i) $(x+y)^n \neq x^n + y^n$

eg: $(2+3)^2 \neq 2^2 + 3^2$

$5^2 \neq 4 + 9$

$25 \neq 13$

ii) $x^m \cdot x^n \neq x^{mn}$

but

$(x^m)^n = x^{mn}$

ie. students often confuse the Product Rule and the Power Rule.

p. 40-43 #1-9, 10-11 for 'fun' + Ch. Review