F.1 What are Factors and How do we find them?

- Factors are values we multiply together to get other values.

eg: $\underset{\text{factor}}{2} \cdot \underset{\text{factor}}{5} = 10$

$\underset{\text{factor}}{1} \cdot \underset{\text{factor}}{10} = 10$

Thus, the factors of 10 are:

<u>1, 2, 5, 10.</u>

- a <u>COMMON</u> factor is a value that is a factor of two or more other values.

eg. 2 is a factor of 10
2 is a factor of 4

so, a COMMON factor of 10 and 4 is 2.

eg1: Find the factors of 12 and 20 and identify any common factors.

$12 \rightarrow 1, 2, 3, 4, 6, 12$

$20 \rightarrow 1, 2, 4, 5, 10, 20$

1, 2, and 4 are common.

eg2: Find the factors of x^2 and x^5 and identify any common factors.

$x^2 \longrightarrow 1, x, x^2$

$x^5 \longrightarrow 1, x, x^2, x^3, x^4, x^5$

1, x, and x^2 are common factors.

- the <u>GREATEST COMMON FACTOR (GCF)</u> is the largest of the common factors.
  - in eg.1, the GCF is <u>4</u>.
  - in eg.2, the GCF is $\underline{x^2}$.

Why is finding the GCF useful?

- one way:

to help simplify fractions.

eg: $\frac{12}{20}$ → GCF of 12 and 20 is 4, so divide each by 4.

$$\frac{12 \div 4}{20 \div 4} = \boxed{\frac{3}{5}}$$

Finding the GCF

Two Methods:

1. List all the factors, then find the GCF.

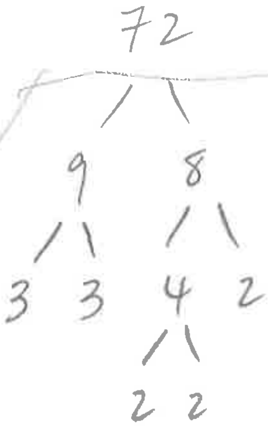
eg: Find GCF of 72 and 124.

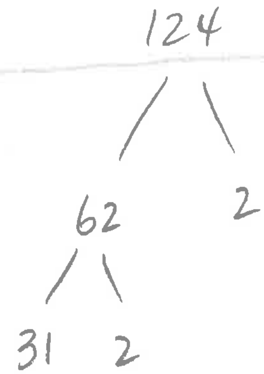
72 → 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

124 → 1, 2, 4, 31, 62, 124

GCF = $\boxed{4}$

② Use Prime Factor Method:





$= 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$

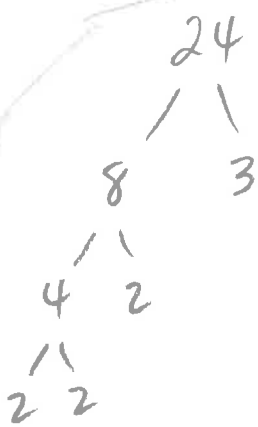
$= 3^2 \cdot 2^3$

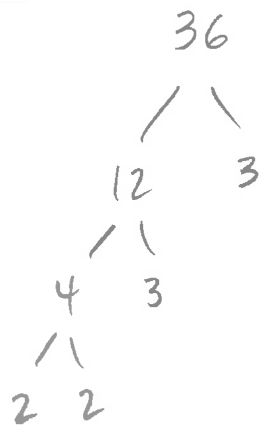
$= 31 \cdot 2 \cdot 2$

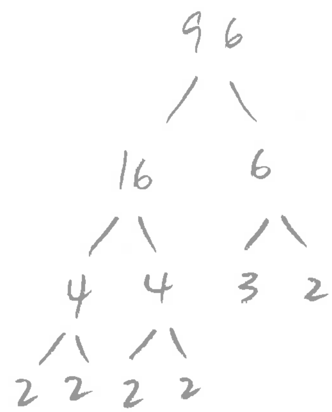
$= 31 \cdot 2^2$

$GCF = 2^2 = \boxed{4}$

Find GCF of 24, 36, 96







$2 \cdot 2 \cdot 2 \cdot 3$

$2 \cdot 2 \cdot 3 \cdot 3$

$3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$GCF = 2 \cdot 2 \cdot 3 = \boxed{12}$

eg: Find the GCF of $21a^2b$, $35a^2b^2c$, and $49ab^2c$

$21a^2b$

$= 7 \cdot 3 \cdot a \cdot a \cdot b$

$35a^2b^2c$

$= 7 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot c$

$49ab^2c$

$= 7 \cdot 7 \cdot a \cdot b \cdot b \cdot c$

$GCF = 7 \cdot a \cdot b = \boxed{7ab}$

Worksheet 1-42

Worksheet 1-42

Write the prime factors of each number.

1. 12 **2.** 16 **3.** 28

4. 63 **5.** 144 **6.** 225

Factor fully.

7. $4xy^2$ **8.** $18a^2b^3$ **9.** $36x^2yz^2$

10. $10x^2y$ **11.** $54x^5$ **12.** $125a^4b^2$

Determine the GCF of each pair.

13. 15, 20 **14.** 16, 24 **15.** 27, 36

16. 28, 42 **17.** 48, 72 **18.** 64, 96

Determine the GCF of each pair.

19. $4a$, $6a$ **20.** $2x^2$, $3x$

21. $12m^3$, $10m^2$ **22.** $12abc$, $3abc$

23. $2x$, $4y$ **24.** $14a$, $7b$

25. $5x^2$, $10x$ **26.** $4xy$, $5xy$

27. $9mn^2$, $8mn$ **28.** $2a^3$, $8a^2$

29. $15bc$, $25b^2c$ **30.** $6x^2y^2$, $9xy$

Determine the GCF of each set.

31. $5xyz$, $10abc$, $25pqr$

32. $20x$, $10x^3$, $8x^2$

33. $12abc$, $18ab$, $6ac$

34. $10x^2y$, $15xy^2$, $25xyz$

35. $21a^2b$, $35a^2b^2c$, $49ab^2c$

36. $12xy$, $16x^2y$, $20xyz$

37. $56abc$, $64a^2b$, $36ab^2c$

Find the GCF.

38. x^2y^2, x^2y^3, x^3y^4

39. $2x^3y$, $4x^2y^4$, $2x^2y^4$

40. $3x^2y^3$, $3x^3y^2$, $6xy^2$

41. $4a^3b^3$, $8a^2b^3$, $16ab^3$

42. $10s^4t^5$, $5s^5t^4$, $15s^3t^4$

ANSWERS:

1. 2, 2, 3 **2.** 2, 2, 2, 2 **3.** 2, 2, 7 **4.** 3, 3, 7 **5.** 2, 2, 2, 2, 3, 3 **6.** 3, 3, 5, 5 **7.** $2 \times 2 \times x \times y \times y$ **8.** $2 \times 3 \times 3 \times a \times a \times b \times b \times b$ **9.** $2 \times 2 \times 3 \times 3 \times x \times x \times y \times z \times z$ **10.** $2 \times 5 \times x \times x \times y$ **11.** $2 \times 3 \times 3 \times 3 \times x \times x \times x \times x \times x$ **12.** $5 \times 5 \times 5 \times a \times a \times a \times a \times b \times b$ **13.** 5 **14.** 8 **15.** 9 **16.** 14 **17.** 24 **18.** 32 **19.** $2a$ **20.** x **21.** $2m^2$ **22.** $3abc$ **23.** 2 **24.** 7 **25.** $5x$ **26.** xy **27.** mn **28.** $2a^2$ **29.** $5bc$ **30.** $3xy$ **31.** 5 **32.** $2x$ **33.** $6a$ **34.** $5xy$ **35.** $7ab$ **36.** $4xy$ **37.** $4ab$ **38.** x^2y^2 **39.** $2x^2y$ **40.** $3xy^2$ **41.** $4ab^3$ **42.** $5s^3t^4$

F.2 - Removing Common Factors

- to factor essentially means to **DIVIDE** (ie. it is the opposite of multiplying).
- thus, to FACTOR a polynomial means to express it as a **PRODUCT**.

(Note: "FACTOR" can be a noun or a verb).

- removing common factors is the most fundamental skill in factoring.
- it is possible that within a polynomial you might have every term possessing a factor that is the same → known as a **COMMON FACTOR**.

eg1: Factor $5x + 10$

Two terms
GCF of 5 & 10 is 5
GCF of x & 1 is 1

$$= 5\left(\frac{5x}{5} + \frac{10}{5}\right)$$

$$= \boxed{5(x+2)}$$

eg2: Factor $-3x^2 - 6$

$$= -3\left(\frac{-3x^2}{-3} - \frac{6}{-3}\right)$$

$$= \boxed{-3(x^2 + 2)}$$

* if you can factor out a NEGATIVE integer, do so.

eg3: Factor $5x^2 + 30x$

$$= 5x\left(\frac{5x^2}{5x} + \frac{30x}{5x}\right)$$

$$= \boxed{5x(x + 6)}$$

eg4: Factor $6x^2 - 3x$

$$= 3x\left(\frac{6x^2}{3x} - \frac{3x}{3x}\right)$$

$$= \boxed{3x(x - 1)}$$

eg5: Factor $12x^4 - 8x^3 + 4x^2$

$$= 4x^2\left(\frac{12x^4}{4x^2} - \frac{8x^3}{4x^2} + \frac{4x^2}{4x^2}\right)$$

$$= \boxed{4x^2(3x^2 - 2x + 1)}$$

eg6: Factor $8x^3 - 6x^2y^2 + 4x^2y$

$$= 2x^2\left(\frac{8x^3}{2x^2} - \frac{6x^2y^2}{2x^2} + \frac{4x^2y}{2x^2}\right)$$

$$= \boxed{2x^2(4x - 3y^2 + 2y)}$$

eg7: Factor $-15x^2y + 10xy^2 - 20x^2y^2$

$$= -5xy\left(\frac{-15x^2y}{-5xy} + \frac{10xy^2}{-5xy} - \frac{20x^2y^2}{-5xy}\right)$$

$$= \boxed{-5xy(3x - 2y + 4xy)}$$

Worksheet # 1-24 and 1-22 and 25-31

- Sometimes, a common factor can be larger than a mere monomial.

eg8: Factor $x(x-2) + 5(x-2)$

$$= (x-2)\left(\frac{x(x-2)}{(x-2)} + \frac{5(x-2)}{(x-2)}\right)$$

$$= (x-2)(x+5)$$

eg 9: Factor $4x(5x+1)(x-3) - 2(x-3)(5x+1)$

$$= \big(2(5x+1)(x-3)\big)(2x-1)$$

$$= 2(5x+1)(x-3)(2x-1)$$

* order of binomials is irrelevant

Factoring by Grouping

When a polynomial has at least four terms, it might be possible to, in the case of 4 terms, group terms with common factors into two groups of two, then factor each.

eg 10: Factor by grouping:

$$ac + bc + ad + bd$$

$$= (ac + bc) + (ad + bd)$$

$$= c(a+b) + d(a+b)$$

$$= \boxed{(a+b)(c+d)}$$

OR

$$= (ac + ad) + (bc + bd)$$

$$= a(c+d) + b(c+d)$$

$$= (c+d)(a+b)$$

eg 11: Factor $2x^3 - 6x^2 + x - 3$

$= (2x^3 - 6x^2) + (x - 3)$

$= 2x^2(x-3) + (x-3)$

$= (x-3)\left(\frac{2x^2(x-3)}{(x-3)} + \frac{(x-3)}{(x-3)}\right)$

$= \boxed{(x-3)(2x^2+1)}$

OR

$= 2x^3 + x - 6x^2 - 3$

$= (2x^3 + x) + (-6x^2 - 3)$

$= x(2x^2+1) - 3(2x^2+1)$

$= \boxed{(2x^2+1)(x-3)}$

HINT: $(x-y)$ and $(y-x)$ differ only by a factor of -1.

eg 12: Factor $5x^2y + 2y - 10x^2 - y^2$

$= 5x^2y - 10x^2 + 2y - y^2$

$= 5x^2(y-2) + y(2-y)$

$= 5x^2(y-2) - y(y-2)$

$\boxed{= (y-2)(5x^2-y)}$

Worksheet #23-36 and

#1-21

Worksheet 1-24, 1-22, 25-31

State the missing factor.

1. $12x + 18y = (\blacksquare)(2x + 3y)$

2. $3x^2 - 5x = (\blacksquare)(3x - 5)$

3. $4ab + 3ac = (\blacksquare)(4b + 3c)$

4. $5x^2 + 10x = (\blacksquare)(x + 2)$

5. $8abc - 12ab = (\blacksquare)(2c - 3)$

Copy and complete.

6. $3y^2 + 18y = 3y(y + \blacksquare)$

7. $14a - 12b = 2(\blacksquare - 6b)$

8. $4a^3 - 8a^2 = 4a^2(\blacksquare - 2)$

9. $10x^3 - 5x^2 + 15x = 5x(2x^2 - \blacksquare + \blacksquare)$

Copy and complete.

10. $33ab - 22b = 11b(\blacksquare - \blacksquare)$

11. $4a^3 - 10a^2 + 6a = 2a(\blacksquare - \blacksquare + \blacksquare)$

12. $27a^2b^2 - 18ab + 9b = 9b(\blacksquare - \blacksquare + \blacksquare)$

13. $6x^2y - 4xy^2 = 2xy(\blacksquare - \blacksquare)$

14. $9a^3b - 12ab^4 = 3ab(\blacksquare - \blacksquare)$

Factor each binomial.

15. $10x + 15$ **16.** $28y - 14$

17. $2mn - n$ **18.** $5x^2 + 10x$

19. $8x^2 + 4x^3$ **20.** $9a^3b^2 - 6a^2b$

21. $4x^2y^2 - 6xy^2z^2$ **22.** $14a^2b^4 - 21b^2c^2$

23. $6x^2y^3z + 12xy^2z$ **24.** $15a^2b^5 - 9b^4c^5$

Answers:

1. 6 **2.** x **3.** a **4.** $5x$ **5.** $4ab$ **6.** 6 **7.** $7a$ **8.** a **9.** x, 3 **10.** $3a$, 2 **11.** $2a^2$, $5a$, 3 **12.** $3a^2b$, $2a$, 1 **13.** $3x$, $2y$ **14.** $3a^2$, $4b^3$ **15.** $5(2x+3)$ **16.** $14(2y-1)$ **17.** $n(2m-1)$ **18.** $5x(x+2)$ **19.** $4x^2(2+x)$ **20.** $3a^2b(3ab-2)$ **21.** $2xy^2(2x-3z^2)$ **22.** $7b^2(2a^2b^2-3c^2)$ **23.** $6xy^2z(xy+2)$ **24.** $3b^4(5a^2b-3c^5)$

$3b^4(5a^2b-3c^5)$

Factor, if possible.

1. $5x + 25$ **2.** $4x + 13$ **3.** $8x + 8$

4. $9y - 9$ **5.** $3x - 15y$ **6.** $25x^2 + 10x$

7. $4ax + 8ay - 6az$ **8.** $5pqr - pqs - 10pqt$

9. $2x^2 - 2x - 6$ **10.** $3y^2 - 9y - 20$

Factor completely, if possible.

11. $9a^3 + 27b^2$

12. $3x^5 - 6x^3 + 3x$

13. $12y - 8y^2 + 24y^3$

14. $24w^5 - 6w^3$

15. $6rst + 3rs - 7t$

16. $33ab + 22bc - 11b^2$

17. $24xy^2 + 16x^2y$

18. $35xy - 10y^2$

19. $5rst - 15ab + 7cd$

20. $24xy^2 - 12xy + 36x^2y$

21. $27a^2b^3 + 9a^2b^2 - 18a^3b^2$

22. $6m^3n^2 + 18m^2n^3 - 12mn^2 + 24mn^3$

Answers:

12. $3x(x^4-2x^2+1)$

1. $5(x+5)$ **2.** not possible **3.** $8(x+1)$ **4.** $9(y-1)$ **5.** $3(x-5y)$ **6.** $5x(5x+2)$ **7.** $2a(2x+4y-3z)$ **8.** $pq(5r-s-10t)$ **9.** $2(x^2-x-3)$ **10.** not possible **11.** $9(a^3+3b^2)$ **12.** $\cancel{3x(x^2-1)^2}$ **13.** $4y(3-2y+6y^2)$ **14.** $\cancel{6w^3(2w+1)(2w-1)}$ $6w^3(4w^2-1)$ **15.** not possible **16.** $11b(3a+2c-b)$ **17.** $8xy(3y+2x)$ **18.** $5y(7x-2y)$ **19.** not possible **20.** $12xy(2y-1+3x)$ **21.** $9a^2b^2(3b+1-2a)$ **22.** $6mn^2(m^2+3mn-2+4n)$

Factor each trinomial.

25. $9a - 6b + 3$

26. $4a - 8b + 16$

27. $12x^3 - 6x^2 + 24x$

28. $10x^3 - 5x^2 + 15x$

29. $24x^4y - 18x^3y + 12x^2y^2$

30. $8a^2b + 16ab - 24a$

31. $25m^3n - 15m^2n^2 + 5mn^3$

Answers:

25. $3(3a-2b+1)$ **26.** $4(a-2b+4)$ **27.** $6x(2x^2-x+4)$ **28.** $5x(2x^2-x+3)$ **29.** $6x^2y(4x^2-3x+2y)$ **30.** $8a(ab+2b-3)$ **31.** $5mn(5m^2-3mn+n^2)$

WORKSHEET (23-36)

Factor, if possible.

23. $5x(a+b)+3(a+b)$
24. $3m(x-1)+5(x-1)$
25. $7x(m+4)-3(m-4)$
26. $4y(p+q)-x(p+q)$
27. $4t(m+7)+(m+7)$
28. $3t(x-y)-(x+y)$
29. $8x(m-n)+6(m-n)$

Factor by grouping.

30. $wx+wy+xz+yz$ **31.** $xy+12+4x+3y$
32. $x^2+x-xy-y$ **33.** $m^2-4n+4m-mn$
34. $2x^2+6y+4x+3xy$ **35.** $5m^2t-10m^2+t^2-2t$
36. $3a^2+6b^2-9a-2ab^2$

ANSWERS:

23. $(a+b)(5x+3)$ **24.** $(x-1)(3m+5)$ **25.** not possible
26. $(p+q)(4y-x)$ **27.** $(m+7)(4t+1)$ **28.** not possible
29. $(m-n)(8x+6)$ **30.** $(w+z)(x+y)$ **31.** $(x+3)(4+y)$
32. $(x+1)(x-y)$ **33.** $(m+4)(m-n)$ **34.** $(x+2)(2x+3y)$
35. $(t-2)(5m^2+t)$ **36.** $(a-3)(3a-2b^2)$

Worksheet 1-21

Factoring By Grouping

Factor each completely.

1) $8r^3 - 64r^2 + r - 8$

2) $12p^3 - 21p^2 + 28p - 49$

3) $12x^3 + 2x^2 - 30x - 5$

4) $6v^3 - 16v^2 + 21v - 56$

5) $63n^3 + 54n^2 - 105n - 90$

6) $21k^3 - 84k^2 + 15k - 60$

7) $25v^3 + 5v^2 + 30v + 6$

8) $105n^3 + 175n^2 - 75n - 125$

9) $96n^3 - 84n^2 + 112n - 98$

10) $28v^3 + 16v^2 - 21v - 12$

11) $4v^3 - 12v^2 - 5v + 15$

12) $49x^3 - 35x^2 + 56x - 40$

13) $24p^3 + 15p^2 - 56p - 35$

14) $24r^3 - 64r^2 - 21r + 56$

15) $56xw + 49xk^2 - 24yw - 21yk^2$

16) $12x^2u + 3x^2v + 28yu + 7yv$

17) $12bc - 4bd - 15xc + 5xd$

18) $56xy - 35x + 16ry - 10r$

19) $5a^2z - 4a^2c + 15xz - 12xc$

20) $21xy - 12b^2 + 14xb - 18by$

21) $28xy + 25 + 35x + 20y$

ANSWERS:

1. $(8r^2+1)(r-8)$
2. $(3p^2+7)(4p-7)$
3. $(2x^2-5)(6x+1)$
4. $(2v^2+7)(3v-8)$
5. $3(3n^2-5)(7n+6)$
6. $3(7k^2+5)(k-4)$
7. $(5v^2+6)(5v+1)$
8. $5(7n^2-5)(3n+5)$
9. $2(6n^2+7)(8n-7)$
10. $(4v^2-3)(7v+4)$
11. $(4v^2-5)(v-3)$
12. $(7x^2+8)(7x-5)$
13. $(3p^2-7)(8p+5)$
14. $(8r^2-7)(3r-8)$
15. $(7x-3y)(8w+7k^2)$
16. $(3x^2+7y)(4u+v)$
17. $(4b-5x)(3c-d)$
18. $(7x+2r)(8y-5)$
19. $(a^2+3x)(5z-4c)$
20. $(7x-6b)(3y+2b)$
21. $(7x+5)(4y+5)$

F.3 - Factoring Quadratic Trinomials

- a quadratic trinomial expression is one of the form:

$$\underline{ax^2 + bx + c,}$$

where $a \neq \underline{0}$ and a, b, and c are REAL numbers.

- When $a = 1$ (or -1), such trinomials are much easier to factor.

(ie. easier to factor $\underline{x^2 + bx + c}$)

Factoring $x^2 + bx + c$ Trinomials

- take a step backwards:

$$(x+m)(x+n)$$
$$= x^2 + nx + mx + mn$$
$$= x^2 + x(n+m) + mn$$
$$\left.\begin{array}{l} = x^2 + (m+n)x + mn \\ (\ x^2 + \quad bx \quad + \quad c\ ) \end{array}\right\}$$

so, $b = m + n$
$c = mn$

Let's try an example:

Factor $x^2 + 5x + 6$

What is the goal? $(x \pm \underset{(m)}{\text{SOMETHING}})(x \pm \underset{(n)}{\text{SOMETHING}})$

$a = 1$ (typical of $x^2 + bx + c$)

$b = 5$

$c = 6$

Since $c = \underline{6}$, and $c = \underline{mn}$,

we need to find Two numbers

that multiply to give $\underline{6}$.

Since $b = \underline{5}$, and $b = \underline{m + n}$,

we need the <u>same two numbers</u>

to add to give 5.

Factors of 6: $\pm 6, \pm 1$ / $\pm 3, \pm 2$

$3 \cdot 2 = 6$

$3 + 2 = 5$

* numbers are 2 and 3

thus, $x^2 + 5x + 6 = \boxed{(x+2)(x+3)}$

Check with F.O.I.L.

- In terms of pattern recognition and simple integer arithmetic, we can draw some conclusions:

Form: i) $\underline{x^2 + bx + c}$;

- find two numbers that multiply to produce a <u>POSITIVE</u> value, and add to produce a <u>POSITIVE</u> value.
  - both numbers, therefore, must be <u>positive</u>.

ii) $\underline{x^2 - bx + c}$;

- find two numbers that multiply to produce a <u>POSITIVE</u> value, but add to produce a <u>NEGATIVE</u> value.
  - both numbers, therefore, must be <u>negative</u>.

iii) $\underline{x^2 + bx - c}$;

- find two numbers that multiply to produce a <u>NEGATIVE</u> value, and add to produce a <u>POSITIVE</u> value.

  - one number must be <u>positive</u>, and one <u>negative</u>.

    - since the sum is positive, the <u>LARGER</u> magnitude number must be positive.

iv) $\underline{x^2 - bx - c}$;

- find two numbers that multiply to produce a <u>NEGATIVE</u> value, but add to also produce a <u>NEGATIVE</u> value.

  - again, one number must be <u>positive</u>, and one <u>negative</u>.

    - since the sum is negative this time, the <u>SMALLER</u> magnitude number must be positive.

eg: Factor each of the following into a product of binomials (if possible):

a) $x^2 + 7x + 10$

$= \boxed{(x+5)(x+2)}$

Form i → two ⊕ numbers

$\times$	$+$
10	7

5, 2

b) $x^2 - 6x + 8$

$= \boxed{(x-4)(x-2)}$

Form ii → two ⊖ numbers

$\times$	$+$
8	−6

−4, −2

c) $x^2 + 5x - 24$

$= \boxed{(x+8)(x-3)}$

Form iii → one ⊕, one ⊖

larger ⊕

$\times$	$+$
−24	5

8, −3

d) $x^2 - 4x - 5$

$= (x+1)(x-5)$

Form iv → one ⊕, one ⊖

smaller ⊕

$\times$	$+$
−5	−4

−5, 1

e) $x^2 + 6x + 9$

Form i

$\times$	$+$
9	6

3, 3

$= (x+3)(x+3)$

$= \boxed{(x+3)^2}$

* a type of PERFECT SQUARE trinomial (where m = n).

f) $x^2 - 7x - 20$

Form iv

$\times$	$+$
-20	-7

$-20, 1$
$20, -1$
$-10, 2$
$10, -2$
$-5, 4$
$5, -4$

$\boxed{\text{not factorable}}$

eg2: Simplify by combining like terms, if possible. Then, remove the GCF and factor fully.

a) $x^2 + 7x - 15 + 9x + x^2 + 45$

$= 2x^2 + 16x + 30$

$= 2(x^2 + 8x + 15)$

$= \boxed{2(x+5)(x+3)}$

b) $3x^2 + 2x + 10 - (12x - 2x^2 + 130)$

$= 5x^2 - 10x - 120$

$= 5(x^2 - 2x - 24)$

$= \boxed{5(x-6)(x+4)}$

Worksheet (1-44)

+

More Trinomials Practice sheet

Worksheet 1-44

Factor:

1. $x^2 + 5x + 4$ **2.** $x^2 + 6x + 9$

State the values of m and n that satisfy the given conditions.

3.	Sum $m + n$	Product mn
a)	7	12
b)	8	15
c)	13	12
d)	18	77
e)	−8	15
f)	−10	25
g)	−7	12

4.	Sum $m + n$	Product mn
a)	−1	−12
b)	1	−12
c)	−3	−40
d)	25	150
e)	4	−5
f)	−1	−42
g)	−7	−60

Factor.

5. $x^2 + 7x + 10$ **6.** $y^2 - 8y + 15$

7. $w^2 - w - 56$ **8.** $z^2 + 3z - 40$

9. $x^2 - x - 30$ **10.** $a^2 - 17a + 16$

11. $x^2 - 9x - 10$ **12.** $x^2 + 12x + 20$

13. $x^2 + 10x + 25$ **14.** $m^2 - 9m + 18$

15. $a^2 - 6a + 9$ **16.** $y^2 + 11y + 30$

17. $x^2 + 10x + 9$ **18.** $x^2 - 15x - 16$

19. $a^2 + 6a - 16$ **20.** $x^2 + 9x + 20$

21. $a^2 - 25a + 24$ **22.** $y^2 - 9y + 14$

23. $y^2 - 7y - 18$ **24.** $x^2 - x - 72$

25. $s^2 - 2s - 80$ **26.** $a^2 - 18a + 81$

Simplify by combining like terms, if possible. Then, remove the GCF and factor fully.

27. $2x^2 - 21x + 36 + x^2$ **28.** $5x^2 - 2x - 10 - 3x$

29. $7x^2 + 35x + 42$ **30.** $b^2 + 3b + 4 + b^2 + 5b$

31. $bx^2 - 28bx + 75b$ **32.** $x^2 - 3x + 8 - 9x + x^2$

33. $5jx^2 - 40jx + 75j$ **34.** $3tx^2 + 12tx + 12t$

35. $t^3 + t^2 - 12t$ **36.** $3k^3 + 15k^2 - 18k$

Problems and Applications

Factor, if possible.

37. $x^2 + x + 1$ **38.** $a^2 - 7a - 8$

39. $b^2 + 14b + 48$ **40.** $y^2 + 7y - 12$

41. $z^2 - 20z + 100$ **42.** $m^2 - 2m + 5$

43. $x^2 - 4x - 4$ **44.** $y^2 + 8y - 20$

Answers:

1. $(x+4)(x+1)$ 2. $(x+3)^2$

3. a) 3, 4 **b)** 3, 5 **c)** 1, 12 **d)** 7, 11 **e)** −3, −5 **f)** −5, −5 **g)** −3, −4 **4. a)** −4, 3 **b)** −3, 4 **c)** −8, 5 **d)** 10, 15 **e)** −1, 5 **f)** −7, 6 **g)** −12, 5
5. $(x+2)(x+5)$ **6.** $(y-3)(y-5)$ **7.** $(w-8)(w+7)$
8. $(z-5)(z+8)$ **9.** $(x-6)(x+5)$ **10.** $(a-1)(a-16)$
11. $(x-10)(x+1)$ **12.** $(x+2)(x+10)$ **13.** $(x+5)(x+5)$
14. $(m-3)(m-6)$ **15.** $(a-3)(a-3)$ **16.** $(y+5)(y+6)$
17. $(x+1)(x+9)$ **18.** $(x-16)(x+1)$ **19.** $(a-2)(a+8)$
20. $(x+4)(x+5)$ **21.** $(a-1)(a-24)$ **22.** $(y-2)(y-7)$
23. $(y-9)(y+2)$ **24.** $(x-9)(x+8)$ **25.** $(s-10)(s+8)$
26. $(a-9)(a-9)$ **27.** $3(x-4)(x-3)$ **28.** $5(x-2)(x+1)$
29. $7(x+2)(x+3)$ **30.** not possible **31.** $b(x-3)(x-25)$
32. not possible **33.** $5j(x-3)(x-5)$ **34.** $3t(x+2)(x+2)$
35. $t(t-3)(t+4)$ **36.** $3k(k-1)(k+6)$ **Problems and Applications** **37.** not possible **38.** $(a-8)(a+1)$
39. $(b+6)(b+8)$ **40.** not possible **41.** $(z-10)(z-10)$
42. not possible **43.** not possible **44.** $(y-2)(y+10)$

MORE TRINOMIALS PRACTICE

Factoring Trinomials (a = 1)

Factor each completely.

1) $b^2 + 8b + 7$

2) $n^2 - 11n + 10$

3) $m^2 + m - 90$

4) $n^2 + 4n - 12$

5) $n^2 - 10n + 9$

6) $b^2 + 16b + 64$

7) $m^2 + 2m - 24$

8) $x^2 - 4x + 24$

9) $k^2 - 13k + 40$

10) $a^2 + 11a + 18$

11) $n^2 - n - 56$

12) $n^2 - 5n + 6$

13) $b^2 - 6b + 8$

14) $n^2 + 6n + 8$

15) $2n^2 + 6n - 108$

16) $5n^2 + 10n + 20$

17) $2k^2 + 22k + 60$

18) $a^2 - a - 90$

19) $p^2 + 11p + 10$

20) $5v^2 - 30v + 40$

21) $2p^2 + 2p - 4$

22) $4v^2 - 4v - 8$

23) $x^2 - 15x + 50$

24) $v^2 - 7v + 10$

25) $p^2 + 3p - 18$

26) $6v^2 + 66v + 60$

Answers:

1) $b^2 + 8b + 7$

$(b+7)(b+1)$

2) $n^2 - 11n + 10$

$(n-10)(n-1)$

3) $m^2 + m - 90$

$(m-9)(m+10)$

4) $n^2 + 4n - 12$

$(n-2)(n+6)$

5) $n^2 - 10n + 9$

$(n-1)(n-9)$

6) $b^2 + 16b + 64$

$(b+8)^2$

7) $m^2 + 2m - 24$

$(m+6)(m-4)$

8) $x^2 - 4x + 24$

Not factorable

9) $k^2 - 13k + 40$

$(k-5)(k-8)$

10) $a^2 + 11a + 18$

$(a+2)(a+9)$

11) $n^2 - n - 56$

$(n+7)(n-8)$

12) $n^2 - 5n + 6$

$(n-2)(n-3)$

More Answers:

13) $b^2 - 6b + 8$

$(b-4)(b-2)$

14) $n^2 + 6n + 8$

$(n+2)(n+4)$

15) $2n^2 + 6n - 108$

$2(n+9)(n-6)$

16) $5n^2 + 10n + 20$

$5(n^2 + 2n + 4)$

17) $2k^2 + 22k + 60$

$2(k+5)(k+6)$

18) $a^2 - a - 90$

$(a-10)(a+9)$

19) $p^2 + 11p + 10$

$(p+10)(p+1)$

20) $5v^2 - 30v + 40$

$5(v-2)(v-4)$

21) $2p^2 + 2p - 4$

$2(p-1)(p+2)$

22) $4v^2 - 4v - 8$

$4(v+1)(v-2)$

23) $x^2 - 15x + 50$

$(x-10)(x-5)$

24) $v^2 - 7v + 10$

$(v-5)(v-2)$

25) $p^2 + 3p - 18$

$(p-3)(p+6)$

26) $6v^2 + 66v + 60$

$6(v+10)(v+1)$

F.4 - Factoring Special Case Trinomials

'Standard' Trinomial:

$$ax^2 + bx + c \quad (\text{with } a \neq 0)$$

* think * if $a = 0$, this would no longer be a quadratic (degree 2) expression. It would be a linear (degree 1) expression.

* recall, in Math 9, $a = \underline{1}$.

However, what if $b = 0$ or $c = 0$?

Let's tackle $c = 0$ first.

If $c = 0$, then $x^2 + bx + c$

is written as:

$$\underline{x^2 + bx}$$

Recall: If $(x + m)(x + n) = x^2 + bx + c$

then $b = \underline{m + n}$. and

$c = \underline{mn}$

Well, if $c = 0$, then $mn = \underline{0}$, meaning one of m or n must equal $\underline{0}$ (assuming $b \neq 0$ in this case).

So... if one of m or n is 0, then the other must equal $\underline{b}$.

Example: Factor $x^2 + 4x$

$$x^2 + 4x + 0 \qquad \frac{\times}{0} \quad \frac{+}{4}$$

$$= (x + 0)(x + 4) \qquad 0, 4$$

$$\boxed{= x(x + 4)}$$

Notice, in fact, when $c = 0$, we can simply factor out x as a COMMON FACTOR:

$$x^2 + 4x$$

$$= x\left(\frac{x^2}{x} + \frac{4x}{x}\right)$$

$$= \boxed{x(x + 4)}$$

eg1: Factor $x^2 - 3x$

$$= x\left(\frac{x^2}{x} - \frac{3x}{x}\right)$$

$$= \boxed{x(x-3)}$$

eg2: Factor $x^2 + 12x$

$$= \boxed{x(x+12)}$$

What if $b = 0$ $(c \neq 0)$ in $x^2 + bx + c$?
(ie. $x^2 + 0x + c$ or $x^2 + c$)

Again: If $(x+m)(x+n) = x^2 + bx + c$,

then $b = \underline{m+n}$ and

$c = \underline{mn}$

Well, if $b = 0$, then $m + n = \underline{0}$.

think for two numbers to add to 0,
What must be true about them?

Answer: They must be OPPOSITES of each other!

ie. the numbers would be m and $-m$.
(or n and $-n$).

Now, what would have to be true about c?

$$c = -m^2 \quad (\text{or } -n^2)$$

ie. c must be negative (because multiplying opposite numbers results in a negative number), and the SQUARE of the number m.

Examples: Factor $x^2 - 4$

x	+
-4	0
2, -2	

$= \boxed{(x+2)(x-2)}$

Factor $x^2 + 4$

x	+
4	0

CANNOT BE FACTORED

Thus, for $x^2 + c$ to be factorable, c MUST be NEGATIVE.

For this reason, we call $x^2 - c$ a DIFFERENCE of TWO SQUARES.

Note: a SUM of two squares is UNFACTORABLE.

eg1: Factor $x^2 - 49$

perfect square

perfect square

SQUARE ROOT of x^2?

SQUARE ROOT of 49?

$\sqrt{x^2} = x$

$\sqrt{49} = 7$

m

So, $-m = -7$

$x^2 - 49$

$= \boxed{(x+7)(x-7)}$

eg2: Factor $x^2 - 36$

$= \boxed{(x+6)(x-6)}$

What if c is not a perfect square number?

eg3: Factor $x^2 - 10$

$= \boxed{(x + \sqrt{10})(x - \sqrt{10})}$

eg4: Factor $2x^2 - 8$

$= 2(x^2 - 4)$

$= \boxed{2(x+2)(x-2)}$

Factoring Special Trinomials Worksheet
#1–20

Factoring Special Trinomials Worksheet

Factor each of the following, if possible:

1. $x^2 + 2x$

2. $x^2 - 9x$

3. $x^2 - 25$

4. $x^2 + 36$

5. $x^2 + 21x$

6. $x^2 - 121$

7. $x^2 - 7x$

8. $x^2 + 1$

9. $x^2 + 5x$

10. $x^2 + 2$

11. $2x^2 - 6x$

12. $3x^2 - 243$

13. $x^2 - \frac{4}{25}$

14. $x^2 + 14x$

15. $x^2 + 49$

16. $4x^2 - 4x$

17. $x^2 - 6$

18. $x^2 + \frac{1}{2}x$

19. $x^2 + \frac{4}{9}$

20. $2x^2 - 1250$

Key

Factoring Special Trinomials Worksheet

Factor each of the following, if possible:

1. $x^2 + 2x$

$= x(x+2)$

2. $x^2 - 9x$

$= x(x-9)$

3. $x^2 - 25$

$= (x+5)(x-5)$

4. $x^2 + 36$

not possible

5. $x^2 + 21x$

$= x(x+21)$

6. $x^2 - 121$

$= (x+11)(x-11)$

7. $x^2 - 7x$

$= x(x-7)$

8. $x^2 + 1$

not possible

9. $x^2 + 5x$

$= x(x+5)$

10. $x^2 + 2$

not possible

11. $2x^2 - 6x$

$= 2x(x-3)$

12. $3x^2 - 243$

$= 3(x+9)(x-9)$

13. $x^2 - \frac{4}{25}$

$= (x+\frac{2}{5})(x-\frac{2}{5})$

14. $x^2 + 14x$

$= x(x+14)$

15. $x^2 + 49$

not possible

16. $4x^2 - 4x$

$= 4x(x-1)$

17. $x^2 - 6$

$= (x+\sqrt{6})(x-\sqrt{6})$

18. $x^2 + \frac{1}{2}x$

$= x(x+\frac{1}{2})$

19. $x^2 + \frac{4}{9}$

not possible

20. $2x^2 - 1250$

$= 2(x+25)(x-25)$