

# 6.1 – Slope of a Line

Name:

Date:

**Goal:** Determine the slope of a line segment and a line.

**Toolkit:**

- Rate of change
- Simplifying fractions

**Main Ideas:**

p. 339-342 #5-9, 13, 16, 17, 24

**Definitions**

**Rise:** the vertical distance between two points.

**Run:** the horizontal distance between two points.

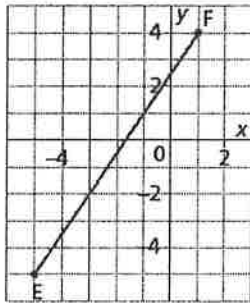
**Slope:** a measure of how one quantity changes with respect to the other, it can be calculated using:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in dependent variable}}{\text{change in independent variable}} = \text{rate of change}$$

**Determining the Slope of a Line Segment**

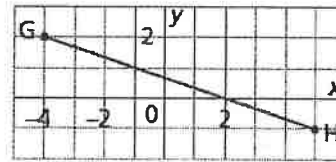
Ex1) Determine the slopes of the following line segments.

- Step 1: Choose two points on the line segment.
- Step 2: Count the units to determine the *rise* and the *run*.
- Step 3: Write the fraction in simplest form.



E to F: rise of 9  
run of 6

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{9}{6} = \frac{3}{2}$$



G to H: rise of -3  
run of 9

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-3}{9} = \frac{-1}{3}$$

or

$$\text{F to E: rise of } -9 \text{ run of } -6 \text{ slope} = \frac{-9}{-6} = \frac{3}{2}$$

or

$$\text{H to G: rise of } 3 \text{ run of } -9 \text{ slope} = \frac{3}{-9} = \frac{-1}{3}$$

When a line segment goes up to the right, both *x* and *y* increase. Both the rise and run are positive, so the slope of the line segment is positive.

When a line segment goes down to the right, *y* decreases and *x* increases. The rise is negative and the run is positive, so the slope of the line segment is negative.

For a **horizontal** line segment, the change in *y* is 0. The rise is 0 and the run is positive.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{0}{\text{run}} = 0$$

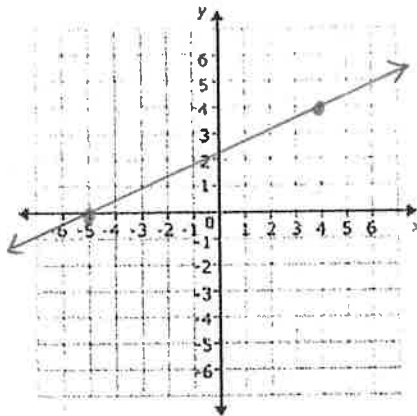
For a **vertical** line segment, *y* increases and the change in *x* is 0. The rise is positive and the run is 0.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{rise}}{0} = \text{undefined}$$

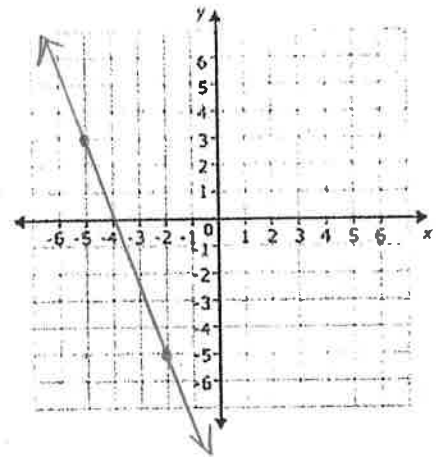
Drawing a line segment with a given slope.

Ex 2) Draw a line segment with the given slope.

a) slope =  $\frac{4}{9}$



b) slope =  $-\frac{8}{3}$



Finding slope when given two points.

Ex 3) Determine the slope of the line that passes through E(4,-5) and F(8,6).

Slope of a line = $\frac{y_2 - y_1}{x_2 - x_1}$	$A(x_1, y_1) B(x_2, y_2)$
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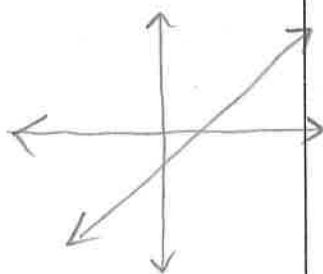
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-5)}{8 - 4} = \frac{11}{4}$$

$$\text{OR} = \frac{-5 - 6}{4 - 8} = \frac{-11}{-4} = \frac{11}{4}$$

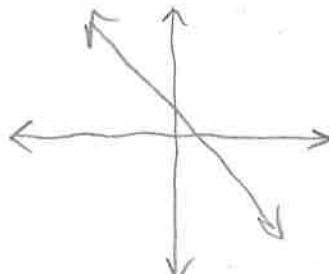
How else could we have found the slope?

*graph and count.*

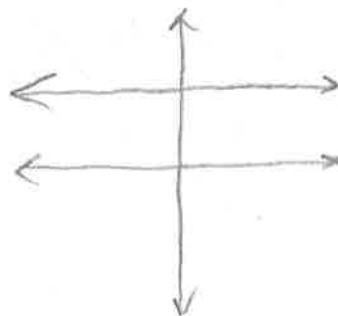
FOUR SLOPE TYPES:



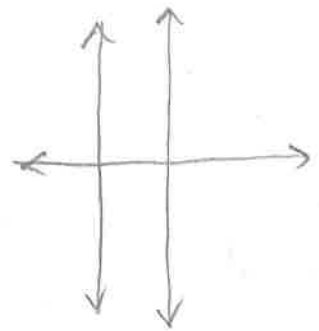
positive



negative



zero  
( $y = \#$ )



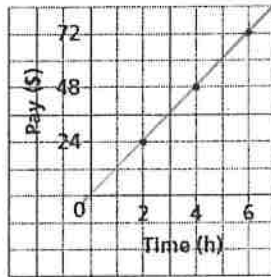
undefined  
( $x = \#$ )

Interpreting the slope of a line

Ex 4)

Tom has a part-time job. He recorded the hours he worked and his pay for 3 different days. Tom plotted these data on a grid.

Graph of Tom's Pay



a) What is the slope of the line through these points?

$$\begin{aligned} \text{rise} &= 24 \\ \text{run} &= 2 \\ \text{Slope} &= \frac{24}{2} = 12 \end{aligned}$$

b) What does the slope represent?

\$12/h → his wage

c) How can the answer to part b be used to determine:

i) how much Tom earned in  $3\frac{1}{2}$  hours?

$$\begin{aligned} \text{c) i) } & 3.5 \text{ h} \times \$12/\text{h} \\ & = \boxed{\$42} \end{aligned}$$

ii) the time it took Tom to earn \$30?

$$\text{ii) } x \cdot \$12/\text{h} = \$30$$

$$12x = 30$$

$$x = \frac{30}{12}$$

$$\boxed{x = 2.5 \text{ h}}$$

**Reflection:** How is the slope of a line related to rate of change?

# 6.4 – Slope-Intercept Form of the Equation for a Linear Function

Name:

Date:

**Goal:** to relate the graph of a linear function to its equation in slope-intercept form.

## Toolkit:

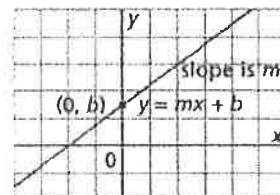
- Slope of a line  $(m) = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{\text{rise}}{\text{run}}$
- The y-intercept (vertical intercept) of a line is  $b$

## Main Ideas:

p. 362-363 # 4-5, 7(a-d), 12, 17.

What is Slope-Intercept Form of the Equation of a Linear Function

The equation of a linear function can be written in the form  $y = mx + b$ , where  $m$  is the slope of the line and  $b$  is its y-intercept (with coordinates  $(0, b)$ ).



$$y = mx + b$$

↑ SLOPE                      ↑ y-INTERCEPT

Writing an Equation Given Slope and y-intercept

Ex. 1) The graph of a linear function has a slope  $\frac{3}{5}$  and y-intercept of  $-4$ . Write an equation for this function.

$$y = mx + b$$

$$y = \frac{3}{5}x - 4$$

Graphing a Linear Function Given the Equation in  $y = mx + b$

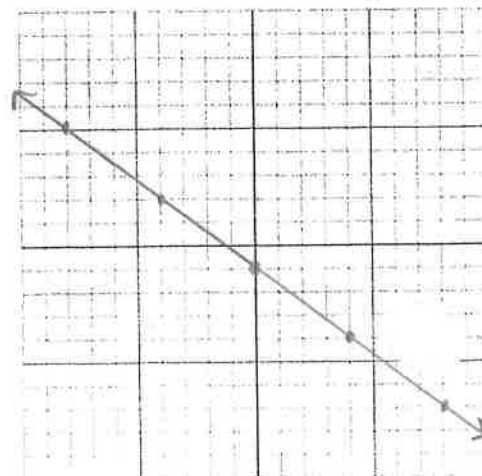
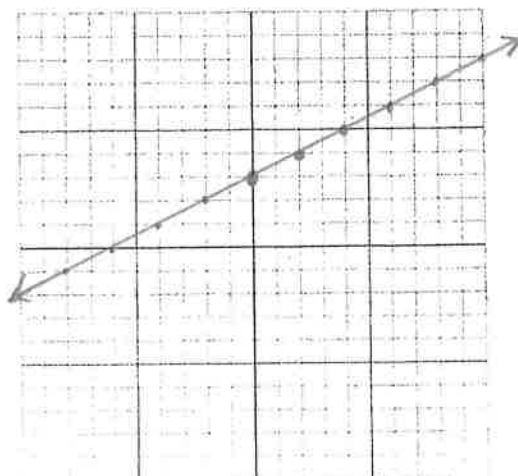
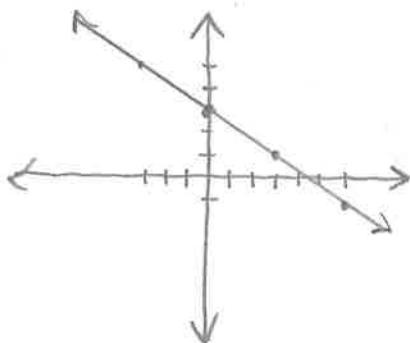
Ex. 2) Graph the linear functions with the following equations:

a)  $y = \frac{1}{2}x + 3$

b)  $y = -\frac{3}{4}x - 1$

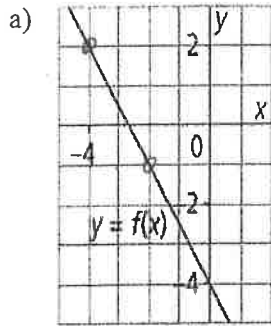
- ① Plot y-intercept
- ② Find other points using slope.
- ③ Connect points

c)  $2x + 3y = 9$   
 $3y = -2x + 9$   
 $y = -\frac{2}{3}x + 3$



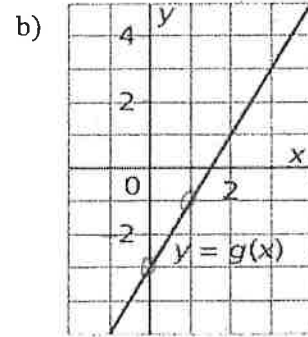
Writing the Equation of a Linear Function Given Its Graph

Ex. 3) Write equations to describe the following functions. Verify the equation.



$y$ -int. =  $-4$   
 $(b = -4)$   
 neg. slope:  $\frac{\text{rise}}{\text{run}} = -\frac{3}{2}$

$$y = -\frac{3}{2}x - 4$$



$y$ -int =  $-3$   
 $(b = -3)$   
 pos. slope:  $\frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

$$y = 2x - 3$$

Using an Equation of a Linear Function to Solve a Problem

Ex. 4) The student council sponsored a dance. A ticket cost \$5 and the cost for the DJ was \$300.

a) Write an equation for the profit,  $P$ , on the sale of  $t$  tickets.

$$P = 5t - 300$$

b) Suppose 123 people bought tickets. Find the profit.

$$P = 5(123) - 300 = \boxed{\$315}$$

c) Suppose the profit was \$350. How many people bought tickets?

$$350 = 5t - 300$$

$$50 = 5t$$

$$\frac{50}{5} = \frac{5t}{5}$$

$$t = 10$$

d) Could the profit be exactly \$146? Justify the answer.

$$146 = 5t - 300$$

$$446 = 5t$$

No. 446 not divisible by 5. Cannot sell part of a ticket.

**Reflection:** How do the values of  $m$  and  $b$  in the linear equation  $y = mx + b$  relate to the graph of the corresponding linear function? Use examples to help.

# 6.6 – General Form of the Equation for a Linear Relation

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Goal:** to relate the graph of a linear function to its equation in general form.

**Toolkit:**

- Slope-Intercept form  $\rightarrow y = mx + b$
- Rearranging Equations

**Main Ideas:**

p. 384-385 #4, 6, 12, 13, 18, 22, 24.

What is General Form of the Equation of a Linear Relation?

How is Standard Form similar?

Rewriting an Equation in General Form

Graphing a Line in General Form

GENERAL FORM of the Equation of a Linear Relation:

$$Ax + By + C = 0$$

where  $A$  is a whole number (not negative!), and  $B$  and  $C$  are integers.

STANDARD FORM of the Equation of a Linear Relation:

$$Ax + By = C$$

Ex. 1) Write each equation in general form and standard form:

a)  $y = -\frac{2}{3}x + 4$

b)  $y - 1 = \frac{3}{5}(x + 2)$

$$3(y) = 3\left(-\frac{2}{3}x + 4\right)$$

$$5(y - 1) = 5\left(\frac{3}{5}\right)(x + 2)$$

$$3y = -2x + 12$$

$$5y - 5 = 3(x + 2)$$

$$2x + 3y = 12 \quad \text{STANDARD}$$

$$5y - 5 = 3x + 6$$

$$2x + 3y - 12 = 0 \quad \text{GENERAL}$$

$$3x - 5y = -11 \quad \text{STD.}$$

$$3x - 5y + 11 = 0 \quad \text{GEN.}$$

Ex. 2) a) Determine the  $x$ - and  $y$ -intercepts of the line whose equation is  $3x + 2y - 18 = 0$

$x$ -int: set  $y = 0$

$y$ -int: set  $x = 0$

$$3x + 2(0) - 18 = 0$$

$$3(0) + 2y - 18 = 0$$

$$3x - 18 = 0$$

$$2y - 18 = 0$$

$$3x = 18$$

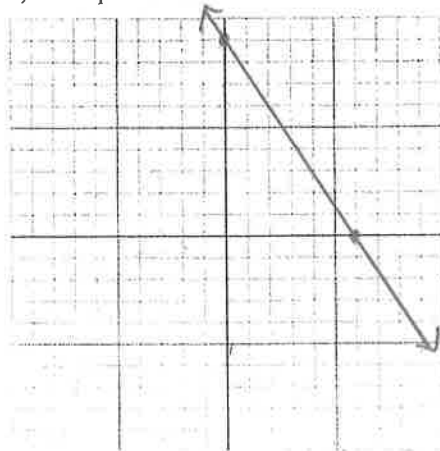
$$2y = 18$$

$$x = 6 \quad (6, 0)$$

$$y = 9 \quad (0, 9)$$

b) Graph the line.

c) Verify that the graph is correct



pick a point and test

Also try:

$$4x - 3y = 12$$

$x$ -int:

$y$ -int:

$$4x = 12$$

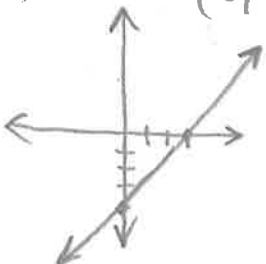
$$-3y = 12$$

$$x = 3$$

$$y = -4$$

$$(3, 0)$$

$$(0, -4)$$



Determining the Slope of a Line Given Its Equation in General Form

(switch to Standard!)

Ex. 3) a) Determine the slope of the line with the equation  $3x - 2y - 16 = 0$

convert to slope/y-int. form:

$$-2y = -3x + 16$$

$$y = \frac{3}{2}x - 8$$

$$\text{slope} = \frac{3}{2}$$

b) Determine the slope of the line with the equation  $5x - 2y + 12 = 0$

$$-2y = -5x - 12$$

$$y = \frac{5}{2}x + 6$$

$$\text{slope} = \frac{5}{2}$$

c) Determine the slope AND the y-intercept of the line with the equation  $4x - 6y = 0$ , then graph the line.

$$6y = 4x$$

$$y = \frac{4}{6}x$$

$$y = \frac{2}{3}x$$

$$\text{slope} = \frac{2}{3}$$

x-int:

$$4x = 0$$

$$x = 0$$

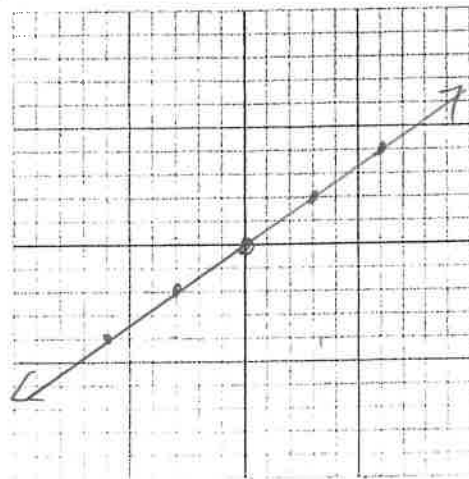
$$(0, 0)$$

y-int:

$$-6y = 0$$

$$y = 0$$

$$(0, 0)$$



Reflection: Why can't you use intercepts to graph the equation  $4x - y = 0$ ? (where  $C = 0$ )

6.7 – Graphing Linear Functions in Two Forms

Name:

Date:

Goal: to recognize the two different forms of linear functions, & to graph them using the easiest method

Toolkit:

- Slope/y-intercept form

$$y = mx + b$$

- General & Standard form

$$Ax + By + C = 0$$

$$Ax + By = C$$

Main Ideas:

p. 362-364

# 6, 7 (a-d), 18, 19, 20

p. 384-385

# 5, 7, 9, 14, 23

Ex 1) Label each linear equation as either “ $y = mx + b$ ”, or “standard”:

$y = -3x + 5$	$2x + 3y = 9$	$2x - y = -4$	$y = \frac{1}{2}x - \frac{3}{4}$	$y = 0.4x - 0.15$
$y = mx + b$	standard	standard	$y = mx + b$	$y = mx + b$

Ex2) Graph the equation  $y = -\frac{3}{2}x + 6$

Step 1: decide what form it is in:  $y = mx + b$  state  $m = -\frac{3}{2}$  and  $b = 6$

Step 2: for  $y = mx + b$ , put a point on the y-axis at “b”

Step 3: use the slope ( $m = \frac{-3}{2} = \frac{\text{rise}}{\text{run}}$ ) to count up/down 3 and over 2 to a new point OR  $\frac{3}{-2} \rightarrow \text{up } 3, \text{ (L) } 2$

Step 4: connect the dots!

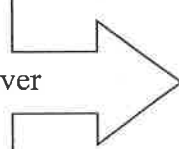
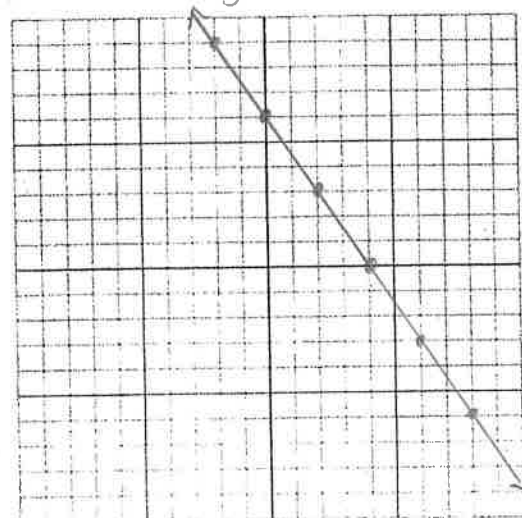
What is the best way to graph an equation in

$y = mx + b$  form?

Plot b then use slope to find more points.

$y = mx + b$

- start at b
- go up/down and over using slope
- connect the dots!

Hint: if you like  $y = mx + b$ , you can change any function to  $y = mx + b$  form and use this method!

Ex 3) Change to  $y = mx + b$  form:  $3x - 2y = 8$  ie. Solve for y.

$$3x - 2y = 8$$

$$-2y = -3x + 8$$

$$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{8}{-2}$$

$$y = \frac{3}{2}x - 4$$



general form

standard form

What is the best way to graph an equation in Standard form? (or general)

Ex 4) Graph the equation  $2x + 3y - 6 = 0 \rightarrow 2x + 3y = 6$

Step 1: decide what form it is in: *Standard*

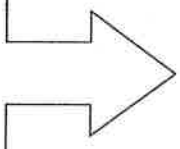
Step 2: for *standard form*, find the intercepts (cover x to get y, cover y to get x)

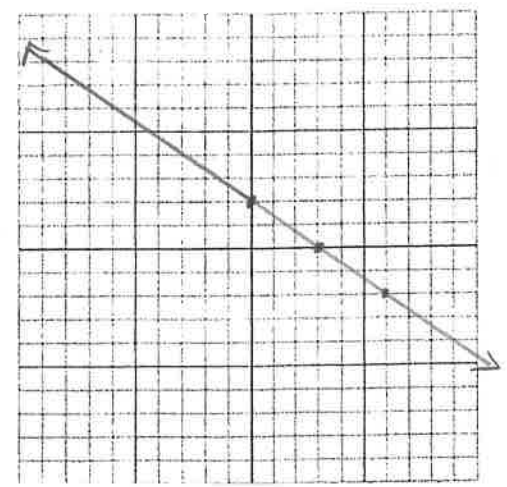
$2x = 6$        $3y = 6$   
x-int = 3    y-int = 2  
(3,0)      (0,2)

Step 3: plot x- and y-intercepts

Step 4: connect the dots! (Can check slope)

The intercepts (cover-up) method (unless the x/y intercept is (0,0))

- standard form*
- get intercepts
  - plot intercepts
  - connect the dots!
- 



**Reflection:** Which form of equation do you prefer to graph?

Would you change every equation to your preferred form, or use the different methods for the different ones? (You may want to try a few in the homework before you answer!)

6.2 – Slopes of Parallel and Perpendicular Lines

Name:

Date:

**Goal:** to use slope to determine whether two lines are parallel or perpendicular.

**Toolkit:**

- Slope
- Simplifying fractions
- Reciprocals

**Main Ideas:**

p. 349-351

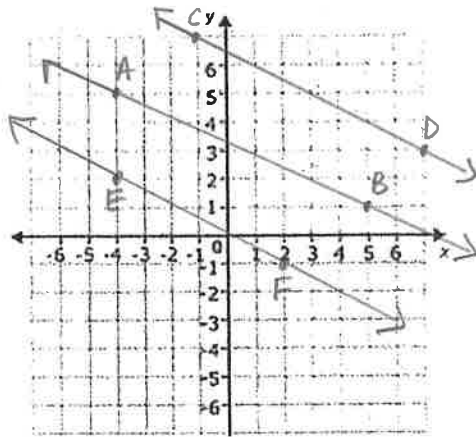
# 3-6, 8-10, 12-13, 17, 19

**Identifying Parallel Lines**

Lines that have equal slopes are parallel.

Ex 1) Line EF passes through E(-4,2) and F(2,-1).  
 Line CD passes through C(-1,7) and D(7,3).  
 Line AB passes through A(-4,5) and B(5,1).

Sketch the lines. Are they parallel? Count out each line's slope.



$$\left. \begin{aligned} \text{slope}_{EF} &= \frac{-3}{6} = -\frac{1}{2} \\ \text{slope}_{CD} &= \frac{-4}{8} = -\frac{1}{2} \end{aligned} \right\} EF \parallel CD$$

$$\text{slope}_{AB} = -\frac{4}{9}$$

**Identifying perpendicular lines**

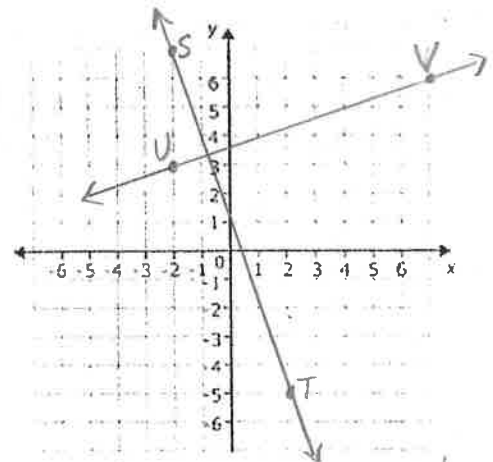
The slopes of two perpendicular lines are negative reciprocals; that is a line with a slope  $a$ ,  $a \neq 0$ , is perpendicular to a line with slope  $-\frac{1}{a}$ .

Ex 2) Line ST passes through S(-2,7) and T(2,-5). Line UV passes through U(-2,3) and V(7,6).

Are these lines parallel, perpendicular or neither? Calculate the slopes, and then sketch the lines to verify your answer.

$$\begin{aligned} \text{slope}_{ST} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 7}{2 - (-2)} \\ &= \frac{-12}{4} \\ &= -3 \end{aligned}$$

$$\begin{aligned} \text{slope}_{UV} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{7 - (-2)} \\ &= \frac{3}{9} \\ &= \frac{1}{3} \end{aligned}$$



Slopes are neg. reciprocals!

**ST ⊥ UV**

eg: What is the neg. reciprocal of each?

a) 2      b)  $-\frac{1}{4}$

$\boxed{-\frac{1}{2}}$        $\boxed{4}$

c) -3      d)  $\frac{3}{2}$

$\boxed{\frac{1}{3}}$        $\boxed{-\frac{2}{3}}$

Identifying a line perpendicular to a given line.

Ex 3)

a) Determine the slope of a line that is perpendicular to the line through G(-2,3) and H(1,-2).

$$\text{slope}_{GH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{1 - (-2)} = \frac{-5}{3}$$

$$\text{slope}_{\text{new line}} = \text{neg. recip. of } \frac{-5}{3} = \boxed{\frac{3}{5}}$$

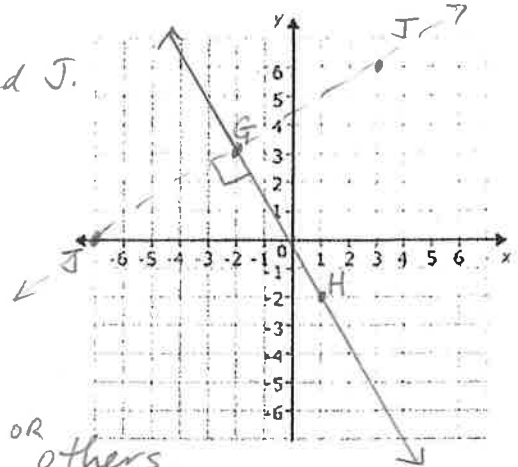
b) Determine the coordinates of J so that line GJ is perpendicular to line GH.

- ① Plot GH
- ② Use slope of GJ to find J.
- ③ Answers may vary.

$$\text{slope}_{GJ} = \frac{3}{5} = \frac{-3}{-5}$$

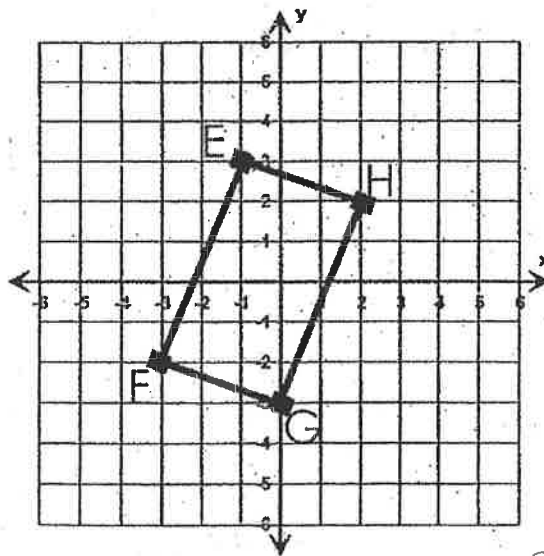
UP 3      DOWN 3  
 @ 5      @ 5

J: (-7, 0) or (3, 6) or others



Using slope to identify a polygon.

Ex 4) Is EFGH a parallelogram? Is it a rectangle?



Parallelogram: opposite sides parallel

Rectangle: parallelogram with all 90° angles.

$$\text{slope}_{EH} = -\frac{1}{3}$$

$$\text{slope}_{FG} = -\frac{1}{3}$$

$$\text{slope}_{EF} = \frac{5}{2}$$

$$\text{slope}_{HG} = \frac{5}{2}$$

} parallelogram!

But, not a rectangle.

Key slopes not negative reciprocals

**Reflection:** What have you learned about parallel and perpendicular lines?

6.8 – Equations of Parallel and Perpendicular Lines

Name:

Date:

**Goal:** to find the equations of lines given information about parallel and perpendicular lines

**Toolkit:**

- slopes of parallel lines are *equal*
- slopes of perpendicular lines are *negative reciprocals*
- to find the equation of a line, you need: *slope and y-int.*

**Main Ideas:**

*Parallel and Perpendicular Lines Worksheet*

+

*Chapter Review: p388-390  
# 3-9, 11-13,  
21, 22, 25, 26*

Ex 1) For a line with the slope 0.7, what is the **slope** of a line that is

a) Parallel?

$$0.7 = \frac{7}{10}$$

$$\text{slope} = \boxed{0.7 = \frac{7}{10}}$$

b) Perpendicular?

$$\text{slope} = \boxed{\frac{-10}{7}} = -1.43$$

Ex 2) State the **slopes** of lines that are:

a) parallel to the line  $3x + 2y - 4 = 0$

$$2y = -3x + 4$$

$$y = -\frac{3}{2}x + 2$$

$$\text{slope} = \boxed{-\frac{3}{2}}$$

b) perpendicular to  $y = \frac{1}{2}x - \frac{3}{4}$

$$\text{slope} = -\frac{2}{1} = \boxed{-2}$$

Ex 3) For this pair of slopes, what is the value of  $k$  if the lines are...

$$\frac{4}{k}, 2$$

a) Parallel?

$$\text{slope of } \frac{4}{k} = \text{slope of } 2$$

$$\frac{4}{k} = 2$$

$$2k = 4$$

$$\boxed{k = 2}$$

b) Perpendicular?

$$\text{slope of } \frac{4}{k} = \text{neg. recip. of } 2$$

$$\frac{4}{k} = -\frac{1}{2}$$

$$-k = 8$$

$$\boxed{k = -8}$$

Ex 4) Are the pairs of lines parallel, perpendicular, or neither?

a)  $2x + 3y + 9 = 0$ ,  $y = \frac{3}{2}x + 6$

$$3y = -2x - 9$$

$$y = -\frac{2}{3}x - 3$$

perpendicular

b)  $y + 1 = \frac{3}{4}(x + 2)$ ,  $6x - 8y + 3 = 0$

$$y = \frac{3}{4}x + \frac{6}{4} - 1$$

$$-8y = -6x - 3$$

$$y = \frac{3}{4}x + \frac{3}{8}$$

$$y = \frac{3}{4}x + \frac{1}{2}$$

parallel

Ex 5) Find the equation of the line (in  $y = mx + b$  form) that is parallel to the line  $2x + 3y + 9 = 0$  and has the same y-intercept as the line  $y = 2x + 4$ .

$$3y = -2x - 9$$

$$y = -\frac{2}{3}x - 3$$

y-int: 4

(0, 4)

parallel → same slope w/ y-int. of 4

$$y = -\frac{2}{3}x + 4$$

Ex 6) Find the equation of the line (in  $Ax + By + C = 0$  form) that is perpendicular to  $y = -3x + 4$  and passes through the point (6, 3).

slope = -3

slope (perpendicular) =  $\frac{1}{3}$

$y = \frac{1}{3}x + b$  (perpendicular line)

passes through (6, 3)

$$3 = \frac{1}{3}(6) + b$$

$$3 = 2 + b$$

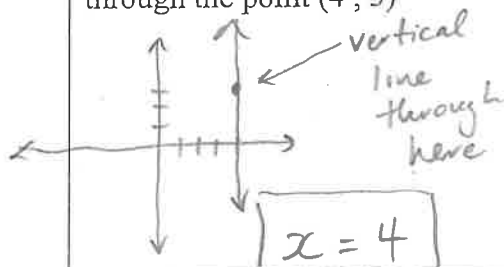
$$b = 1$$

$$y = \frac{1}{3}x + 1$$

$$0 = \frac{1}{3}x - y + 1$$

$$0 = x - 3y + 3$$

Ex 7) Find the equation of the line that is perpendicular to the x-axis and passes through the point (4, 3)



equation of x-axis:  $y = 0$   
slope = 0

neg. reciprocal of 0:  $-\frac{1}{0}$

undefined slope (vertical line)

Reflection: What short-cuts have you picked up this unit to make answering the questions faster?

