

Ch. 6.1 - Linear Equations

- within the context of mathematics (including algebra), an 'equal' sign ($=$) is considered a 'balance' sign, in that it verifies that the expressions on either side of the equal sign are identical in value (ie. represent the same number).

Are the following equations true, false, or neither?

i) $4 + 3 = 7$ TRUE

ii) $7 - 3 = 5$ FALSE

iii) $3 \times 4 = 12$ TRUE

iv) $x + 2 = 7$ NEITHER, since x is unknown.

- replacing the variable, x , in iv with a constant that makes the equation true would SOLVE the equation.

What # + 2 = 7?

$$\boxed{5}$$

$$\therefore \underline{x = 5}$$

- we need to develop skills that allow us to isolate a variable on one side of an equation, in order to solve for it.

The Addition Principle

For any real numbers a , b , and c :

$a = b$ implies that

$$\underline{a + c = b + c}$$

eg!: Solve each of the following using the Addition Principle:

a) $x + 2 = 5$ * to "solve" means to solve for x

What can be added to both sides to isolate x ?

$$x + 2 + (-2) = 5 + (-2)$$
$$\boxed{x = 3}$$

over \rightarrow

* What could be subtracted from either side to isolate x ?

$$x + 2 - 2 = 5 - 2$$
$$\boxed{x = 3}$$

so...

$a = b$ implies that

$$\underline{a - c = b - c}$$

Check: 'Plug' 3 in for x in the original question:

$$(3) + 2 = 5$$
$$5 = 5 \quad \text{TRUE!}$$

b) $x - 5 = -3$

$$x - 5 + 5 = -3 + 5$$

$$\boxed{x = 2}$$

Check: $(2) - 5 = -3$

$$-3 = -3 \quad \checkmark$$

c) $4 = 5 + x$

$$4 + (-5) = 5 + x + (-5)$$

$$4 - 5 = 5 + x - 5$$

$$-1 = x$$

SAME AS:

$$\boxed{x = -1}$$

Check: $4 = 5 + (-1)$

$$4 = 5 - 1$$

$$4 = 4 \quad \checkmark$$

$$d) \quad 3 - x = 5$$

$$3 - x + (-3) = 5 + (-3)$$

$$3 - x - 3 = 5 - 3$$

$$-x = 2 \quad \text{hmm...}$$

Multiply (or divide) both sides by -1 .

Golden Rule: What you do to one side of an equation, you do to the other.

$$\frac{-x}{-1} = \frac{2}{-1}$$

$$\boxed{x = -2}$$

OR

$$3 - x = 5$$

$$3 - x + x = 5 + x$$

$$3 = 5 + x$$

$$3 + (-5) = 5 + x + (-5)$$

$$3 - 5 = 5 + x - 5$$

$$-2 = x$$

$$\boxed{x = -2}$$

$$e) \quad 2x - 3 = x + 5$$

$$2x - 3 + 3 = x + 5 + 3$$

$$2x = x + 8$$

$$2x - x = x + 8 - x$$

$$\boxed{x = 8}$$

OR

$$2x - 3 = x + 5$$

$$2x - 3 - 5 = x + 5 - 5$$

$$2x - 8 = x$$

$$2x - 8 - 2x = x - 2x$$

$$-8 = -x$$

$$\frac{-8}{-1} = \frac{-x}{-1}$$

$$8 = x$$

$$\boxed{x = 8}$$

The Multiplication Principle

For any real numbers a , b , and c :

$$a = b \text{ implies that } \underline{a \times c = b \times c}$$

Likewise:

$$a = b \text{ implies that } \underline{a \div c = b \div c}$$

$(c \neq 0)$

Connection?

Requires knowledge of RECIPROCALs:

Two fractions are reciprocals if their product is 1.

ie. The reciprocal of $\frac{a}{b}$ is $\boxed{\frac{b}{a}}$
where $a, b \neq 0$.

eg2: Find the reciprocal of each of the following:

a) $\frac{2}{3} \rightarrow \boxed{\frac{3}{2}}$ b) $-\frac{3}{4} \rightarrow \frac{4}{-3} = \boxed{-\frac{4}{3}}$

c) $4 = \frac{4}{1} \rightarrow \boxed{\frac{1}{4}}$ d) $-\frac{1}{2} \rightarrow \frac{2}{-1} = \boxed{-2}$

So: dividing a value by a number is the same as multiplying that value by the number's reciprocal.

$$\text{eg: } 3 \div \frac{3}{4} = \underline{\underline{4}}$$
$$3 \times \frac{4}{3} = \frac{3}{1} \times \frac{4}{3} = \frac{12}{3} = \underline{\underline{4}}$$

eg3: Solve each of the following by the Multiplication (or Division) Principle:

a) $3x = 15$ (ie. 3 times what equals 15?)

i) Using Mult. Principle: (3 times what is 1?)
(to isolate x)

$$3x \times \frac{1}{3} = 15 \times \frac{1}{3}$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$\boxed{x = 5}$$

Check: $3(5) = 15$
 $15 = 15 \checkmark$

ii) Using Div. Principle:

$$3x = 15$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$\boxed{x = 5}$$

ie. $\times \frac{1}{3}$ same as $\div 3$

$$b) \quad -\frac{2}{3}x = 6 \quad \text{Same as: } \begin{cases} -\frac{2}{3}x = 6 \\ (-\frac{2x}{3} = 6) \end{cases}$$

$$i) \quad -\frac{2}{3}x = 6$$

$$-\frac{2}{3}x \times \frac{-3}{2} = 6 \times \frac{-3}{2}$$

$$\frac{6x}{6} = \frac{-18}{2}$$

$$\boxed{x = -9}$$

$$\text{Check: } \left(-\frac{2}{3}\right)(-9) = 6$$

$$\frac{18}{3} = 6$$

$$6 = 6 \quad \checkmark$$

$$ii) \quad -\frac{2}{3}x = 6$$

$$-\frac{2}{3}x = 6$$

$$\left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right)$$

$$-\frac{2}{3}x \cdot \frac{-3}{2} = 6 \cdot \frac{-3}{2}$$

$$\boxed{x = -9}$$

$$c) \quad \frac{2}{7x} = 4$$

$$\frac{2}{7x} \times \frac{7x}{1} = 4 \times \frac{7x}{1}$$

$$\frac{14x}{7x} = \frac{28x}{1}$$

$$2 = 28x$$

$$2 \times \frac{1}{28} = (28x) \left(\frac{1}{28}\right)$$

$$\frac{2}{28} = \frac{28x}{28}$$

$$\boxed{\frac{1}{14} = x}$$

$$\text{Check: } \frac{2}{7\left(\frac{1}{14}\right)} = 4$$

$$\frac{2}{\left(\frac{7}{14}\right)} = 4$$

$$\frac{2}{\left(\frac{1}{2}\right)} = 4$$

$$2 \cdot 2 = 4 \quad \checkmark$$

Putting It All Together

eg 4: Solve the following:

a) $5x - 4 = 2x - 1$

$$5x - 4 - 2x = 2x - 1 - 2x$$

$$3x - 4 = -1$$

$$3x - 4 + 4 = -1 + 4$$

$$3x = 3$$

$$\frac{3x}{3} = \frac{3}{3}$$

$$\boxed{x = 1}$$

Check: $5(1) - 4 = 2(1) - 1$
 $5 - 4 = 2 - 1$
 $1 = 1 \checkmark$

b) $5x - 3x + 7 = 8x + 5$

$$2x + 7 = 8x + 5$$

$$2x + 7 - 2x = 8x + 5 - 2x$$

$$7 = 6x + 5$$

$$7 - 5 = 6x + 5 - 5$$

$$2 = 6x$$

$$\frac{6x}{6} = \frac{2}{6}$$

$$\boxed{x = \frac{1}{3}}$$

Check:

$$5\left(\frac{1}{3}\right) - 3\left(\frac{1}{3}\right) + 7$$

$$= 8\left(\frac{1}{3}\right) + 5$$

$$\frac{5}{3} - 1 + 7 = \frac{8}{3} + 5$$

$$\frac{5}{3} + 6 = \frac{8}{3} + 5$$

$$\frac{5}{3} + \frac{18}{3} = \frac{8}{3} + \frac{15}{3}$$

$$\frac{23}{3} = \frac{23}{3} \checkmark$$

$\boxed{p. 203-207 \# 1-5, 7, 8}$

Ch. 6.2 - Removing Fractions, Decimals, and Parentheses

Helpful Hints:

- ① Eliminate any fraction or decimal by multiplying each term on both sides by the lowest common denominator;
- ② Remove brackets by multiplication (Distributive Property);
- ③ Combine like terms;
- ④ Use the Addition Principle to get all terms with a variable to one side and all constants to the other side;
- ⑤ Combine terms;
- ⑥ Use the Multiplication/Division Principle to make the variable's coefficient 1.
- ⑦ Check answer!

eg1: Solve $\frac{2}{3}x - \frac{1}{6} = \frac{3}{4}x$

$$\frac{2x}{3} - \frac{1}{6} = \frac{3x}{4} \quad \text{LCD} = 12$$

$$\frac{12(2x)}{3} - \frac{12(1)}{6} = \frac{12(3x)}{4}$$

$$4(2x) - 2(1) = 3(3x)$$

$$8x - 2^{-8x} = 9x^{-8x}$$

$$-2 = x$$

$$\boxed{x = -2}$$

Check:

$$\left(\frac{2}{3}\right)(-2) - \frac{1}{6} = \left(\frac{3}{4}\right)(-2)$$

$$-\frac{4}{3} - \frac{1}{6} = -\frac{6}{4}$$

$$-\frac{16}{12} - \frac{2}{12} = -\frac{18}{12}$$

$$-\frac{18}{12} = -\frac{18}{12} \quad \checkmark$$

Another strategy:

$3x? = 12$ → $\frac{2}{3}x - \frac{1}{6} = \frac{3}{4}x$ LCD = 12

$\boxed{4}$

$$4(2x) - 2(1) = 3(3x)$$

eliminates 'middle' step.

eq 2: Solve: $\frac{2x+5}{3} = \frac{1}{2} + \frac{x}{6}$

$$\frac{6(2x+5)}{3} = \frac{6(1)}{2} + \frac{6(x)}{6} \quad \text{LCD} = 6$$

$$2(2x+5) = 3 + x$$

$$4x + 10 = 3 + x$$

$$3x = -7$$

$$x = \frac{-7}{3}$$

Check:

$$2\left(\frac{-7}{3}\right) + 5 = \frac{1}{2} + \left(\frac{-7}{6}\right)$$

$$\frac{-14}{3} + \frac{15}{3} = \frac{1}{2} + \left(\frac{-7}{3}\right)\left(\frac{1}{6}\right)$$

$$\left(\frac{1}{3}\right) = \frac{9}{18} - \frac{7}{18}$$

$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{18}$$

$$\frac{1}{9} = \frac{1}{9} \quad \checkmark$$

eq 3: Solve $0.002x + 0.05 = 0.03x - 0.006$

* convert decimals to fractions (PreCalc. vs. Science)

$$\frac{2}{1000}x + \frac{5}{100} = \frac{3}{100}x - \frac{6}{1000}$$

LCD = 1000

$$\frac{1000(2x)}{1000} + \frac{1000(5)}{100} = \frac{1000(3x)}{100} - \frac{1000(6)}{1000}$$

$$2x + 10(5) = 10(3x) - 6$$

$$2x + 50 = 30x - 6 - 2x + 6$$

OVER →