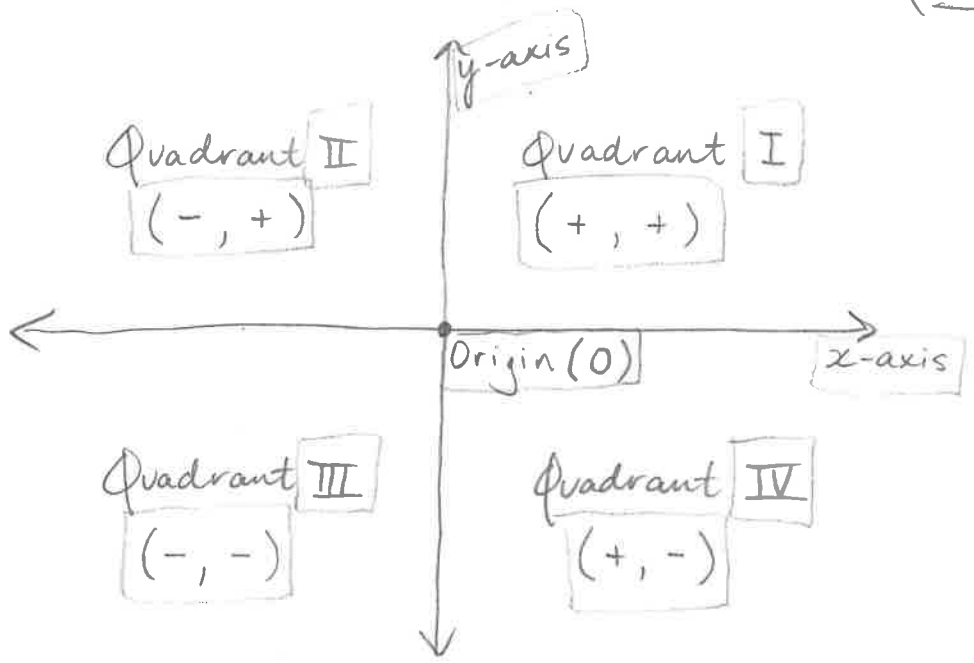
Ch. 4.2 - Linear Systems

Coordinate System (Cartesian Plane)

- recall that real numbers are able to be visually represented on a number line.
- an ORDERED PAIR can be represented as points in a Cartesian Plane also known as the rectangular coordinate system. (two-dimensional)

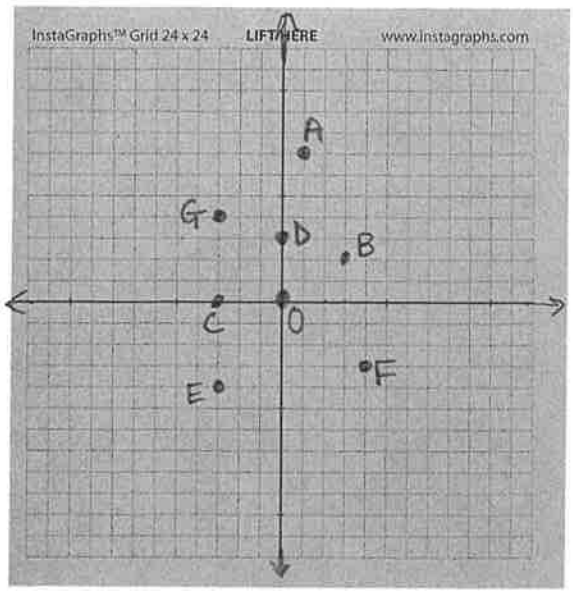


For each ordered pair, there is a unique point in the plane → (x, y)

* the plane is two-dimensional

- horizontal ("run") (x)
- vertical ("rise") (y)

The ordered pair (0,0) is located at the ORIGIN (O)



Plot the following points:

A (1, 7)

B (3, 2)

C (-3, 0)

D (0, 3)

E (-3, -4)

F (4, -3)

G (-3, 4)

* notice that (4, -3) and (-3, 4) plot different points.

They are called "ordered" pairs because order matters

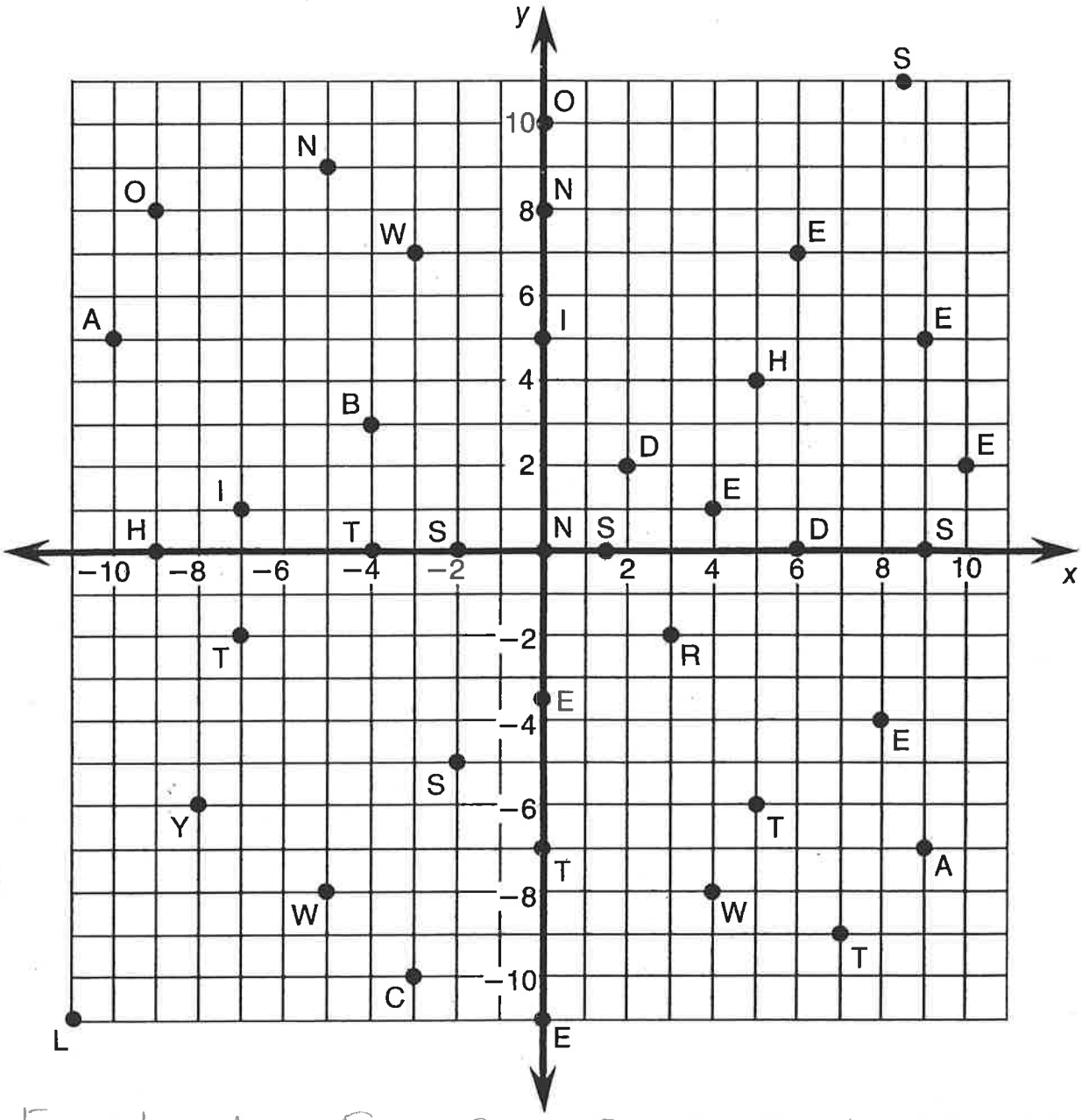
(x, y) not (y, x)

Linear Equations

A linear equation is an algebraic representation of a STRAIGHT LINE on a plane.

What Happened After a Burglar Broke Into a Tuba Factory ?

Each ordered pair at the bottom of the page represents a point on the coordinates below. Above each ordered pair, write the letter that appears at that point.



H E W A S C R E D I T E D

(5, 4)(10, 2)(−3, 7)(−10, 5)(−2, −5)(−3, −10)(3, −2)(8, −4)(6, 0)(0, 5)(−4, 0)(0, −11)(2, 2)

W I T H T W E N T Y O N E

(−5, −8)(−7, 1)(7, −9)(−9, 0)(−7, −2)(4, −8)(6, 7)(−5, 9)(0, −7)(−8, −6)(0, 10)(0, 0)(9, 5)

S T O L E N B A S S E S

(9, 0)(5, −6)(−9, 8)(−11, −11)(4, 1)(0, 8)(−4, 3)(9, −7)(−2, 0)(8.5, 11)(0, −3.5)(1.5, 0)

  OBJECTIVE 5–a: To locate a point in a coordinate plane given its coordinates.

Two types of Linear Equations:

i) Slope / y-intercept form (this section)

ii) Standard form (next section)

Slope / y-intercept form:

$$y = mx + b$$

x and y represent all points (x, y) on the line.

$$m = \text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$ } given two points on line.

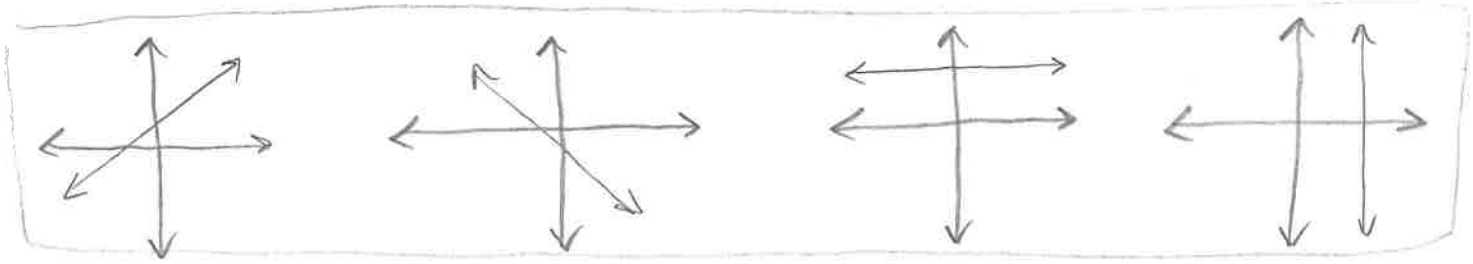
* m can be positive, negative, zero, or undefined.

b = y-intercept = where the line crosses the y-axis

* b can be positive, negative, zero, or may not exist

Slope

There are four types of slopes (lines):



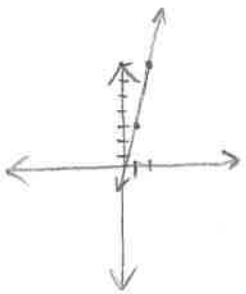
- a positive-slope line rises from (L) to (R)
- a negative-slope line falls from (L) to (R)
- a zero-slope line is exactly HORIZONTAL
  (has the equation $y = b$ (since $m = 0$))
- an undefined-slope line is exactly VERTICAL
  (has the equation $x = \#$ (# is the x-int.)

Calculating slope given two points:

ex1: Find the slope of each of the following lines possessing the points:

a) (1, 3) and (2, 7)

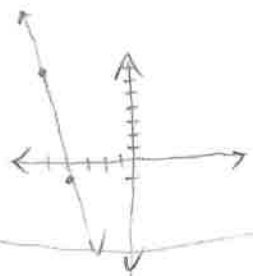
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{2-1} = \frac{4}{1} = \boxed{4}$$



$$= \frac{3-7}{1-2} = \frac{-4}{-1} = \boxed{4}$$

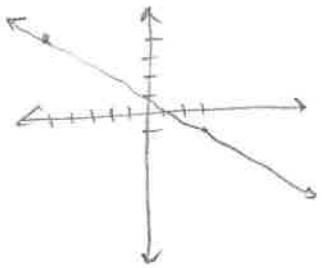
b) (−5, 7) and (−4, −1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-7}{-4-(-5)} = \frac{-8}{1} = \boxed{-8}$$



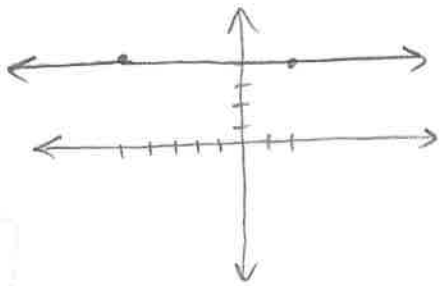
c) $(-5, 4)$ and $(3, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{3-(-5)} = \boxed{\frac{-5}{8}}$$



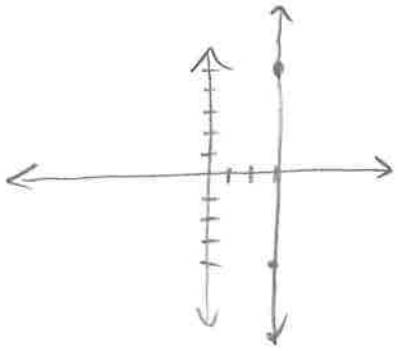
d) $(2, 4)$ and $(-5, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{-5-2} = \frac{0}{-7} = \boxed{0}$$



e) $(3, 5)$ and $(3, -4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-5}{3-3} = \frac{-9}{0} = \boxed{\text{UNDEFINED}}$$



eg2: The slope of a line is 2. The line passes through the points (4, 8) and (-1, k). Find k.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2 = \frac{8 - k}{4 - (-1)}$$

$$2 = \frac{8 - k}{5}$$

$$10 = 8 - k$$

$$\boxed{k = -2}$$

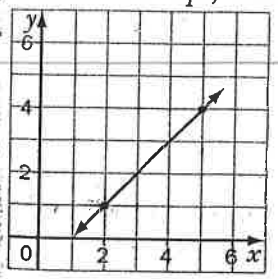
HANDOUT: # 1-6, 7-23 (odds), 24-30

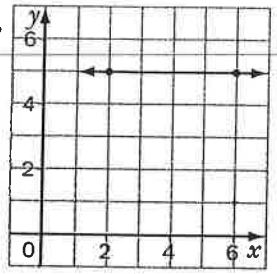
+

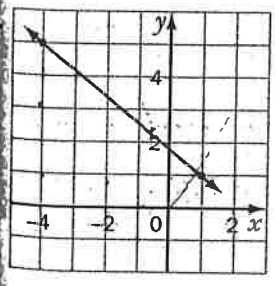
p. 129 # 1-5

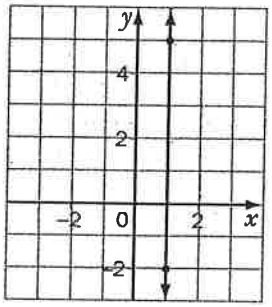
Practice

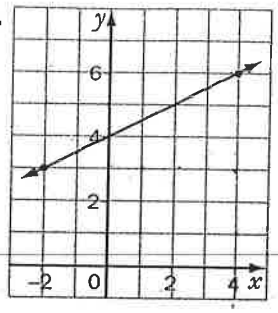
a) *Without calculating the slope, state whether the slope of each line is positive, negative, zero, or undefined.*
b) *State the rise.*
c) *State the run.*
d) *Calculate the slope, where possible.*

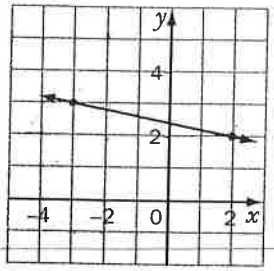
1. 

2. 

3. 

4. 

5. 

6. 

Find the slope of the line passing through the points.

7. (0, 0) and (2, 3)
8. (0, 0) and (–2, –4)
9. (1, 3) and (2, 7)
10. (–4, 5) and (6, 5)
11. (5, –2) and (–3, 4)
12. (0, 6) and (4, 0)
13. (–5, 7) and (–4, –2)
14. (–2, 5) and (0, 8)
15. (–6, –5) and (0, 0)
16. (–8, –7) and (–4, –3)
17. (5, 7) and (5, –3)
18. (–6, –1) and (–2, 5)

Find the slope of the line passing through the points.

19. (3.4, 1.6) and (5.4, 2.2)
20. (0, –1.7) and (0.3, –3.8)
21. (11.9, –2.3) and (15.4, 8.2)
22. $\left(\frac{1}{2}, 4\right)$ and (2, –6)
23. $\left(\frac{1}{3}, 1\frac{1}{2}\right)$ and $\left(2, 3\frac{1}{2}\right)$

24. The slope of a line is 3. The line passes through (2, k) and (4, 1). Find the value of k.

25. The slope of a line is –2. The line passes through (t, –1) and (–4, 9). Find the value of t.

Write the coordinates of two points on a line that satisfies the given condition.

26. The line rises from left to right.
27. The line is horizontal.
28. The line falls from left to right.
29. The line is vertical.

Applications and Problem Solving

30. Visualization Given a point on the line and the slope, sketch the graph of the line.

a) (2, 3), $m = 2$
b) (–1, 1), $m = 3$
c) (0, 4), $m = -2$
d) (–3, 0), $m = \frac{1}{2}$
e) (–3, –2), $m = \frac{2}{3}$
f) (–3, 4), $m = \frac{-4}{3}$
g) (4, –1), $m = 0$
h) (–4, 5), m is undefined

ANSWERS

Section 6.3 pp. 267–269

Practice **1. a)** positive **b)** 3 **c)** 3 **d)** 1 **2. a)** zero **b)** 0 **c)** 4 **d)** 0 **3. a)** negative **b)** –4 **c)** 5 **d)** $-\frac{4}{5}$ **4. a)** undefined **b)** 7 **c)** 0 **d)** undefined **5. a)** positive **b)** 3 **c)** 6 **d)** $\frac{1}{2}$ **6. a)** negative **b)** –1 **c)** 5 **d)** $-\frac{1}{5}$ **7.** $\frac{3}{2}$ **8.** 2 **9.** 4 **10.** 0 **11.** $-\frac{3}{4}$ **12.** $-\frac{3}{2}$ **13.** –9 **14.** $\frac{3}{2}$ **15.** $\frac{5}{6}$ **16.** 1 **17.** undefined **18.** $\frac{3}{2}$ **19.** 0.3 **20.** –7 **21.** 3 **22.** $-\frac{20}{3}$ **23.** $\frac{6}{5}$ **24.** –5 **25.** 1 **26.** Answers may vary. (0, 0), (1, 1) **27.** Answers may vary. (0, 0), (3, 0) **28.** Answers may vary. (0, 0), (1, –1) **29.** Answers may vary. (0, 0), (0, 7)

Applications and Problem Solving **31.** Answers may vary. **a)** (3, 3) **b)** (3, 1) **c)** (3, 4) **d)** (1, 4) **e)** (4, 2) **f)** (4, 3) **g)** (2, 4) **h)** (4, 16) **i)** (1, 6) **j)** (5, 1) **k)** (4, 7)

Graphing a Linear Equation

Given a linear equation of the form $y = mx + b$ (slope/y-intercept form), there are two methods by which to graph the line:

Method 1:

(1) Make sure the equation is in $y = mx + b$ form;

(2) Plot the y-intercept (if it exists)

↳ b-value

* NOTE: only a vertical line other than $x = 0$ will NOT have a y-intercept

(eg: $x = 3$, $x = -1$, $x = \pi$)

(3) Using the slope, plot a second point relative to the y-intercept. Remember, slope $= \frac{\text{RISE}}{\text{RUN}} = \frac{\Delta y}{\Delta x}$. Also remember that a negative fraction has its negative sign in the NUMERATOR.

* NOTE: only TWO points are required to graph a line!

Method 2 :

① Make sure the equation is in $y = mx + b$ form;

② Create a table of values and let x equal two values that are divisible by the 'run' (denominator) of the slope.

* NOTE :- if slope is 0, find y-intercept and graph the horizontal line.

- if slope is undefined, graph the vertical line.

③ Solve for y in the table of values

④ Plot the two points and draw a line through them.

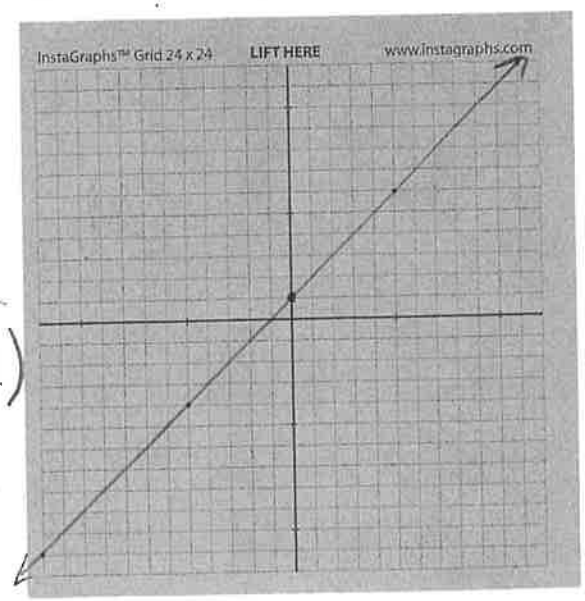
* NOTE: You may want a third point as a CHECK!

eg 1: Graph each of the following using both methods:

a) $y = x + 1$

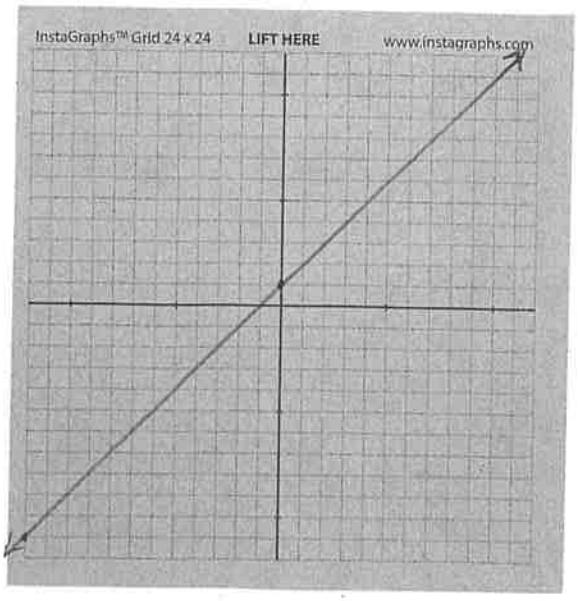
Method ①: y-intercept = 1

Slope = $1 = \frac{1}{1}$ (up 1, right 1)



Method ②:

x	y
0	1
2	3
-3	-2



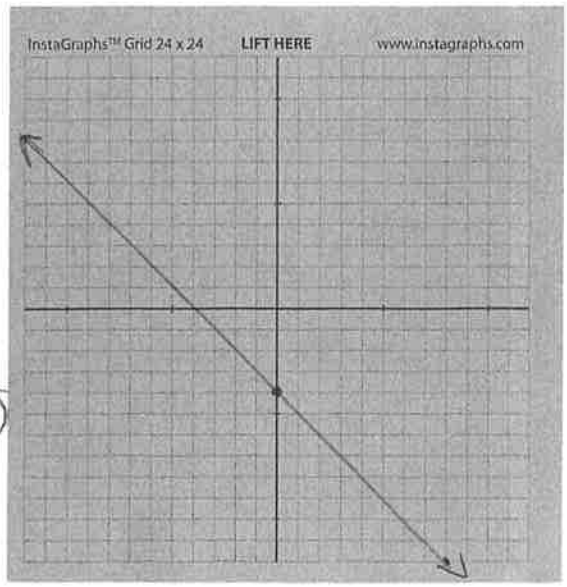
b) $y = -x - 4$

Method ①:

y-intercept = -4

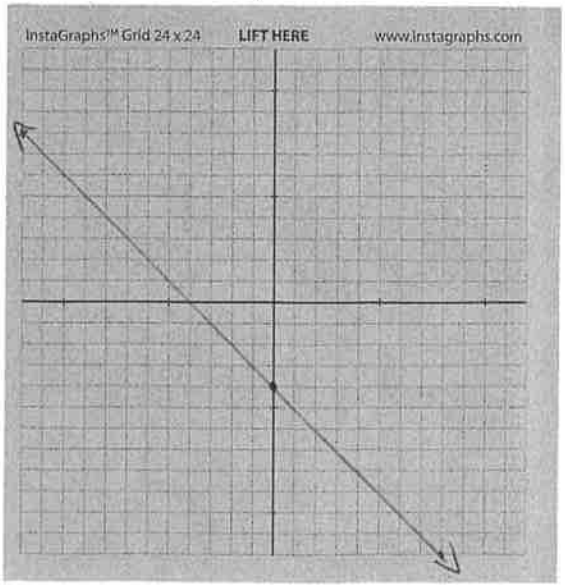
Slope = $-1 = \frac{-1}{1}$

(down 1, right 1)



Method ②:

x	y
0	-4
3	-7
-2	-2

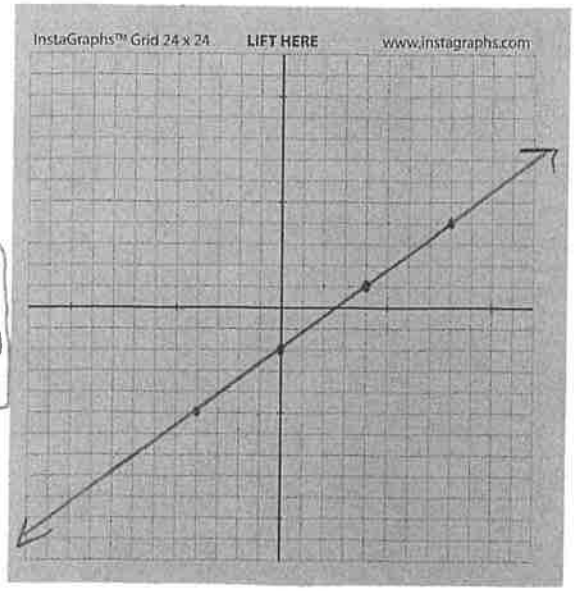


c) $y = \frac{3}{4}x - 2$

Method ①:

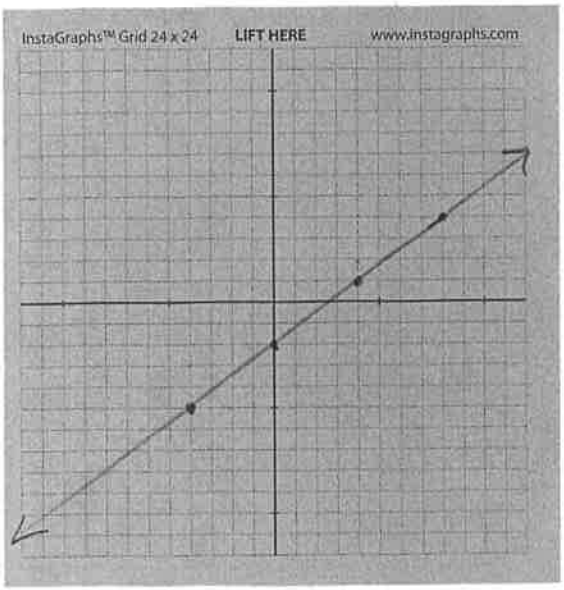
y-int. = -2

Slope = $\frac{3}{4}$ (up 3, right 4)



Method ②:

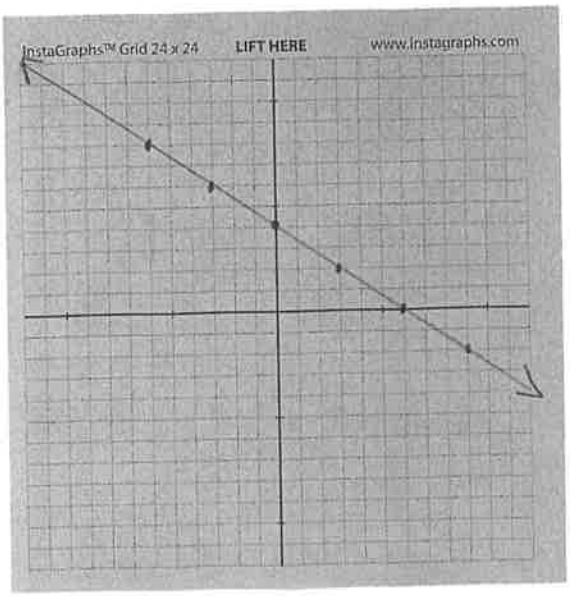
x	y	
0	-2	$(0, -2)$
4	1	$(4, 1)$
-4	-5	$(-4, -5)$



d) $y = -\frac{2}{3}x + 4$

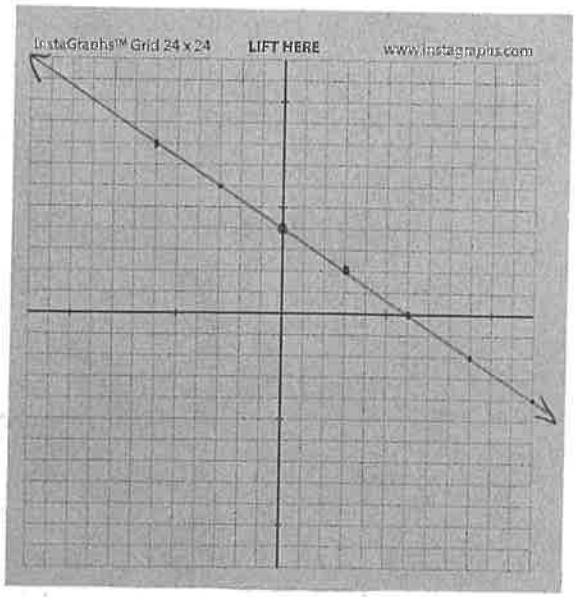
Method ①: y-int. = 4

Slope = $\frac{-2}{3}$ (down 2, right 3)



Method ②:

x	y
0	4
3	2
-3	6

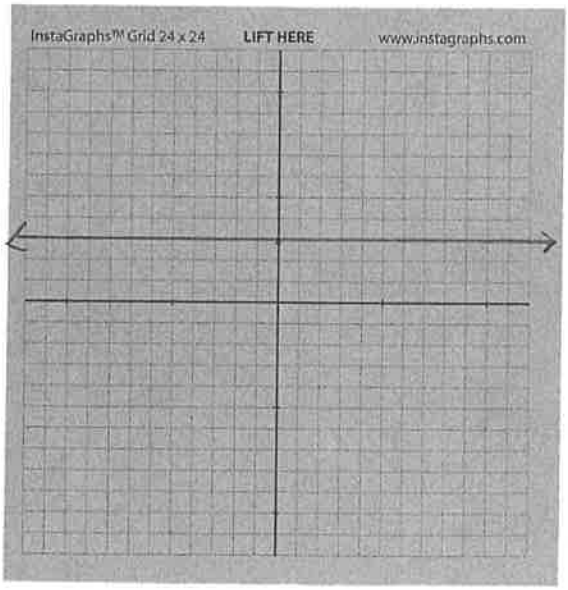


e) $y = 3$

$y = 0x + 3$

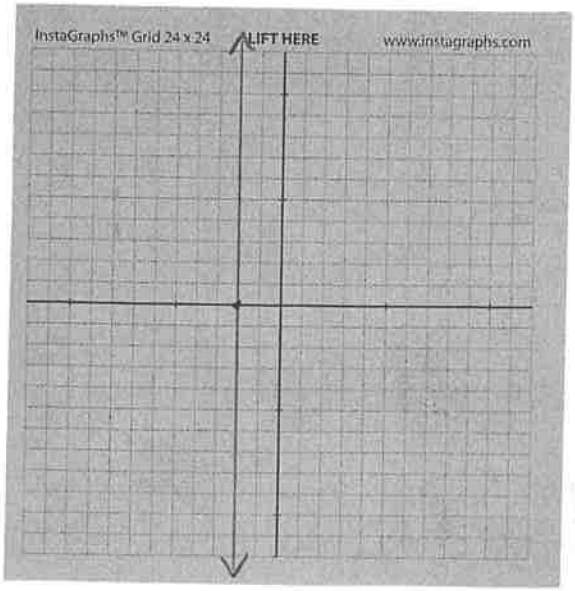
y-int. = 3

Slope = 0 (horizontal line)



f) $x = -2$

- undefined slope } vertical line
- no y-int.



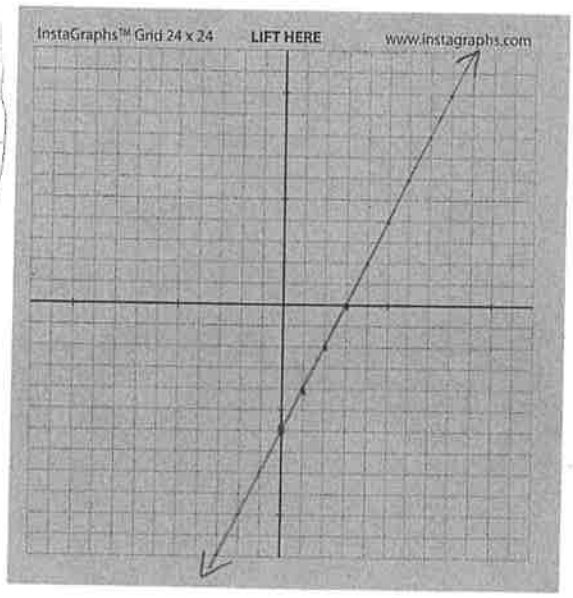
eg2: Graph $y = 2(x-3)$ using either method

$y = 2(x-3)$

$y = 2x - 6$

(1): y-int $= -6$

Slope $= 2 = \frac{2}{1} =$ 2up, 1 right



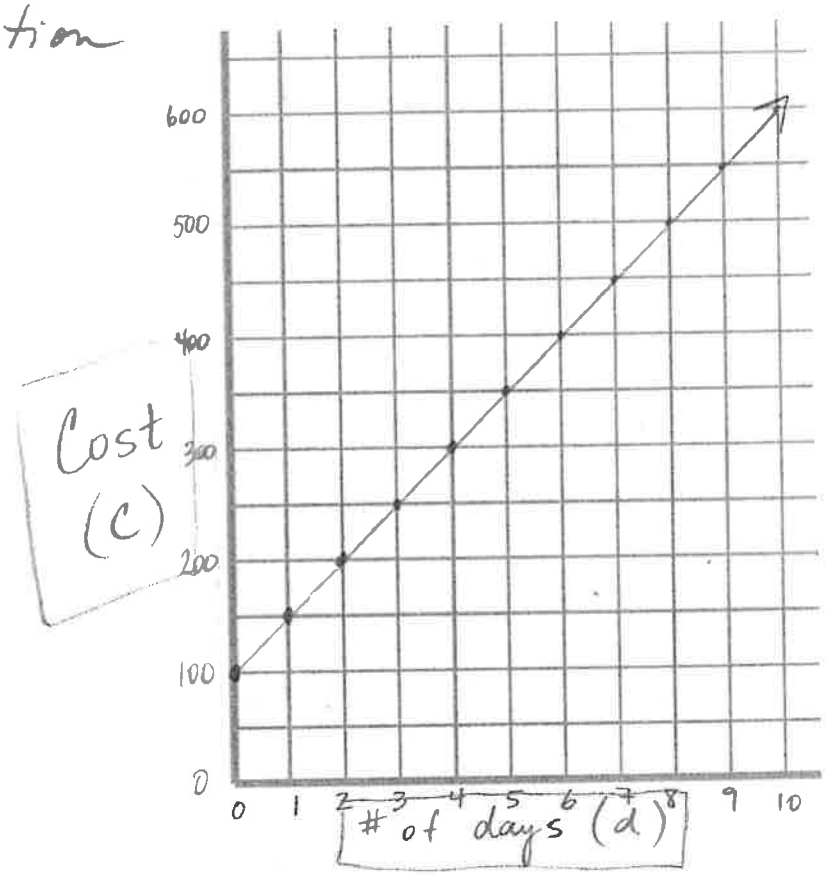
(2)

x	y
0	-6
2	-2
-2	-10

eg3: The cost of renting a car is $100 up front, plus an extra $50/day ⇒ see equation:

$$C = 50d + 100$$

a) Graph the equation



b) How much would it cost to rent a car for 6 days? 10 days?

$C = 50(6) + 100 = 300 + 100 = \boxed{\$400}$

$C = 50(10) + 100 = 500 + 100 = \boxed{\$600}$

p. 130 - 134 #6 - 13

Ch. 4.3 - Graphing in the Form $Ax + By = C$

Two Types of Linear Equations:

i) Slope/y-intercept form: $y = mx + b$

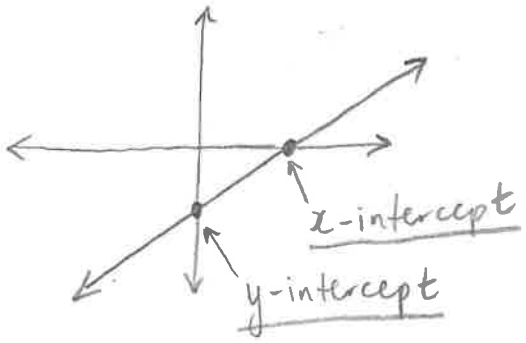
ii) Standard form $\Rightarrow$ $Ax + By = C$

Useful terminology:

y-intercept - where a line crosses the y-axis (x-value is 0)

x-intercept - where a line crosses the x-axis (y-value is 0).

eg:



Question: i) All lines have a y-intercept except VERTICAL LINES. (excluding $x = 0$)

ii) All lines have an x-intercept except HORIZONTAL LINES. (excluding $y = 0$)

Graphing a Linear Equation in Standard Form: ($Ax + By = C$)

Step 1: a) Find the y-intercept (set $x = 0$)

b) Find the x-intercept (set $y = 0$)

* TWO points!

Step 2: Pick another x-value and solve for y to get third point.

Step 3: Draw a line through the 3 points.

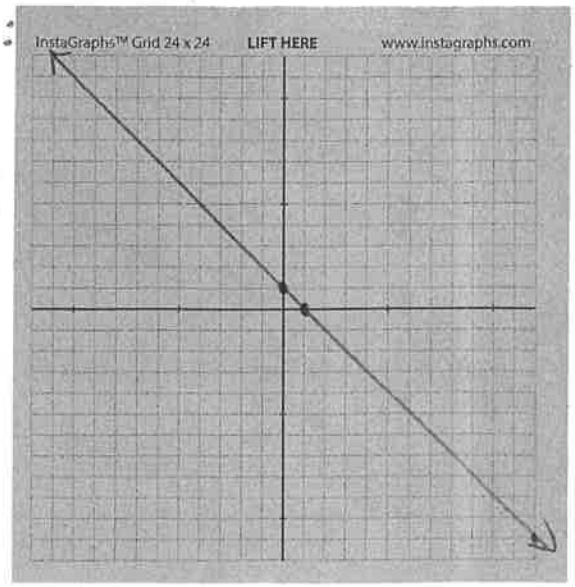
OR

Convert Standard form to Slope/y-int. form and graph.

eg: Graph each of the following:

a) $x + y = 1$

x	y	
0	1	$0 + y = 1$
1	0	$x + 0 = 1$
4	-3	$4 + y = 1$
		$y = -3$



b) $2x - y = -4$

x	y
0	4
-2	0
-5	-6

$2(0) - y = -4$

$y = 4$

$2x - 0 = -4$

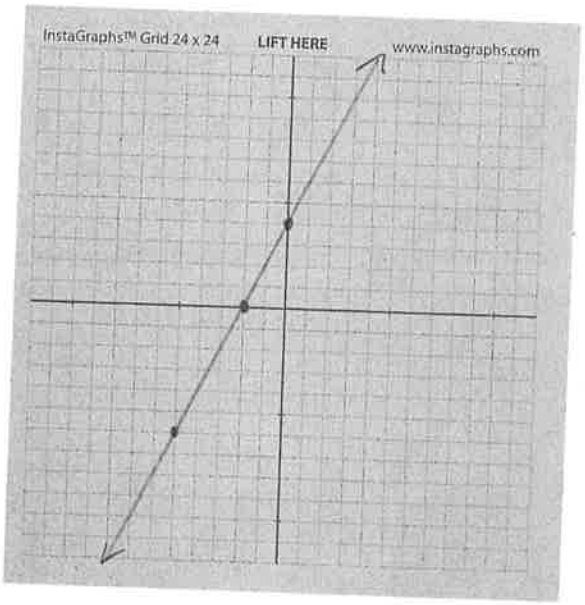
$x = -2$

$2(-5) - y = -4$

$-10 - y = -4$

$-y = 6$

$y = -6$



c) $3x + 2y = 6$

x	y
0	3
2	0
-2	6

$3(0) + 2y = 6$

$2y = 6$

$y = 3$

$3x + 2(0) = 6$

$3x = 6$

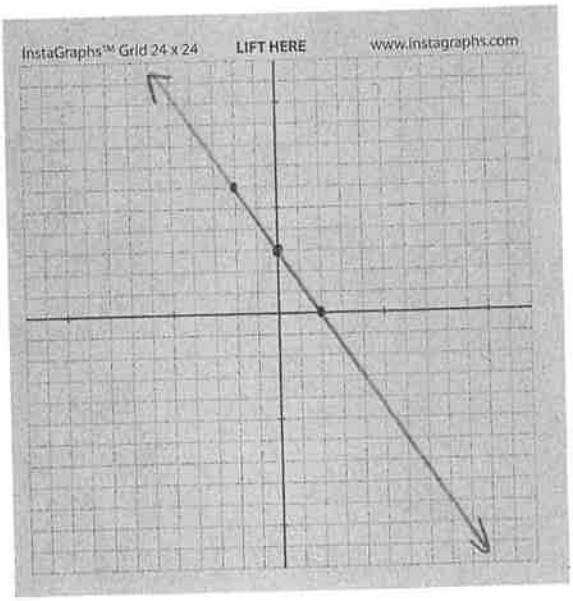
$x = 2$

$3(-2) + 2y = 6$

$-6 + 2y = 6$

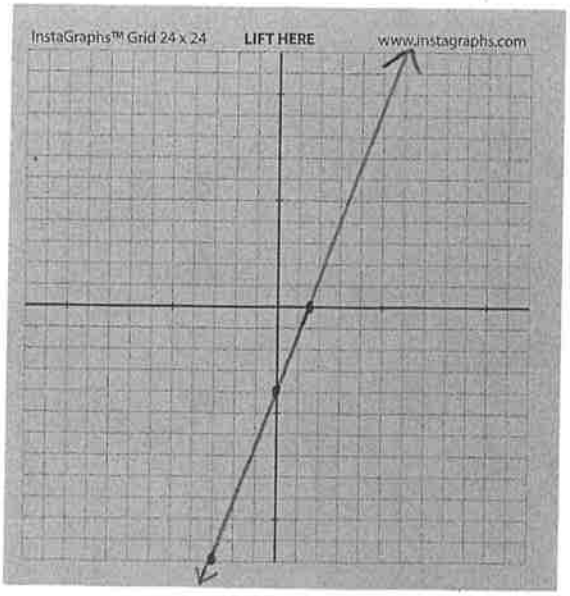
$2y = 12$

$y = 6$



d) $\frac{2}{3}x - \frac{1}{4}y = 1$

x	y	
0	-4	$\frac{2}{3}(0) - \frac{1}{4}y = 1$ $-\frac{1}{4}y = 1$ $y = -4$
$\frac{3}{2}$	0	$\frac{2}{3}x - \frac{1}{4}(0) = 1$ $\frac{2}{3}x = 1$ $x = \frac{3}{2}$
-3	-12	$\frac{2}{3}(-3) - \frac{1}{4}y = 1$ $-2 - \frac{1}{4}y = 1$ $-\frac{1}{4}y = 3$ $y = -12$



Horizontal and/or Vertical Lines Re-visited

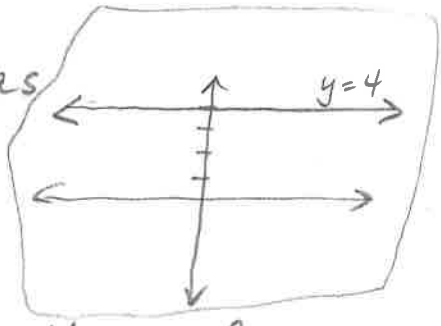
Horizontal Lines: $y = \#$

Slope = 0

so... can be written as:

$0x + y = \#$

eg: $y = 4$ can be written as



$$0x + y = 4$$

* pick any value for x and the value of y will always be 4.

Key notes:
1. The line is horizontal
2. The line is PARALLEL to the x-axis
3. The y-intercept is 4 → (0, 4)
4. Any point (x, 4) is a solution to the equation.

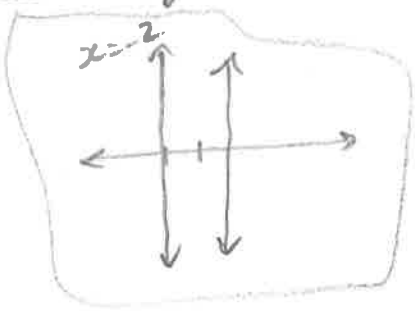
Vertical Lines: $x = \#$

Slope = UNDEFINED

so... can be written as: $x + 0y = \#$

eg: $x = -2$ can be written as

$$x + 0y = -2$$



* pick any value for y and the value of x will always be -2.

Key notes:
1. The line is vertical
2. The line is PARALLEL to the y-axis
3. The x-intercept is -2 → $(-2, 0)$
4. Any point $(-2, y)$ is a solution to the equation

Summary:

1. The graph $y = a$ is a HORIZONTAL line with y-intercept $(0, a)$.

2. The graph $x = b$ is a VERTICAL line with x-intercept $(b, 0)$.

p. 137-140
1-5

Ch. 4.4 - Matching Equations of Graphs

- equations can be matched to a particular graph by testing points that exist on the graph in the equation.
  - test at least <u>two</u> points (to be 'safe', test <u>three</u>.)

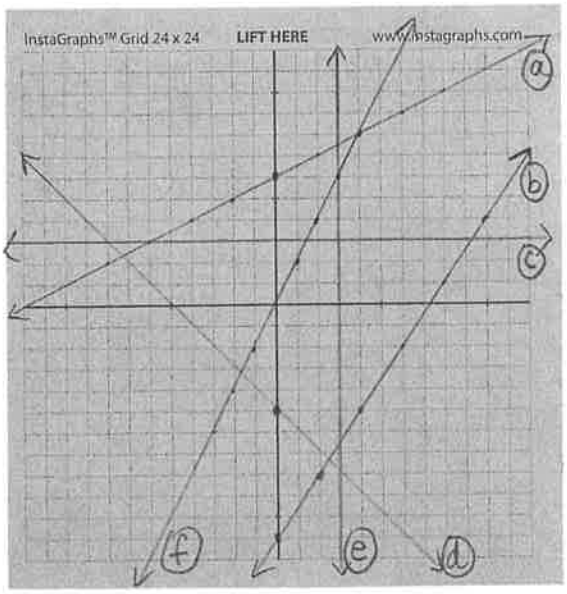
Steps

1. Select two points on the graph (if possible, select the x-intercept $(a, 0)$ and the y-intercept $(0, b)$ → do so if a and b are INTEGERS), then plug into the equation.
2. Pick a third point and plug in to be sure
3. To be a solution, all test points must 'work' (be TRUE).

* helpful hint (in certain situations):

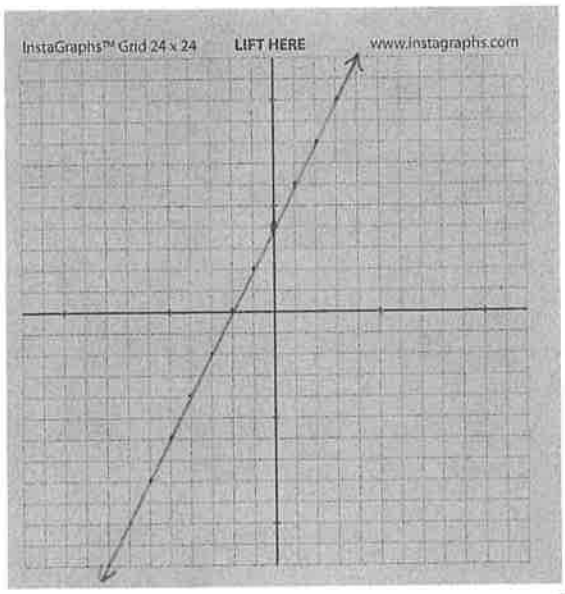
  If equation(s) are in slope/y-intercept form, use <u>b-value (y-intercept)</u> as a selection criterion.

eg 1: Match each of the following graphs to its equation:



$x = 3$	e
$y = 2x$	f
$y = \frac{1}{2}x + 6$	a
$y = 3$	c
$y = -x - 5$	d
$y = \frac{3}{2}x - 11$	b

eg 2: Which one of the equations match the graph? a) $y = -2x + 4$ b) $y = 4x + 4$ c) $y = 2x + 4$



POINTS TO TEST:

$(0, 4)$ $(-2, 0)$

a) $4 = -2(0) + 4$ a) $0 = -2(-2) + 4$

$4 = 4$ ✓ $0 = 8$ ×

b) $4 = 4(0) + 4$ b) $0 = 4(-2) + 4$

$4 = 4$ ✓ $0 = -4$ ×

c) $4 = 2(0) + 4$ c) $0 = 2(-2) + 4$

$4 = 4$ ✓ $0 = 0$ ✓

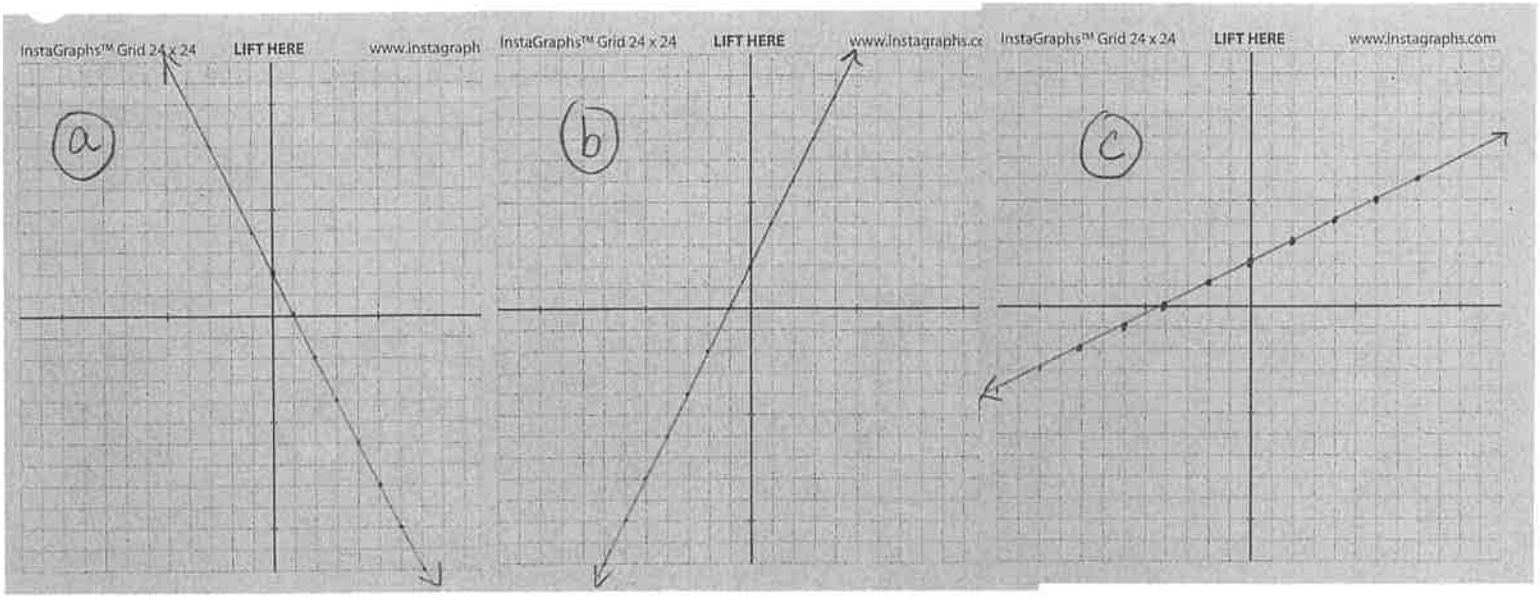
Third point for (c): $(-1, 2)$: $2 = 2(-1) + 4$

$2 = -2 + 4$

$2 = 2$ ✓

(c)

eg3: Which one of the graphs matches the equation $x - 2y = -4$?



@: (0, 2)

$0 - 2(2) = -4$

$-4 = -4$ ✓

(1, 0)

$1 - 2(0) = -4$

$1 = -4$ ×

or:

$-2y = -x - 4$

$y = \frac{1}{2}x + 2$

slope, y-int!

(b) (0, 2)

$0 - 2(2) = -4$

$-4 = -4$ ✓

(-1, 0)

$-1 - 2(0) = -4$

$-1 = -4$ ×

(c) (0, 2)

$0 - 2(2) = -4$

$-4 = -4$ ✓

(-4, 0)

$-4 - 2(0) = -4$

$-4 = -4$ ✓

(2, 3)

$2 - 2(3) = -4$

$2 - 6 = -4$

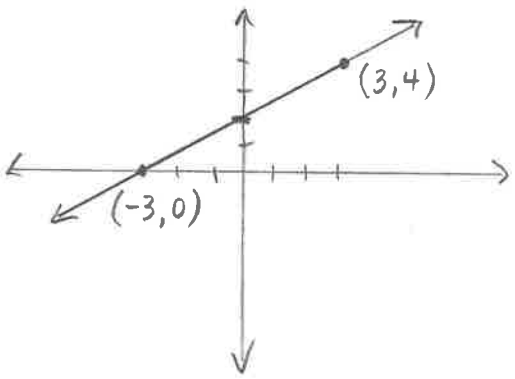
$-4 = -4$ ✓

(c)

Determining the Equation of a Graph

Given the graph,

find the equation of the graphed line.



$y = mx + b$

SLOPE — y-INTERCEPT

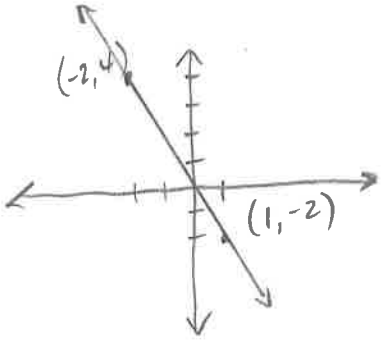
$b = \underline{2}$ (from graph)

$m = \frac{\text{RISE}}{\text{RUN}} = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{4}{6} = \frac{2}{3}$

$y = \frac{2}{3}x + 2$

Given:



Find equation of line.

$b = \underline{0}$ $m = \frac{-6}{3} = -2$

$y = -2x$

p. 144 – 152 # 1–10, 12 *11 for a challenge.

+ Chapter Review! ← OMIT #5 in review