

15.  $f(x) = \frac{x^2 - 4}{4 - x^2} = -1$  has holes when  $x = 2, -2$ . Answer is a.

16.  $f(x) = \frac{3x+2}{2x+3} = \frac{\frac{3x}{x} + \frac{2}{x}}{\frac{2x}{x} + \frac{3}{x}} = \frac{3 + \frac{2}{x}}{2 + \frac{3}{x}} = \frac{3 + \frac{2}{\infty}}{2 + \frac{3}{\infty}} = 1.5^+$ . Answer is c.

17. To have a horizontal asymptote of  $y = 0, n < m$ . Answer is d.

18. By definition, answer is c.

19. By definition, answer is c.

20. Symmetry about the  $y$ -axis requires an even power of  $x$ . Answer is c.

21. The only function with a horizontal asymptote of  $y = -2$  is a. Answer is a.

22, 23, 24, 25. If  $h > 0$ , graph in quadrant I, IV; if  $k > 0$ , in quadrant I, II. If  $a > 0$ , graph opens up; if  $b > 0$ , graph opens right.

22. Answer is d.

23. Answer is a.

24. Answer is c.

25. Answer is a.

26. By definition, answer is b.

27. Volume  $= x^2 y = 30 \rightarrow y = \frac{30}{x^2}$ , Surface Area  $x^2 + 4xy = x^2 + 4x \cdot \frac{30}{x^2} = x^2 + \frac{120}{x}$ . Answer is d.

28. Cost/km  $= 0.20 + \text{Rental Cost per } x \text{ km} = 0.20 + \frac{25}{x}$  Answer is d.

29.  $x$ -intercept:  $\frac{ax+b}{cx+d} = 0 \rightarrow ax+b=0 \rightarrow x = -\frac{b}{a}$ ;  $y$ -intercept:  $f(0) = \frac{a(0)+b}{c(0)+d} = \frac{b}{d}$ . Answer is b.

30. Horizontal asymptote:  $f(x) = \frac{ax+b}{cx+d} = \frac{\frac{ax}{x} + \frac{b}{x}}{\frac{cx}{x} + \frac{d}{x}} = \frac{a + \frac{b}{x}}{c + \frac{d}{x}} = \frac{a + \frac{\infty}{\infty}}{c + \frac{\infty}{\infty}} = \frac{a+0}{c+0} = \frac{a}{c}$

Vertical asymptote:  $cx+d=0 \rightarrow cx=-d \rightarrow x = \frac{-d}{c}$ . Answer is d.

## Logarithms Solutions

### 5.1 Exercise Set

1. a)  $\frac{(3^{\frac{1}{3}})^{10}}{9} = \frac{3^2 \cdot 3^{-3}}{3^2} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

b)  $\frac{(-4x^2y^{-2})^{-3}}{x^{-1}y^2} = \frac{(-4)^{-3}x^{-6}y^6}{x^{-1}y^2} = \frac{-y^4}{64x^5}$

c)  $\frac{125^{3x-1} \cdot 25^{1-2x}}{(\frac{1}{5})^{2x-3}} = \frac{5^{3(3x-1)} \cdot 5^{2(1-2x)}}{5^{-1(2x-3)}} = \frac{5^{9x-3} \cdot 5^{2-4x}}{5^{-2x+3}} = 5^{9x-3+2-4x+2x-3} = 5^{7x-4}$

d)  $\frac{2x^4 \cdot 3^{5x} - 4x^3 \cdot 3^{5x}}{x^3 - 2x^2} = \frac{2x^3 \cdot 3^{5x}(x-2)}{x^2(x-2)} = 2x \cdot 3^{5x}$

e)  $(4^{-x} \cdot 8^x)^2 = 4^{-2x} \cdot 8^{2x} = (2^2)^{-2x} \cdot (2^3)^{2x} = 2^{-4x} \cdot 2^{6x} = 2^{-4x+6x} = 2^{2x} = 4^x$

f)  $\frac{2^x(2^x+2^{-x}) - 2^y(2^x-2^{-x})}{2^{-2}} = \frac{2^{2x} + 2^0 - 2^{2x} + 2^0}{2^{-2}} = \frac{1+1}{2^{-2}} = 2 \cdot 2^2 = 2^3 = 8$

2. a)  $4^{x^2-x} = 1 \rightarrow 4^{x^2-x} = 4^0 \rightarrow x^2 - x = 0 \rightarrow x(x-1) = 0 \rightarrow x = 0, 1$

b)  $3^{x^2} = 9 \cdot 3^{-x} \rightarrow 3^{x^2} = 3^2 \cdot 3^{-x} \rightarrow 3^{x^2} = 3^{-x+2} \rightarrow x^2 = -x+2 \rightarrow x^2+x-2=0 \rightarrow (x+2)(x-1)=0 \Rightarrow x=-2, 1$

c)  $4^{\sqrt{x+1}} = 2^{3x-2} \rightarrow 2^{2\sqrt{x+1}} = 2^{3x-2} \rightarrow 2\sqrt{x+1} = 3x-2 \rightarrow (2\sqrt{x+1})^2 = (3x-2)^2 \rightarrow$

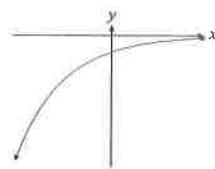
$4(x+1) = 9x^2 - 12x + 4 \rightarrow 9x^2 - 16x = 0 \rightarrow x(9x-16) = 0 \rightarrow x = 0, \frac{16}{9}$  Check, reject 0,  $\therefore x = \frac{16}{9}$

d)  $4^{-|x+1|} = \frac{1}{16} \rightarrow 4^{-|x+1|} = 4^{-2} \rightarrow -|x+1| = -2 \rightarrow |x+1| = 2 \rightarrow x+1 = 2 \text{ or } x+1 = -2 \rightarrow x = 1, -3$

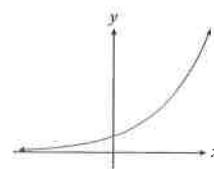
e)  $4^{-2x+1} = 8^{x-4} \rightarrow 2^{2(-2x+1)} = 2^{3(x-4)} \rightarrow 2^{-4x+2} = 2^{3x-12} \rightarrow -4x+2 = 3x-12 \rightarrow -7x = -14 \rightarrow x = 2$

f)  $9^{2x-1} = \left(\frac{1}{27}\right)^{x+2} \rightarrow 3^{2(2x-1)} = 3^{-3(x+2)} \rightarrow 2(2x-1) = -3(x+2) \rightarrow 4x-2 = -3x-6 \rightarrow 7x = -4 \rightarrow x = -\frac{4}{7}$

3. a)  $y = -ab^x = -f(x)$  will reflect the graph over the  $x$ -axis.



- b)  $y = ab^{-x} = f(-x)$  will reflect the graph over the  $y$ -axis.



4. a) Graph  $y = 3^{x+2} - 3$  is shifted left two units and down three units.

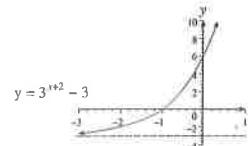
Domain: all real numbers

Range:  $y > -3$

$x$ -intercept: -1

$y$ -intercept: 6

Asymptote:  $y = -3$



- b) Graph  $y = 3^{-x} + 2$  is reflected about the  $y$ -axis and up two units.

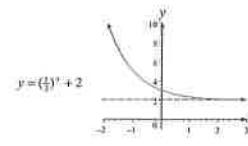
Domain: all real numbers

Range:  $y > 2$

$x$ -intercept: no  $x$ -intercept

$y$ -intercept: 3

Asymptote:  $y = 2$



- c) Graph of  $y = -3^{-x}$  is reflected about both the  $x$ -axis and  $y$ -axis.

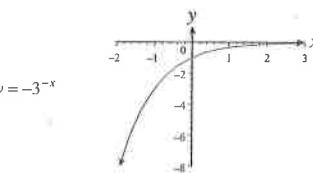
Domain: all real numbers

Range:  $y < 0$

$x$ -intercept: no  $x$ -intercept

$y$ -intercept: -1

Asymptote:  $y = 0$



5. a) D

b) C

c) F

d) A

e) E

f) B

6. a)  $y = b^x \rightarrow 3 = b^{-1} \rightarrow b = \frac{1}{3}$

b)  $y = b^x \rightarrow 27 = b^{\frac{3}{2}} \rightarrow b = 27^{\frac{1}{2}} \rightarrow b = (3^3)^{\frac{1}{2}} \rightarrow b = 3^{\frac{3}{2}} \rightarrow b = 9$

c)  $y = b^x \rightarrow \frac{1}{9} = b^{-\frac{3}{2}} \rightarrow b = \left(\frac{1}{9}\right)^{-\frac{1}{2}} \rightarrow b = (3^{-2})^{-\frac{1}{2}} \rightarrow b = 3^3 \rightarrow b = 27$

7.  $y = c \cdot 2^{kx} \rightarrow 4 = c \cdot 2^{k \cdot 0} \rightarrow c = 4 \rightarrow y = 4 \cdot 2^{kx} \rightarrow 256 = 4 \cdot 2^{12k} \rightarrow 64 = 2^{12k} \rightarrow 2^6 = 2^{12k} \rightarrow 12k = 6 \rightarrow k = \frac{1}{2}$ ,

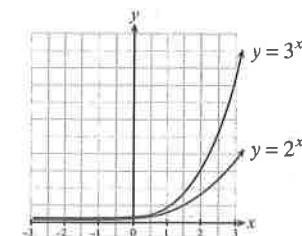
therefore,  $y = 4 \cdot 2^{\frac{1}{2}x}$  or  $y = 2^{\frac{1}{2}x+2}$

8. When  $y = 2^x$  and  $y = 3^x$  has  $x < 0$ ,  $2^x > 3^x$

When  $y = 2^x$  and  $y = 3^x$  has  $x > 0$ ,  $3^x > 2^x$

$x$	-3	-2	-1	0	1	2	3
$2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$3^x$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

9. a)  $\frac{10^{8.9}}{10^{6.4}} = 10^{2.5} = 316$  times as strong



- b)  $1000 = 10^3$ . A 4.9 earthquake is a  $10^{4.9}$  measure. So the San Francisco earthquake has a  $10^{4.9} \cdot 10^3 = 10^{4.9+3} = 10^{7.9}$  or a Richter scale measure of 7.9

c)  $A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow A = 1000\left(1 + \frac{0.06}{4}\right)^{4 \times 8} \rightarrow A = 1000(1.015)^{32} \rightarrow A = \$1610.32$

d)  $A = A_0(x)^{\frac{t}{T}} \rightarrow A = 84\left(\frac{1}{2}\right)^{\frac{23}{4}} \rightarrow A = 1.56$  grams of argon-39 remains

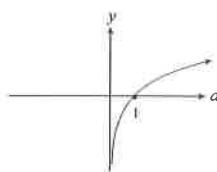
e)  $A = P(1 + \frac{r}{n})^{nt} \rightarrow A = 12250(1 + \frac{0.096}{12})^{12 \times 10} = \$31871.31$

f)  $A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow A = 30000000\left(1 + \frac{0.019}{1}\right)^{1 \times 32} \rightarrow A = 30000000(1.019)^{32} = 54789223 = 55$  million to the nearest million.

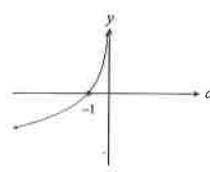
## 5.2 Exercise Set

1. a)  $4^2 = 16$    b)  $3^4 = 81$    c)  $6^{-2} = \frac{1}{36}$    d)  $10^{-2} = \frac{1}{100}$    e)  $32^{\frac{3}{5}} = 8$    f)  $8^{\frac{1}{3}} = 2$    g)  $5^0 = 1$    h)  $10^3 = 1000$    i)  $8^{\frac{2}{3}} = 4$    j)  $4^{-\frac{3}{2}} = \frac{1}{8}$
2. a)  $\log_2 16 = 4$    b)  $\log_8 64 = 2$    c)  $\log_{16} 2 = \frac{1}{4}$    d)  $\log_3 \frac{1}{9} = -2$    e)  $\log_3 1 = 0$   
f)  $\log_{10} 0.01 = -2$    g)  $\log_5 5 = 1$    h)  $\log_9 27 = \frac{3}{2}$    i)  $\log_8 16 = \frac{4}{3}$    j)  $\log_{\frac{2}{3}} \frac{81}{16} = -4$
3. a)  $f(x) = \log_2 8$   
 $2^y = 8$   
 $2^y = 2^3$   
 $y = 3$
- b)  $f(x) = \log_4 16$   
 $4^y = 16$   
 $4^y = 4^2$   
 $y = 2$
- c)  $f(x) = \log_8 2$   
 $8^y = 2$   
 $2^{3y} = 2$   
 $3y = 1$   
 $y = \frac{1}{3}$
- d)  $f(x) = \log_{16} 4$   
 $16^y = 4$   
 $4^{2y} = 4^1$   
 $2y = 1$   
 $y = \frac{1}{2}$
- e)  $f(x) = \log_5 1$   
 $5^y = 1$   
 $5^y = 5^0$   
 $y = 0$
- f)  $f(x) = \log_2 7$   
 $7^y = 7$   
 $7^y = 7^1$   
 $y = 1$
- g)  $f(x) = \log_a a$   
 $a^y = a$   
 $a^y = a^1$   
 $y = 1$
- h)  $f(x) = \log_a a^3$   
 $a^y = a^3$   
 $y = 3$
- i)  $f(x) = \log_b b^{-4}$   
 $b^y = b^{-4}$   
 $y = -4$
- j)  $f(x) = \log_s 0$   
 $s^y = 0$   
undefined  
log cannot be zero
4. a)  $x^3 = 27$   
 $x^3 = 3^3$   
 $x = 3$
- b)  $x = 4^{-3}$   
 $x = \frac{1}{64}$
- c)  $10^x = 1000$   
 $10^x = 10^3$   
 $x = 3$
- d)  $x^1 = 8$   
 $x = 8$
- e)  $x = 7^{-2}$   
 $x = \frac{1}{49}$
- f)  $9^x = 27$   
 $3^{2x} = 3^3$   
 $2x = 3$   
 $x = \frac{3}{2}$
- g)  $x^2 = 32$   
 $x = \sqrt{32}$   
 $x = 4\sqrt{2}$   
(reject  $x = -\sqrt{32}$ )
- h)  $x = 4^0$   
 $x = 1$
- i)  $32^x = 8$   
 $2^{5x} = 2^3$   
 $5x = 3$   
 $x = \frac{3}{5}$
- j)  $x^4 = 625$   
 $x^4 = 5^4$   
 $x = 5$
- k)  $x = 4^{\frac{3}{2}}$   
 $= (2^2)^{\frac{3}{2}}$   
 $= 2^3$   
 $x = 8$
- l)  $4^x = 0.25$   
 $4^x = \frac{1}{4}$   
 $4^x = 4^{-1}$   
 $x = -1$
- m)  $x = \sqrt[8]{2}$   
 $= (2^{\frac{1}{8}})^8$   
 $= 2^4$   
 $x = 16$
- n)  $x = \sqrt[4]{3}$   
 $= (3^{\frac{1}{2}})^4$   
 $= 3^2$   
 $x = 9$
- o)  $x^{\frac{1}{2}} = \sqrt{3}$   
 $x^{\frac{1}{2}} = 3^{\frac{1}{2}}$   
 $x = 3$
- p)  $(3x)^2 = 36$   
 $9x^2 = 36$   
 $x^2 = 4$   
 $x = 2$   
(reject  $x = -2$ )
- q)  $\sqrt{2^x} = 16$   
 $2^{\frac{1}{2}x} = 2^4$   
 $\frac{1}{2}x = 4$   
 $x = 8$
- r)  $\sqrt[3]{2^x} = 9$   
 $3^{\frac{1}{2}x} = 3^2$   
 $\frac{1}{2}x = 2$   
 $x = 4$
- s)  $x^2 + 24 = 7^2$   
 $x^2 = 25$   
 $x = \pm 5$
- t)  $(x-2)^2 = 10^{-2}$   
 $(x-2)^2 = \frac{1}{100}$   
 $x-2 = \pm \frac{1}{10}$   
 $x = 2 \pm \frac{1}{10}$   
 $x = 1.9 \text{ or } 2.1$
5. a)  $x-1 > 0$   
 $x > 1$
- b)  $x > 0$
- c)  $2-x > 0; \quad 2-x \neq 1$   
 $-x > -2; \quad -x \neq -1$   
 $x < 2; \quad x \neq 1$
- d)  $-x > 0$   
 $x < 0$
- e)  $y = \log_{x+1}(x-2)$  The base  $x+1 > 0$  and  $x+1 \neq 1$   
 $\therefore x > -1$  and  $x \neq 0$ .  
The log term is  $(x-2) > 0$  so  $x > 2$ .  
Take the intersection of  $x > -1, x \neq 0$  with  $x > 2$  which is  $x > 2$ .  
Therefore, the domain of  $y = \log_{x+1}(x-2)$  is  $x > 2$ .
- f)  $y = \log_{x-2}(x+1)$  The base is  $x-2 > 0$  and  $x-2 \neq 1$   
 $x > 2$  and  $x \neq 3$   
The log term is  $x+1 > 0$   
 $x > -1$   
Take the intersection of  $x > -1$  with  $x > 2$  which is  $x > 2$ .  
Therefore, the domain of  $y = \log_{x-2}(x+1)$  is  $x > 2, x \neq 3$ .
6. a) E      b) B      c) C      d) F      e) A      f) D

7. a)  $y = -\log_b a = -f(a)$  will reflect the graph over the  $x$ -axis.



b)  $y = \log_b(-a) = f(-a)$  will reflect the graph over the  $y$ -axis.



c)  $y = \log_{\frac{1}{b}} a = -\log_b a$ , which makes the graph the same as 7 a), above. (See question 10 as to why  $y = \log_{\frac{1}{b}} a = -\log_b a$ )

8.  $y = 5^x$  and  $y = \log_5 x$  are the inverse of each other. Therefore, if  $(a, b)$  is a point on  $y = 5^x$  then  $(b, a)$  must be a point on  $y = \log_5 x$ .

Two other points on  $y = \log_5 x$  are  $(5, 1)$  and  $(1, 0)$ .

9.  $f(x) = \log_2 x \rightarrow -f(x) = -\log_2 x : -f(x)$  is reflected over the  $x$ -axis. So putting a negative in front of a logarithmic statement reflects the equation over the  $x$ -axis. The point  $(1, 0)$  is on the  $x$ -axis, so the point will not change, therefore the answer is  $(1, 0)$ .

10. If  $y = \log_{\frac{1}{b}} a$  then  $a = \left(\frac{1}{b}\right)^y = b^{-y}$ , thus  $-y = \log_b a \rightarrow y = -\log_b a$ . So if  $(c, d)$  is a point on the graph  $y = \log_b a$  then  $(c, -d)$  must be on the graph  $y = \log_{\frac{1}{b}} a$ . Two other points on  $y = \log_{\frac{1}{b}} a$  are  $(\frac{1}{b}, 1)$  and  $(1, 0)$ .

11. a)  $\log 1253$

$$\log 1000 = x \rightarrow 10^x = 1000; \log 10000 = x \rightarrow 10^x = 10000$$

$$10^x = 10^3$$

$$x = 3$$

Thus  $3 < \log 1253 < 4$

b)  $\log 0.025$

$$\log 0.01 = x \rightarrow 10^x = 0.01; \log 0.1 = x \rightarrow 10^x = 0.1$$

$$10^x = 10^{-2}$$

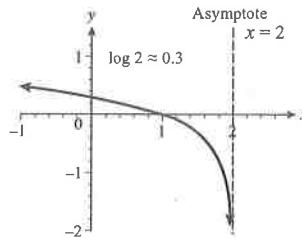
$$10^x = 10^{-1}$$

$$x = -2$$

$$x = -1$$

Thus  $-2 < \log 0.025 < -1$

12. Graph  $y = \log(2 - x)$



13. a)  $y = 8^{x-2} \rightarrow f^{-1}: x = 8^{y-2} \rightarrow \log_8 x = y-2 \rightarrow y = 2 + \log_8 x \rightarrow f^{-1}(x) = 2 + \log_8 x$

b)  $f(x) = 5^{4x-1} + 6 \rightarrow f^{-1}: x = 5^{4y-1} + 6 \rightarrow x - 6 = 5^{4y-1} \rightarrow \log_5(x-6) = 4y-1 \rightarrow$

$$4y = \log_5(x-6) + 1 \rightarrow f^{-1}(x) = \frac{1}{4}\log_5(x-6) + \frac{1}{4}$$

c)  $f: y+1 = \log_3(x-2) \rightarrow f^{-1}: x+1 = \log_3(y-2) \rightarrow y-2 = 3^{x+1} \rightarrow y = 3^{x+1} + 2 \rightarrow f^{-1}(x) = 3^{x+1} + 2$

d)  $f(x) = 2 + \log(5x-3) \rightarrow f^{-1}(x): x = 2 + \log(5y-3) \rightarrow x-2 = \log(5y-3) \rightarrow$

$$5y-3 = 10^{x-2} \rightarrow 5y = 10^{x-2} + 3 \rightarrow y = \frac{10^{x-2} + 3}{5} \rightarrow f^{-1}(x) = \frac{10^{x-2} + 3}{5}$$

### 5.3 Exercise Set

1. a)  $\log 6 = \log(2 \cdot 3) = \log 2 + \log 3$

b)  $\log 12 = \log(2^2 \cdot 3) = 2\log 2 + \log 3$

c)  $\log 72 = \log(2^3 \cdot 3^2) = 3\log 2 + 2\log 3$

d)  $\log 3200 = \log(2^5 \cdot 100) = 5\log 2 + 2$

e)  $\log 0.36 = \log \frac{36}{100} = \log\left(\frac{2^2 \cdot 3^2}{100}\right) = 2\log 2 + 2\log 3 - 2$

f)  $\log_2 216 = \log_2(2^3 \cdot 3^3) = 3\log_2 2 + 3\log_2 3 = 3 + \frac{3\log 3}{\log 2}$

g)  $\log 5.4 = \log \frac{2 \cdot 3^3}{10} = \log 2 + 3\log 3 - 1$

1. h)  $\log_6 180 = \frac{\log 180}{\log 6} = \frac{\log(2 \cdot 3^2 \cdot 10)}{\log(2 \cdot 3)} = \frac{\log 2 + 2 \log 3 + 1}{\log 2 + \log 3}$

i)  $\log_{18} 2160 = \frac{\log(2^3 \cdot 3^3 \cdot 10)}{\log(2 \cdot 3^2)} = \frac{3 \log 2 + 3 \log 3 + 1}{\log 2 + 2 \log 3}$

j)  $\log_{12} 0.108 = \log_{12} \left( \frac{108}{1000} \right) = \log_{12} 108 - \log_{12} 1000 = \frac{\log 108 - \log 1000}{\log 12} = \frac{\log(2^2 \cdot 3^3) - 3}{\log(2^2 \cdot 3)} = \frac{2 \log 2 + 3 \log 3 - 3}{2 \log 2 + \log 3}$

2. a)  $\log_3 81 = \log_3 3^4 = 4 \log_3 3 = 4$

b)  $\log_2 \frac{1}{32} = \log_2 2^{-5} = -5 \log_2 2 = -5$

c)  $\log_2 \sqrt[4]{8} = \log_2 2^{\frac{3}{4}} = \frac{3}{4} \log_2 2 = \frac{3}{4}$

d)  $\log_5 \sqrt{125} = \log_5 5^{\frac{3}{2}} = \frac{3}{2} \log_5 5 = \frac{3}{2}$

e)  $\log_9 27^{22} = 2.2 \frac{\log 3^3}{\log 3^2} = \frac{6.6 \log 3}{2 \log 3} = 3.3$

f)  $\log_4 \frac{1}{32} = \frac{\log 2^{-5}}{\log 2^2} = \frac{-5 \log 2}{2 \log 2} = \frac{-5}{2}$

g)  $(\log_4 8)(\log_{16} 32) = \frac{\log 2^3}{\log 2^2} \cdot \frac{\log 2^5}{\log 2^4} = \frac{(3 \log 2)(5 \log 2)}{(2 \log 2)(4 \log 2)} = \frac{15}{8}$

h)  $\frac{\log_{27} 81}{\log_{15} 125} = \frac{\log 3^4}{\log 3^3} \cdot \frac{\log 5^2}{\log 5^3} = \left( \frac{4 \log 3}{3 \log 3} \right) \left( \frac{2 \log 5}{3 \log 5} \right) = \frac{8}{9}$  i)  $\log_4 2 + \log_2 32 = \frac{\log 2}{2 \log 2} + \frac{\log 2^5}{\log 2} = \frac{\log 2}{2 \log 2} + \frac{5 \log 2}{\log 2} = \frac{11}{2}$

j)  $\log_9 16 - 2 \log_3 2 = \frac{\log 2^4}{\log 3^2} - 2 \log_3 2 = \frac{4 \log 2}{2 \log 3} - 2 \log_3 2 = 2 \log_3 2 - 2 \log_3 2 = 0$

3. a)  $\log 100x^2y^3 = \log 100 + \log x^2 + \log y^3 = 2 + 2 \log x + 3 \log y$  b)  $\log \frac{x^3}{1000y^2} = \log x^3 - \log 1000 - \log y^2 = 3 \log x - 2 \log y - 3$

c)  $\log(x^2 + y^3)^4 = 4 \log(x^2 + y^3)$

d)  $\log^4(x^2 + y^3)$  cannot be expanded

e)  $\log_s \frac{25x^2y^3}{z} = \log_s 25 + \log_s x^2 + \log_s y^3 - \log_s z = 2 \log_s x + 3 \log_s y - \log_s z + 2$

f)  $\log \sqrt{x^2(x+2)} = \frac{1}{2} \log [x^2(x+2)] = \frac{1}{2} \log x^2 + \frac{1}{2} \log(x+2) = \log x + \frac{1}{2} \log(x+2)$

g)  $4 \log_2(2x)^{12} = 48 \log_2 2x = 48 [\log_2 2 + \log_2 x] = 48 + 48 \log_2 x$

h)  $\log_a \sqrt{\frac{x^2y+1}{a^3}} = \frac{1}{2} [\log_a(x^2y+1) - \log_a a^3] = \frac{1}{2} \log_a(x^2y+1) - \frac{3}{2}$

i)  $\log \frac{(x^3+y)^3}{x^3} = \log(x^3+y)^3 - \log x^3 = 3 \log(x^3+y) - 3 \log x$

j)  $\log_3 \sqrt[3]{\frac{xy^3}{z^6}} = \frac{1}{3} \log \frac{xy^3}{z^6} = \frac{1}{3} [\log x + 3 \log y - 6 \log z] = \frac{1}{3} \log x + \log y - 2 \log z$

4. a)  $\log_5 x - \log_5 25 = \log_5 \frac{x}{25}$

b)  $\log_3 x - 2 \log_3 27 = \log_3 x - \log_3 x - \log_3 (3^3)^2 = \log_3 \frac{x}{729}$

c)  $\log \sqrt{x} + \log x^{\frac{3}{2}} = \log x^{\frac{1}{2}} + x^{\frac{3}{2}} = \log x^2$

d)  $\log(x^2 - 1) - \log(x+1) - \log x = \log \frac{(x^2 - 1)}{x(x+1)} = \log \frac{(x-1)(x+1)}{x(x+1)} = \log \left( \frac{x-1}{x} \right)$

e)  $\log(3x^2 - 5x - 2) - \log(x^2 - 4) - \log(3x+1) = \log \frac{(3x+1)(x-2)}{(x-2)(x+2)(3x+1)} = \log \left( \frac{1}{x+2} \right) = \log 1 - \log(x+2) = -\log(x+2)$

f)  $\log_3(2x-3) - \log_3(2x^2-x-3) + \log_3 3(x+1) = \log_3 \left[ \frac{3(2x-3)(x+1)}{(2x-3)(x+1)} \right] = \log_3 3 = 1$

g)  $2[\log(x^2 - 1) - \log(x+1) - \log(x-1)] = 2 \log \left[ \frac{(x-1)(x+1)}{(x-1)(x+1)} \right] = 2 \log 1 = 0$  h)  $\frac{3}{2} \log 4x^4 - \frac{1}{2} \log y^6 = \log(4x^4)^{\frac{3}{2}} - \log(y^6)^{\frac{1}{2}} = \log \left( \frac{8x^6}{y^3} \right)$

i)  $\frac{1}{4} [\log(x^2 - 4) - \log(x-2)] - \log x = \frac{1}{4} \log \frac{(x-2)(x+2)}{x-2} - \log x = \log \frac{\sqrt[4]{x+2}}{x}$

j)  $\log(x^2 - 4) - [\log(x-2) + \log(x+2)] = \log \frac{(x-2)(x+2)}{(x-2)(x+2)} = \log 1 = 0$

5. a)  $\log_b x^{\log_x a} = \log_x a \cdot \log_b x = \frac{\log a}{\log x} \cdot \frac{\log x}{\log b} = \frac{\log a}{\log b} = \log_b a$

b)  $x^{\log_x 20 - \log_x 4} = x^{\log_x \frac{20}{4}} = x^{\log_x 5} = 5$  (rule #7, p. 220)

5. c)  $(\log_2 10)(\log_4 48 - \log_4 3) = \frac{\log 10}{\log 2} \cdot \log 16 = \frac{1}{\log 2} \cdot \log 2^4 = \frac{4 \log 2}{\log 2} = 4$

d) Method 1:  $\frac{\log x^3 + \log x^5}{\log x^6 - \log x^3} = \frac{\log x^3 \cdot x^5}{\log x^6} = \frac{\log x^8}{\log x^3} = \frac{8 \log x}{3 \log x} = \frac{8}{3}$       Method 2:  $\frac{\log x^3 + \log x^5}{\log x^6 - \log x^3} = \frac{3 \log x + 5 \log x}{6 \log x - 3 \log x} = \frac{8 \log x}{3 \log x} = \frac{8}{3}$

e)  $\left(\frac{a}{b}\right)^{\log 0.5} \cdot \left(\frac{a}{b}\right)^{\log 0.2} = \left(\frac{a}{b}\right)^{\log 0.5 + \log 0.2} = \left(\frac{a}{b}\right)^{\log(0.5)(0.2)} = \left(\frac{a}{b}\right)^{\log 0.1} = \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

f) Method 1:  $4^{-2 \log_4 3} = x \rightarrow \log_4 x = -2 \log_4 3 \rightarrow \log_4 x = \log_4 3^{-2} \rightarrow x = 3^{-2} \rightarrow x = \frac{1}{9}$

Method 2:  $4^{\log_4 3^{-2}} = 3^{-2} \rightarrow x = \frac{1}{9}$ . (rule #7, p. 220)

g)  $10 \log_4 x - 12 \log_8 x = \frac{10 \log x}{\log 4} - \frac{12 \log x}{\log 8} = \frac{10 \log x}{\log 2^2} - \frac{12 \log x}{\log 2^3} = \frac{10 \log x}{2 \log 2} - \frac{12 \log x}{3 \log 2} = 5 \log_2 x - 4 \log_2 x = \log_2 x$

h) Method 1:  $\log \pi + \log \frac{\sqrt{2}}{\pi} + \frac{1}{2} \log \frac{3}{2} - \log \frac{\sqrt{3}}{10} = \log \left( \frac{\pi \cdot \frac{\sqrt{2}}{\pi} \cdot \left(\frac{3}{2}\right)^{\frac{1}{2}}}{\frac{\sqrt{3}}{10}} \right) \rightarrow \log \left( \frac{3^{\frac{1}{2}}}{\frac{\sqrt{3}}{10}} \right) = \log 10 = 1$

Method 2:  $\log \pi + \log \sqrt{2} - \log \pi + \log \sqrt{3} - \log \sqrt{2} - \log \sqrt{3} + \log 10 = \log 10 = 1$

i)  $\log(1-x^3) - \log(1+x+x^2) - \log(1-x) = \log \frac{(1-x^3)}{(1+x+x^2)(1-x)} = \log \frac{(1-x^3)}{(1-x^3)} = \log 1 = 0$

j)  $\frac{\log_a x}{\log_{ab} x} - \frac{\log_a x}{\log_b x} = \frac{\log x}{\log a} - \frac{\log x}{\log b} = \frac{\log ab}{\log a} - \frac{\log b}{\log a} = \frac{\log a + \log b - \log b}{\log a} = 1$

k)  $\frac{1}{\log_a x} + \frac{1}{\log_b x} = \log_x a + \log_x b = \log_x a b$  (rule #8, p. 220)

l)  $(\log_5 9)(\log_3 7)(\log_7 5) = \frac{\log 9}{\log 5} \cdot \frac{\log 7}{\log 3} \cdot \frac{\log 5}{\log 7} = \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2 \log 3}{\log 3} = 2$

## 5.4 Exercise Set

1. a)  $\log_5(2x-1) + \log_5(x-2) = 1 \rightarrow \log_5(2x-1)(x-2) = 1 \rightarrow (2x-1)(x-2) = 5^1 \rightarrow$

$2x^2 - 5x + 2 = 5 \rightarrow 2x^2 - 5x - 3 = 0 \rightarrow (2x+1)(x-3) = 0 \rightarrow x = -\frac{1}{2}, 3$

Check:  $(2x-1) \rightarrow 2\left(-\frac{1}{2}\right) - 1 = -2$ , logarithm must be positive, therefore, reject  $x = -\frac{1}{2}$

$(x-2) \rightarrow 3 - 2 = 1$ , o.k.  $(2x-1) \rightarrow 2 \cdot 3 - 1 = 5$ , o.k. Therefore,  $x = 3$

b)  $\log_2(2-2x) + \log_2(1-x) = 5 \rightarrow \log_2(2-2x)(1-x) = 5 \rightarrow (2-2x)(1-x) = 2^5 \rightarrow$

$2-4x+2x^2 = 32 \rightarrow 2x^2 - 4x - 30 = 0 \rightarrow x^2 - 2x - 15 = 0 \rightarrow (x-5)(x+3) = 0 \rightarrow x = 5, -3$

Check:  $(1-x) \rightarrow 1-5 = -4$  logarithm must be positive, therefore, reject  $x = 5$

$(1-x) \rightarrow 1 - (-3) = 4$  o.k.,  $(2-2x) \rightarrow (2-2(-3))$  o.k. Therefore,  $x = -3$

c)  $\frac{1}{2} - \log_{16}(x-3) = \log_{16} x \rightarrow \log_{16} x + \log_{16}(x-3) = \frac{1}{2} \rightarrow \log_{16} x(x-3) = \frac{1}{2} \rightarrow x(x-3) = 16^{\frac{1}{2}} \rightarrow$

$x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \rightarrow x = 4, -1$  Check and reject  $x = -1$ . Therefore,  $x = 4$

d)  $\log_2(3x+1) + \log_2(x-1) = \log_2(10x+14) \rightarrow \log_2(3x+1)(x-1) = \log_2(10x+14) \rightarrow$

$(3x+1)(x-1) = 10x+14 \rightarrow 3x^2 - 2x - 1 = 10x+14 \rightarrow 3x^2 - 12x - 15 = 0 \rightarrow x^2 - 4x - 5 = 0 \rightarrow$

$(x-5)(x+1) = 0 \rightarrow x = -1, 5$  Check and reject  $x = -1$  Therefore,  $x = 5$

e)  $\log_4(3x^2 - 5x - 2) - \log_4(x-2) = 1 \rightarrow \log_4 \frac{(3x+1)(x-2)}{(x-2)} = 1 \rightarrow \frac{(3x+1)(x-2)}{(x-2)} = 4 \rightarrow 3x+1=4 \rightarrow 3x=3 \rightarrow x=1$

Check and reject. Therefore, answer is  $\phi$

f)  $\log x + \log(29-x) = 2 \rightarrow \log x(29-x) = 2 \rightarrow x(29-x) = 10^2 \rightarrow x^2 - 29x + 100 = 0 \rightarrow (x-25)(x-4) = 0 \rightarrow x = 4 \text{ and } 25$  Check answers, both work. Therefore,  $x = 4, 25$

1. g)  $\log_{25}(x-1) + \log_{25}(x+3) = \log_7 \sqrt{7} \rightarrow \log_{25}(x-1)(x+3) = \frac{1}{2} \log_7 7 = \frac{1}{2} \rightarrow$   
 $(x-1)(x+3) = 25^{\frac{1}{2}} \rightarrow x^2 + 2x - 3 = 5 \rightarrow x^2 + 2x - 8 = 0 \rightarrow (x+4)(x-2) = 0 \rightarrow$   
 $x = -4, 2 \quad \text{Check and reject } -4 \quad \text{Therefore, } x = 2$

h)  $2 \log(4-x) - \log 3 = \log(10-x) \rightarrow \log \frac{(4-x)^2}{3} = \log(10-x) \rightarrow$   
 $\frac{(4-x)^2}{3} = 10-x \rightarrow 16-8x+x^2 = 30-3x \rightarrow x^2 - 5x - 14 = 0 \rightarrow$   
 $(x-7)(x+2) = 0 \rightarrow x = -2, 7 \quad \text{Check and reject } 7 \quad \text{Therefore, } x = -2$   
i)  $2 \log_2(x+2) - \log_2(3x-2) = 2 \rightarrow \log_2 \frac{(x+2)^2}{(3x-2)} = 2 \rightarrow \frac{(x+2)^2}{(3x-2)} = 2^2 \rightarrow x^2 + 4x + 4 = 12x - 8 \rightarrow$   
 $x^2 - 8x + 12 = 0 \rightarrow (x-6)(x-2) = 0, x = 2, 6 \quad \text{Check answers, both work} \quad \text{Therefore, } x = 2, 6$   
j)  $2 \log_4 x + \log_4(x-2) - \log_4 2x = 1 \rightarrow \log_4 \frac{x^2(x-2)}{2x} = 1 \rightarrow \frac{x(x-2)}{2} = 4 \rightarrow x^2 - 2x - 8 = 0 \rightarrow$   
 $(x-4)(x+2) = 0, x = -2, 4 \quad \text{Check and reject } -2 \quad \text{Therefore, } x = 4$

2. a)  $\log \frac{x^3}{y^2} \rightarrow \log x^3 - \log y^2 \rightarrow 3 \log x - 2 \log y \rightarrow 3a - 2b$   
b)  $\log_{16} 81 = \frac{\log 81}{\log 16} = \frac{\log 3^4}{\log 2^4} = \frac{4 \log 3}{4 \log 2} = \log_2 3 = a \quad$  c)  $\log \frac{9}{5} = \log \frac{3^2}{25^{\frac{1}{2}}} = \log 3^2 - \log 25^{\frac{1}{2}} = 2 \log 3 - \frac{1}{2} \log 25 = 2a - \frac{1}{2}b$   
d)  $\log \frac{25}{9} = \log 5^2 - \log 3^2 = 2 \log 5 - 2 \log 3 = 2 \log \frac{10}{2} - 2 \log 3 = 2(\log 10 - \log 2) - 2 \log 3 = 2(1-a) - 2b = 2 - 2a - 2b$   
e) (i)  $\log \frac{A}{B^2} = \log A - 2 \log B = 2 - 2(3) = 2 - 6 = -4 \quad$  (ii)  $(\log AB)^2 = (\log A + \log B)^2 = (2+3)^2 = 25$   
f)  $\log_5 12 = \frac{\log 12}{\log 5} = \frac{\log 2^2 \cdot 3}{\log \frac{10}{2}} = \frac{2 \log 2 + \log 3}{\log 10 - \log 2} = \frac{2a+b}{1-a}$   
g)  $\log AB = 8 \rightarrow \log A + \log B = 8, \log A - 4 = 8 \rightarrow \log A = 12 \quad \text{Therefore, } A = 10^{12}$   
h)  $\log_2 \sqrt[3]{12.6} = \log_2 \left( \frac{63}{5} \right)^{\frac{1}{3}} = \frac{\frac{1}{3}(\log 63 - \log 5)}{\log 2} = \frac{\frac{1}{3}(\log 3^2 \cdot 7 - \log 5)}{\log \left( \frac{10}{2} \right)} = \frac{\frac{1}{3}(2 \log 3 + \log 7 - \log 5)}{\log 10 - \log 5} = \frac{\frac{1}{3}(2x+z-y)}{1-y} = \frac{2x-y+z}{3-3y}$   
i)  $a = \log_8 3 \rightarrow a = \frac{\log 3}{\log 8}, b = \log_3 5 \rightarrow b = \frac{\log 5}{\log 3} \quad \text{Therefore, } ab = \frac{\log 3}{\log 8} \cdot \frac{\log 5}{\log 3} = \frac{\log 5}{\log 8}$   
so  $\log 5 = ab \log 8 = ab \log 2^3 = 3ab \log 2 = 3ab \log \frac{10}{5} = 3ab(\log 10 - \log 5) = 3ab(1 - \log 5) =$   
 $3ab - 3ab \log 5, \text{ so, } 3ab \log 5 + \log 5 = 3ab \rightarrow \log 5(3ab + 1) = 3ab \rightarrow \log 5 = \frac{3ab}{3ab+1}$

3. a)  $A = \log 3B - \log C \rightarrow A = \log \frac{3B}{C} \rightarrow 10^A = \frac{3B}{C} \rightarrow B = \frac{C \cdot 10^A}{3}$   
b)  $1 + \log(AB) = \log C \rightarrow \log(AB) - \log C = -1 \rightarrow \log \left( \frac{AB}{C} \right) = -1 \rightarrow 10^{-1} = \frac{AB}{C} \rightarrow A = \frac{C}{10B}$   
c)  $3 \log A + \log B = \log C \rightarrow \log A^3 + \log B = \log C \rightarrow \log A^3 \cdot B = \log C \rightarrow A^3 B = C \rightarrow A = \sqrt[3]{\frac{C}{B}}$   
d)  $\log A = \log B - C \log x \rightarrow \log A = \log B - \log x^c \rightarrow \log A = \log \frac{B}{x^c} \rightarrow x^c = \frac{B}{A} \rightarrow x = \left( \frac{B}{A} \right)^{\frac{1}{c}} \text{ or } c \sqrt[c]{\frac{B}{A}}$

4. a)  $2^{3x} = 5^{x-1} \rightarrow \log 2^{3x} = \log 5^{x-1} \rightarrow 3x \log 2 = (x-1) \log 5 \rightarrow$   
 $3x \log 2 = x \log 5 - \log 5 \rightarrow x \log 5 - 3x \log 2 = \log 5 \rightarrow x(\log 5 - 3 \log 2) = \log 5 \rightarrow$   
 $x = \frac{\log 5}{\log 5 - 3 \log 2} \quad (\text{acceptable answer}) \rightarrow x = \frac{\log 5}{\log \frac{5}{2^3}} = \log_{\frac{5}{8}} 5 \quad (\text{better answer})$   
b)  $7^{2x-1} = 17^x \rightarrow \log 7^{2x-1} = \log 17^x \rightarrow (2x-1) \log 7 = x \log 17 \rightarrow 2x \log 7 - \log 7 = x \log 17 \rightarrow$   
 $2x \log 7 - x \log 17 = \log 7 \rightarrow x(2 \log 7 - \log 17) = \log 7 \rightarrow x = \frac{\log 7}{2 \log 7 - \log 17} \text{ or } x = \log_{\frac{49}{17}} 7$

4. c)  $3^{x-1} = 9 \cdot 10^x \rightarrow \frac{3^{x-1}}{9} = 10^x \rightarrow \frac{3^{x-1}}{3^2} = 10^x \rightarrow 3^{x-3} = 10^x \rightarrow \log 3^{x-3} = \log 10^x \rightarrow$

$$(x-3)\log 3 = x\log 10 \rightarrow x\log 3 - 3\log 3 = x \rightarrow x\log 3 - x = 3\log 3 \rightarrow$$

$$x(\log 3 - 1) = \log 3^3 \rightarrow x = \frac{\log 27}{\log 3 - 1} \text{ or } \frac{\log 27}{\log 3 - \log 10} \text{ or } \frac{\log 27}{\log \frac{1}{10}} \text{ or } \log_{\frac{1}{10}} 27$$

d)  $7^{x-1} = 2 \cdot 5^{1-2x} \rightarrow \log 7^{x-1} = \log 2 \cdot 5^{1-2x} \rightarrow (x-1)\log 7 = \log 2 + (1-2x)\log 5 \rightarrow$

$$x\log 7 - \log 7 = \log 2 + \log 5 - 2x\log 5 \rightarrow x\log 7 + 2x\log 5 = \log 2 + \log 5 + \log 7 \rightarrow$$

$$x(\log 7 + 2\log 5) = \log 2 + \log 5 + \log 7 \rightarrow x = \frac{\log 2 + \log 5 + \log 7}{\log 7 + 2\log 5} \text{ or } \frac{\log 70}{\log 175} \text{ or } \log_{175} 70$$

5. a)  $\log_2(\log_8 x) = -1 \rightarrow \log_8 x = 2^{-1} \rightarrow \log_8 x = \frac{1}{2} \rightarrow x = 8^{\frac{1}{2}} = (2^3)^{\frac{1}{2}} \rightarrow x = 2^{\frac{3}{2}} \text{ or } 2\sqrt{2}$

b)  $\log_2(\log_x(\log_3 27)) = -1 \rightarrow \log_x(\log_3 27) = 2^{-1} \rightarrow \log_x(\log_3 3^3) = \frac{1}{2} \rightarrow$

$$\log_x 3(\log_3 3) = \frac{1}{2} \rightarrow \log_x 3 = \frac{1}{2} \rightarrow x^{\frac{1}{2}} = 3 \rightarrow x = 3^2 = 9$$

c)  $\log_{\frac{1}{2}}(\log_4(\log_2 x)) = 1 \rightarrow \log_4(\log_2 x) = \frac{1}{2} \rightarrow \log_2 x = 4^{\frac{1}{2}} = 2 \rightarrow x = 2^2 = 4$

d)  $\log x = \frac{2}{3}\log 27 + 2\log 2 - \log 3 = \log \frac{27^{\frac{2}{3}} \cdot 2^2}{3} \rightarrow x = \frac{27^{\frac{2}{3}} \cdot 2^2}{3} = \frac{9 \cdot 4}{3} = 12$

e)  $\log x = \log 2 + 3\log_{\sqrt{10}} y - \log 2z = \log 2 + \frac{\log y^3}{\log \sqrt{10}} - \log 2z \rightarrow \log x = \log 2 + \frac{\log y^3}{\frac{1}{2}\log 10} - \log 2z = \log 2 + 2\log y^3 - \log 2z \rightarrow$

$$\log x = \log 2 + \log y^6 - \log 2z = \log \frac{2y^6}{2z} \Rightarrow x = \frac{y^6}{z}$$

f)  $2\log x = -\log a + 3\log b + 4\log \frac{1}{c} \rightarrow \log x^2 = \log a^{-1} + \log b^3 + \log \left(\frac{1}{c}\right)^4 \rightarrow$

$$\log x^2 = \log a^{-1} \cdot b^3 \cdot \left(\frac{1}{c}\right)^4 \rightarrow x^2 = \frac{b^3}{ac^4} \rightarrow x = \sqrt{\frac{b^3}{a \cdot c^4}} \rightarrow x = \sqrt{\frac{b^2 \cdot b \cdot a}{a^2 \cdot c^4}} = \frac{b\sqrt{ab}}{ac^2}$$

6. a)  $x = \frac{a^{\frac{2}{3}}}{b^{\frac{3}{2}}c^{\frac{1}{2}}} \rightarrow \log x = \log \frac{a^{\frac{2}{3}}}{b^{\frac{3}{2}}c^{\frac{1}{2}}} \rightarrow \log x = 2\log a - 3\log b - \frac{1}{2}\log c$

b)  $x = \frac{a^{-2}b^3}{c^{\frac{1}{2}}} \rightarrow x = \frac{b^3c^{\frac{1}{2}}}{a^2} \rightarrow \log x = \log \frac{b^3c^{\frac{1}{2}}}{a^2} \rightarrow \log x = 3\log b + \frac{1}{2}\log c - 2\log a$

c)  $x = \frac{\sqrt[3]{a^2} \cdot b^{\frac{-3}{2}}}{c^{\frac{1}{2}}} \rightarrow \log x = \log \frac{\sqrt[3]{a^2} \cdot b^{\frac{-3}{2}}}{c^{\frac{1}{2}}} \rightarrow \log x = \frac{2}{3}\log a - \frac{3}{2}\log b - \frac{1}{2}\log c$

d)  $x = \frac{\sqrt{a^5}b^{\frac{-1}{3}}}{c^3d^{\frac{2}{3}}} = \frac{a^{\frac{5}{2}}d^{\frac{2}{3}}}{b^{\frac{1}{3}}c^3} \rightarrow \log x = \log \frac{a^{\frac{5}{2}}d^{\frac{2}{3}}}{b^{\frac{1}{3}}c^3} \rightarrow \log x = \frac{5}{2}\log a + \frac{2}{3}\log d - \frac{1}{3}\log b - 3\log c$

7. a)  $\log_2 16^{2x+1} = 8 \rightarrow 2^8 = 16^{2x+1} \rightarrow 2^8 = 2^{4(2x+1)} \rightarrow 2^8 = 2^{8x+4} \rightarrow 8x+4=8 \rightarrow 8x=4 \rightarrow x=\frac{1}{2}$

b)  $\log_{16} x + \log_4 x + \log_2 x = 1 \rightarrow \frac{\log_2 x}{\log_2 16} + \frac{\log_2 x}{\log_2 4} + \log_2 x = 1 \rightarrow \frac{\log_2 x}{4} + \frac{\log_2 x}{2} + \log_2 x = 1 \rightarrow \frac{7}{4}\log_2 x = 1 \rightarrow \log_2 x = \frac{4}{7} \rightarrow x = 2^{\frac{4}{7}} = 16$

c)  $\log_9 x + 3\log_3 x = 7 \rightarrow \frac{\log_3 x}{\log_3 9} + 3\log_3 x = 7 \rightarrow \frac{\log_3 x}{\log_3 3^2} + 3\log_3 x = 7 \rightarrow \frac{1}{2}\log_3 x + 3\log_3 x = 7 \rightarrow \frac{7}{2}\log_3 x = 7 \rightarrow \log_3 x = 7 \cdot \frac{2}{7} = 2 \rightarrow x = 3^2 = 9$

d)  $2\log_4 x - 3\log_x 4 = 5 \rightarrow 2\log_4 x - \frac{3}{\log_4 x} = 5 \rightarrow 2(\log_4 x)^2 - 3 = 5\log_4 x \rightarrow$

$$2(\log_4 x)^2 - 5\log_4 x - 3 = 0 \rightarrow (2\log_4 x + 1)(\log_4 x - 3) = 0 \rightarrow \log_4 x = -\frac{1}{2} \rightarrow x = 4^{-\frac{1}{2}} = \frac{1}{2} \text{ and } \log_4 x = 3 \rightarrow x = 4^3 = 64$$

Therefore,  $x = \frac{1}{2}$  or 64

e)  $(\log_4 a)(\log_a 2a)(\log_{2a} x) = \log_a a^3 \rightarrow \frac{\log a}{\log 4} \cdot \frac{\log 2a}{\log a} \cdot \frac{\log x}{\log 2a} = 3 \rightarrow \frac{\log x}{\log 4} = 3 \rightarrow \log_4 x = 3 \rightarrow x = 4^3 \rightarrow x = 64$

f)  $\sqrt{\log x} = \log \sqrt{x} \rightarrow \sqrt{\log x} - \frac{1}{2} \log x = 0 \rightarrow \sqrt{\log x} \left(1 - \frac{1}{2} \sqrt{\log x}\right) = 0$

$$\sqrt{\log x} = 0 \text{ or } 1 - \frac{1}{2} \sqrt{\log x} = 0$$

$$\log x = 0 \text{ or } \log x = 4$$

$$x = 10^0 \text{ or } x = 10^4, \text{ therefore, } x = 1,10000$$

8. a) In step 6, you are dividing by  $\log \frac{1}{2}$  which is a negative number, therefore, the direction of the inequality **must be changed**.

b) When you multiply by  $\log \frac{1}{2}$  in step 2, the value of two **positive** numbers is changed to two **negative** numbers, without changing the direction of the inequality.

## 5.5 Exercise Set

1.  $A = A_0(x)^{\frac{t}{T}} \rightarrow 10000 = 40000(0.85)^{\frac{t}{T}} \rightarrow 0.25 = 0.85^{\frac{t}{T}} \rightarrow \log_{0.85} 0.25 = t$

$$t = \frac{\log 0.25}{\log 0.85} = 8.53 \quad \text{It takes 8.53 years to depreciate to \$10 000.}$$

2. a)  $A = P \left(1 + \frac{r}{n}\right)^{nt} \rightarrow 1000000 = 10000 \left(1 + \frac{0.12}{4}\right)^{4t} \rightarrow 100 = 1.03^{4t} \rightarrow \log_{1.03} 100 = 4t \rightarrow t = \frac{\log 100}{4 \log 1.03} = 38.95 \text{ years}$

b)  $A = Pe^{rt} \rightarrow 1000000 = 10000e^{0.12t} \rightarrow 100 = e^{0.12t} \rightarrow \ln 100 = 0.12t \rightarrow t = \frac{\ln 100}{0.12} = 38.38 \text{ years}$

3. a)  $A = P \left(1 + \frac{r}{n}\right)^{nt} \rightarrow 3P = P \left(1 + \frac{r}{2}\right)^{2 \cdot 15} \rightarrow 3 = \left(1 + \frac{r}{2}\right)^{30} \rightarrow 1 + \frac{r}{2} = 3^{\frac{1}{30}} \rightarrow \frac{r}{2} = 3^{\frac{1}{30}} - 1 \rightarrow r = 2(3^{\frac{1}{30}} - 1) = 7.46\%$

b)  $A = Pe^{rt} \rightarrow 3P = Pe^{15r} \rightarrow 3 = e^{15r} \rightarrow \ln 3 = 15r \rightarrow r = \frac{\ln 3}{15} = 7.32\%$

4. Method 1:  $A = A_0(x)^{\frac{t}{T}} \rightarrow 0.8 = 1 \left(\frac{1}{2}\right)^{\frac{30}{T}} \quad (\text{if 20% lost, 80% remains}) \rightarrow \log_{\frac{1}{2}} 0.8 = \frac{30}{T} \rightarrow$

$$T = \frac{30}{\log_{\frac{1}{2}} 0.8} = \frac{30}{\frac{\log 0.8}{\log \frac{1}{2}}} = \frac{30 \log \frac{1}{2}}{\log 0.8} = 93.2 \text{ hours}$$

Method 2:  $A = A_0 e^{kt} \rightarrow 0.8 = 1 \cdot e^{k \cdot 30} \rightarrow \ln 0.8 = 30k \rightarrow k = \frac{\ln 0.8}{30}$

$$A = A_0 e^{\left(\frac{\ln 0.8}{30}\right)t} \rightarrow \frac{1}{2} = 1 \cdot e^{\left(\frac{\ln 0.8}{30}\right)t} \rightarrow \ln 0.5 = \left(\frac{\ln 0.8}{30}\right)t \rightarrow t = \frac{30 \ln 0.5}{\ln 0.8} = 93.2 \text{ hours}$$

5. Remember, the **smaller** the pH values of an acidic solution, the **stronger** the acidity. The **larger** the pH value of an alkaline solution, the **stronger** the alkalinity.

a)  $4.8 - 2.1 = 2.7$ , then  $10^{2.7} = 501$ , therefore, lemon juice is 501 times more acidic than black coffee.

b)  $10^x = 75 \rightarrow x = \log 75 \rightarrow x = 1.9$ , therefore,  $4.2 + 1.9 = 6.1$  is the pH of milk.

6.  $A = A_0(x)^{\frac{t}{T}} \rightarrow 400000(1.02)^t = 300000(1.03)^t \rightarrow 1.02^t = \frac{3}{4}(1.03)^t \rightarrow \log 1.02^t = \log \frac{3}{4}(1.03)^t = \log \frac{3}{4} + \log 1.03^t \rightarrow$

$$t \log 1.02 = \log \frac{3}{4} + t \log 1.03 \rightarrow t(\log 1.02 - \log 1.03) = \log \frac{3}{4} \rightarrow t = \frac{\log \frac{3}{4}}{\log \frac{1.02}{1.03}} \approx 29.5 \quad \text{Surrey catches up in population to Vancouver in 29.5 years.}$$

7. a)  $A = P \left(1 + \frac{r}{n}\right)^{nt} \rightarrow 3 = 1 \left(1 + \frac{0.08}{365}\right)^{365t} \rightarrow 365t = \frac{\log 3}{\log \left(1 + \frac{0.08}{365}\right)} \rightarrow t = \frac{\log 3}{365 \log \left(1 + \frac{0.08}{365}\right)} = 13.73 \text{ years}$

b)  $A = Pe^{rt} \rightarrow 3 = 1e^{0.08t} \rightarrow \ln 3 = 0.08t \rightarrow t = \frac{\ln 3}{0.08} = 13.73 \text{ years}$

8.  $C = 8e^{0.3t} \rightarrow 100 = 8e^{0.3t} \rightarrow 12.5 = e^{0.3t} \rightarrow 0.3t = \ln 12.5 \rightarrow t = \frac{\ln 12.5}{0.3} = 8.42^\circ \text{C} \quad t = 8.42 \text{ degrees Celsius}$

9.  $P(t) = 4\,000\,000 e^{0.012t} \rightarrow 6\,400\,000 = 4\,000\,000 e^{0.012t} \rightarrow 1.6 = e^{0.012t} \rightarrow$

$\ln 1.6 = 0.012t \rightarrow t = \frac{\ln 1.6}{0.012} = 39.2$  Therefore, in year 2039 the population will reach 6 400 000.

10. Method 1:  $A = A_0(2)^{\frac{t}{4}} \rightarrow 100\,000 = 1200(2)^{\frac{t}{4}} \rightarrow \frac{250}{3} = 2^{\frac{t}{4}} \rightarrow \log_2 \frac{250}{3} = \frac{t}{4} \rightarrow t = \frac{4 \log \frac{250}{3}}{\log 2} = 25.5$  days

Method 2:  $A = A_0 e^{kt} \rightarrow 2 = 1 \cdot e^{k \cdot 4} \rightarrow \ln 2 = 4k \rightarrow k = \frac{\ln 2}{4} \rightarrow A = A_0 e^{\left(\frac{\ln 2}{4}\right)t} \rightarrow$

$$100\,000 = 1200 e^{\left(\frac{\ln 2}{4}\right)t} \rightarrow \frac{250}{3} = e^{\left(\frac{\ln 2}{4}\right)t} \rightarrow \ln \left(\frac{250}{3}\right) = \left(\frac{\ln 2}{4}\right)t \rightarrow t = \frac{4 \ln \left(\frac{250}{3}\right)}{\ln 2} = 25.5 \text{ days}$$

11.  $A = A_0 e^{kt} \rightarrow \frac{1}{2} = 1 \cdot e^{k \cdot 5570} \rightarrow \ln 0.5 = 5570k \rightarrow k = \frac{\ln 0.5}{5570} \quad A = A_0 e^{\left(\frac{\ln 0.5}{5570}\right)t} = 500 e^{\left(\frac{\ln 0.5}{5570}\right)2500} = 366.3 \text{ grams}$

12. Let  $q = 2^{74207281} - 1$  with  $2 = 10^x \rightarrow x = \log 2 \rightarrow 2 = 10^{\log 2}$ . Substitute  $q = (10^{\log 2})^{74207281} - 1 = 10^{22338617.48} - 1$

But  $10^n$  has  $n + 1$  digits so  $q$  has 22 338 618 digits. (By the way, printing this number would take about 3700 pages!)

## 5.6 Chapter Review

### Logarithms – Multiple-choice Answers

- |      |       |       |       |       |       |       |       |       |       |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. a | 8. d  | 15. b | 22. b | 29. d | 36. b | 43. a | 50. b | 57. a | 64. d |
| 2. d | 9. c  | 16. a | 23. a | 30. a | 37. d | 44. b | 51. b | 58. b | 65. a |
| 3. c | 10. c | 17. b | 24. c | 31. a | 38. a | 45. c | 52. d | 59. d |       |
| 4. b | 11. a | 18. c | 25. c | 32. d | 39. c | 46. b | 53. d | 60. d |       |
| 5. b | 12. b | 19. c | 26. c | 33. d | 40. b | 47. d | 54. a | 61. d |       |
| 6. d | 13. a | 20. b | 27. b | 34. d | 41. c | 48. c | 55. b | 62. a |       |
| 7. d | 14. a | 21. c | 28. c | 35. c | 42. b | 49. c | 56. a | 63. a |       |

### Logarithms – Multiple-choice Solutions

1. Basic definition. Answer is a.

2.  $\log 5 = \log x - \log 2 \rightarrow \log 5 = \log \frac{x}{2} \rightarrow 5 = \frac{x}{2} \rightarrow x = 10$ . Answer is d.

3.  $x - 2 = 0 \rightarrow x = 2$ . Answer is c.

4.  $\frac{\log 10^x}{10^{\log x}} = \frac{x \log 10}{x} = \log 10 = 1$ . Answer is b.

Note: If you don't understand that  $10^{\log x} = x$ , review Helpful log rule #7, p. 220.

5.  $y = \log x$  then  $y + 2 = \log x + 2 = \log x + \log 100 = \log 100x$ . Answer is b.

6.  $x = \frac{\sqrt{A}}{3B} \rightarrow \log x = \log \frac{\sqrt{A}}{3B} \rightarrow \log x = \frac{1}{2} \log A - \log 3 - \log B$ . Answer is d.

7. If  $\log_a 2 = b$  then  $a^b = 2$  and if  $\log_c 5 = d$  then  $c^d = 5$ , therefore,  $a^b \cdot c^d = 2 \cdot 5 = 10$ . Answer is d.

8.  $4 \log a^2 - 2 \log a = \log a^8 - \log a^2 = \log \frac{a^8}{a^2} = \log a^6$ . Answer is d.

9. Basic definition. Answer is c.

10.  $2 - x > 0 \rightarrow -x > -2 \rightarrow x < 2$ . Answer is c.

11.  $3^{\log x} = \frac{1}{27} \rightarrow 3^{\log x} = 3^{-3} \rightarrow \log x = -3 \rightarrow x = 10^{-3} \rightarrow x = \frac{1}{1000}$ . Answer is a

12.  $\left(\frac{1}{9}\right)^{2x-1} = 27^{2-x} \rightarrow 3^{-2(2x-1)} = 3^{3(2-x)} \rightarrow -4x+2 = 6-3x \rightarrow -x = 4 \rightarrow x = -4$ . Answer is b.

13.  $4^{x^2-2x} = 8^{1-x} \rightarrow 2^{2(x^2-2x)} = 2^{3(1-x)} \rightarrow 2x^2 - 4x = 3 - 3x \rightarrow 2x^2 - x - 3 = 0 \rightarrow (2x-3)(x+1) = 0 \rightarrow x = \frac{3}{2}, -1$ . Answer is a.

14. x-intercept has  $y = 0 \rightarrow x = \log_3(0+5)-2 \rightarrow x = \frac{\log 5}{\log 3} - 2 \rightarrow x = -0.535$

y-intercept has  $x = 0 \rightarrow 0 = \log_3(y+5)-2 \rightarrow \log_3(y+5) = 2 \rightarrow y+5 = 3^2 \rightarrow y = 4$ . Answer is a.

15.  $y = -\log_4(x+8) + \frac{1}{2} = 0 \rightarrow \log_4(x+8) = \frac{1}{2} \rightarrow x+8 = 4^{\frac{1}{2}} \rightarrow x+8 = 2 \rightarrow x = -6$ . Answer is b.

16.  $y = \log_2(0+8) - 3 \rightarrow y = \log_2 8 - 3 \rightarrow y = \log_2 2^3 - 3 \rightarrow y = 3 \log_2 2 - 3 = 0$ . Answer is a.

17.  $x+2 > 0 \rightarrow x > -2$ . Answer is b.

18.  $y = -3 \cdot 2^{x-1} + 4$  has the basic graph  $y = 2^x$  reflected in the x-axis and shifted up 4, therefore, range is  $y < 4$ . Answer is c.

19.  $2\log_x\left(\frac{1}{\sqrt{x}}\right) = 2\log_x x^{-\frac{1}{2}} = -\log_x x = -1$ . Answer is c.

20.  $\frac{10^{8.1}}{10^{7.4}} = 10^{0.7} = 5.01$ . Answer is b

21.  $x = \frac{\sqrt[3]{a}}{bc^2} \rightarrow \log x = \log \frac{\sqrt[3]{a}}{bc^2} \rightarrow \log x = \frac{1}{3}\log a - \log b - 2\log c$ . Answer is c.

22. The inverse of  $y = \log\left(\frac{x}{2}\right)$  is  $x = \log\left(\frac{y}{2}\right) \rightarrow 10^x = \frac{y}{2} \rightarrow y = 2 \cdot 10^x$ . Answer is b.

23.  $\log_{\frac{1}{a}}(\sqrt{a})^a = \frac{\log a^{\frac{a}{2}}}{\log \frac{1}{a}} = \frac{\frac{a}{2}\log a}{-\log a} = \frac{-a}{2}$ . Answer is a.

24. The restriction on  $y = \log_x a$  is  $a > 0$ ,  $x > 0$ ,  $x \neq 1$ , therefore,  $y = \log_x(x+2)$  has restriction  $x+2 > 0$  and base  $x > 0$ ,  $x \neq 1 \rightarrow x > -2$  and  $x > 0$ ,  $x \neq 1$ , the intersection is  $x > 0$ ,  $x \neq 1$ . Answer is c.

25.  $\log_3(x+5) - \log_3(x-3) = 2 \rightarrow \log_3\left(\frac{x+5}{x-3}\right) = 2 \rightarrow \frac{x+5}{x-3} = 3^2 \rightarrow x+5 = 9x-27 \rightarrow -8x = -32 \rightarrow x = 4$ . Answer is c.

26.  $3 - 2\log a + \log b \rightarrow \log 1000 - \log a^2 + \log b \rightarrow \log \frac{1000b}{a^2}$ . Answer is c.

27. If  $(m, n)$  is on  $f(x) = \log_a x$ , then  $(n, m)$  is on  $h(x) = a^x$ , so  $h(x) = a^{-x}$  reflects the graph on the  $y$ -axis, therefore point is  $(-n, m)$ . Answer is b.

28.  $(\log_9 x)(\log_5 3) = 1 \rightarrow \frac{\log x}{\log 9} \cdot \frac{\log 3}{\log 5} = 1 \rightarrow \frac{\log x}{\log 3^2} \cdot \frac{\log 3}{\log 5} = 1 \rightarrow \frac{\log x}{2\log 3} \cdot \frac{\log 3}{\log 5} = 1 \rightarrow \frac{\log x}{2\log 5} = 1 \rightarrow$

$\log_5 x = 2 \rightarrow x = 5^2 \rightarrow x = 25$ . Answer is c.

29.  $f(x) = 2^{-x}$  has inverse  $x = 2^{-y} \rightarrow \log_2 x = -y \rightarrow y = -\log_2 x \rightarrow y = \log_2 x^{-1} \rightarrow y = \log_2\left(\frac{1}{x}\right)$ . Answer is d.

30.  $2^{3\log_8 5} = x \rightarrow 8^{\log_8 5} = x \rightarrow x = 5$ . Answer is a.

31. The inverse of  $f(x) = 6^{x+1} - 2$  is  $x = 6^{y+1} - 2 \rightarrow x+2 = 6^{y+1} \rightarrow \log_6(x+2) = y+1 \rightarrow y = \log_6(x+2) - 1$ . Answer is a.

32. The inverse of  $f(x) = \log_5(x-1) - 2$  is  $x = \log_5(y-1) - 2 \rightarrow x+2 = \log_5(y-1) \rightarrow y-1 = 5^{x+2} \rightarrow y = 5^{x+2} + 1$ . Answer is d.

33.  $\log_{27} x = a \rightarrow x = 81^a = 3^{4a} \quad \log_{27} x = \log_{27}(3^{4a}) = 4a \log_{27} 3 = 4a \log_{27} 27^{\frac{1}{3}} = \frac{4a}{3}$ . Answer is d.

34.  $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{T}} \rightarrow A = 50\left(\frac{1}{2}\right)^{\frac{t}{14}}$ . Answer is d.

35.  $A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow 1000 = P\left(1 + \frac{0.12}{4}\right)^{4 \cdot 5} \rightarrow 1000 = P(1.03)^{20} \rightarrow P = \frac{1000}{1.03^{20}}$ . Answer is c.

36.  $10^x = 240 \rightarrow x = \log 240 \rightarrow x = 2.4$ , so  $7.3 - 2.4 = 4.9$ . Answer is b.

37.  $P = 30e^{0.019t} \rightarrow P = 30e^{(0.019)(50)} \rightarrow P = 77.57$ . Answer is d.

38.  $A = A_0(3)^{\frac{t}{T}} \rightarrow 200 = A_0(3)^{\frac{t}{5}} \rightarrow A_0 = \frac{200}{3^{\frac{t}{5}}}$ . Answer is a.

39.  $9.8 - 8.2 = 1.6$ , therefore,  $10^{1.6} = 39.8$ . Answer is c.

40.  $10^x = 160 \rightarrow x = \log 160 \rightarrow x = 2.2$ , therefore  $8.7 + 2.2 = 10.9$ . Answer is b.

41.  $\log_3\left(\frac{a}{9b^2}\right) = \log_3 a - \log_3 9 - \log_3 b^2 = \log_3 a - \log_3 3^2 - 2\log_3 b = \log_3 a - 2 - 2\log_3 b$ . Answer is c.

42.  $\log_2\left(\frac{25}{72}\right) = \log_2 \frac{5^2}{2^3 \cdot 3^2} = 2\log_2 5 - 3\log_2 2 - 2\log_2 3 = 2a - 3 - 2b$ . Answer is b.

43.  $1 - \log \frac{3}{b} - \log c = \log 10 - \log \frac{3}{b} - \log c = \log \frac{10}{\frac{3}{b} \cdot c} = \log \frac{10b}{3c}$ . Answer is a.

44.  $y = ab^x \rightarrow \frac{y}{a} = b^x \rightarrow x = \log_b\left(\frac{y}{a}\right)$ . Answer is b.

45.  $\log_2(a-3) = b \rightarrow a-3 = 2^b \rightarrow a = 2^b + 3$ . Answer is c.

46.  $\log\left(\frac{2}{9}\right) = \log\left(\frac{\sqrt{4}}{3^2}\right) = \log\left(\frac{4^{\frac{1}{2}}}{3^2}\right) = \frac{1}{2}\log 4 - 2\log 3 = \frac{a}{2} - 2b$ . Answer is b.

47.  $\log_b a$  has restrictions  $a > 0$ ,  $b > 0$ ,  $b \neq 1$ , therefore,  $2 - x > 0 \rightarrow x < 2$ ;  $x + 1 > 0 \rightarrow x > -1$ ;  $x + 1 \neq 1 \rightarrow x \neq 0$ , therefore,  $-1 < x < 2$ ,  $x \neq 0$ . Answer is d.
48.  $2\log(3-x) = \log 2 + \log(22-2x) \rightarrow \log(3-x)^2 = \log 2(22-2x) \rightarrow (3-x)^2 = 2(22-2x) \rightarrow 9-6x+x^2 = 44-4x \rightarrow x^2-2x-35=0 \rightarrow (x-7)(x+5)=0 \rightarrow x=7, -5$ , reject 7. Answer is c.
49.  $\log_x 12 - \log_x(x-1) = 1 \rightarrow \log_x\left(\frac{12}{x-1}\right) = 1 \rightarrow x = \frac{12}{x-1} \rightarrow x^2 - x = 12 \rightarrow x^2 - x - 12 = 0 \rightarrow (x-4)(x+3) = 0 \rightarrow x = -3, 4$ , reject -3. Answer is c.
50.  $\log_5(2x+1) = 1 - \log_5(x+2) \rightarrow \log_5(2x+1) + \log_5(x+2) = 1 \rightarrow \log_5(2x+1)(x+2) = 1 \rightarrow (2x+1)(x+2) = 5 \rightarrow 2x^2 + 5x + 2 = 5 \rightarrow 2x^2 + 5x - 3 = 0 \rightarrow (2x-1)(x+3) = 0 \rightarrow x = \frac{1}{2}, -3$ , reject -3. Answer is b.
51.  $a = 3\log_8 c \rightarrow \frac{a}{3} = \log_8 c \rightarrow c = 8^{\frac{a}{3}} \rightarrow c = 2^a \quad b = \log_4 d \rightarrow d = 4^b \rightarrow d = 2^{2b}$ , therefore,  $\frac{c}{d} = \frac{2^a}{2^{2b}} = 2^{a-2b}$ . Answer is b.
52.  $x = 2^a$ ,  $y = a$ , in answer d,  $y = \log_2 x \rightarrow a = \log_2 2^a \rightarrow a = a \log_2 2 \rightarrow a = a$ . Answer is d.
53.  $\log_3[\log_x(\log_2 8)] = -1 \rightarrow \log_x(\log_2 8) = 3^{-1} = \frac{1}{3} \rightarrow x^{\frac{1}{3}} = \log_2 8 \rightarrow x^{\frac{1}{3}} = \log_2 2^3 = 3 \log_2 2 = 3 \rightarrow x^{\frac{1}{3}} = 3 \rightarrow x = 3^3 = 27$ . Answer is d.
54.  $\log 4 = x \rightarrow \log 2^2 = x \rightarrow 2\log 2 = x \rightarrow \log 2 = \frac{x}{2} \quad \log \frac{1}{3} = y \rightarrow \log 1 - \log 3 = y$   
 $\rightarrow \log 3 = -y$ ,  $\log 6 = \log(2 \cdot 3) = \log 2 + \log 3 = \frac{x}{2} - y$ . Answer is a.
55.  $2^{x-1} = 3^x \rightarrow \log 2^{x-1} = \log 3^x \rightarrow (x-1)\log 2 = x\log 3 \rightarrow x\log 2 - \log 2 = x\log 3 \rightarrow x\log 2 - x\log 3 = \log 2 \rightarrow x(\log 2 - \log 3) = \log 2 \rightarrow x = \frac{\log 2}{\log 2 - \log 3}$ . Answer is b.
56.  $a = \log 2 \rightarrow a = \log 4^{\frac{1}{2}} \rightarrow a = \frac{1}{2}\log 4 \rightarrow \log 4 = 2a \quad b = \log 9 \rightarrow b = \log 3^2 \rightarrow b = 2\log 3 \rightarrow \log 3 = \frac{b}{2}$   
 $\log 12 = \log(3 \cdot 4) = \log 3 + \log 4 = 2a + \frac{b}{2}$ . Answer is a.
57.  $\frac{1}{\log_3 x} - \log_x 27 = 2 \rightarrow \log_x 3 - \log_x 27 = 2 \rightarrow \log_x \frac{3}{27} = 2 \rightarrow x^2 = \frac{1}{9} \rightarrow x = \frac{1}{3}$ . Answer is a.
58.  $\log_4 3 = x \rightarrow 4^x = 3 \rightarrow 2^{2x} = 3$ ;  $\log_8 7 = y \rightarrow 8^y = 7 \rightarrow 2^{3y} = 7$   
 $\log_2 21 = \log_2 3 \cdot 7 = \log_2 2^{2x} \cdot 2^{3y} = \log_2 2^{2x+3y} = (2x+3y)\log_2 2 = 2x+3y$ . Answer is b.
59.  $A = A_0(0.99)^{\frac{t}{T}} \rightarrow 0.15 = 1(0.99)^{\frac{t}{100}} \rightarrow \log_{0.99} 0.15 = \frac{t}{100} \rightarrow t = 100 \frac{\log 0.15}{\log 0.99} = 18876$ . Answer is d.
60.  $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{T}} \rightarrow 0.75 = 1\left(\frac{1}{2}\right)^{\frac{40}{T}} \rightarrow \log_{\frac{1}{2}} 0.75 = \frac{40}{T} \rightarrow T = 40 \frac{\log 0.5}{\log 0.75} = 96.38$ . Answer is d.
61.  $\log_3(2-4x) - \log_3(3-x) = 2 \rightarrow \log_3\left(\frac{2-4x}{3-x}\right) = 2 \rightarrow \frac{2-4x}{3-x} = 3^2 \rightarrow 2-4x = 27-9x \rightarrow 5x = 25 \rightarrow x = 5$ , check solution, 5 rejected. No solution. Answer is d.
62.  $3a^{x-1} = b^x \rightarrow \log 3a^{x-1} = \log b^x \rightarrow \log 3 + (x-1)\log a = x\log b \rightarrow \log 3 + x\log a - \log a = x\log b \rightarrow x\log a - x\log b = \log a - \log b \rightarrow x(\log a - \log b) = \log a - \log b \rightarrow x = \frac{\log a - \log b}{\log a - \log b}$ . Answer is a.
63.  $2\log_3(-x) = 2 - \log_3 4 \rightarrow 2\log_3(-x) + \log_3 4 = 2 \rightarrow \log_3 4(-x)^2 = 2 \rightarrow 4(-x)^2 = 3^2 \rightarrow x = \pm \frac{3}{2}$   
check solution, reject  $\frac{3}{2}$ , accept  $-\frac{3}{2}$ . Answer is a.
64.  $A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow 3P = P\left(1 + \frac{r}{12}\right)^{12 \times 10} \rightarrow 3 = \left(1 + \frac{r}{12}\right)^{120} \rightarrow 1 + \frac{r}{12} = 3^{\frac{1}{120}} \rightarrow \frac{r}{12} = 3^{\frac{1}{120}} - 1 \rightarrow r = 12(3^{\frac{1}{120}} - 1) \rightarrow r = 0.11036 = 11.0\%$ . Answer is d.
65.  $A = A_0(x)^{\frac{t}{T}} \rightarrow 200 = 600\left(\frac{1}{2}\right)^{\frac{10}{t}} \rightarrow \left(\frac{1}{2}\right)^{\frac{10}{t}} = \frac{1}{3} \rightarrow \log_{\frac{1}{2}}\left(\frac{1}{3}\right) = \frac{10}{t} \rightarrow t = \frac{10 \log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{3}\right)} = 6.3$ . Answer is a.