CONICS – OPEN-ENDED SOLUTIONS

1. a) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \rightarrow d=\sqrt{(-3-6)^2+(5+2)^2} = \sqrt{130} \approx 11.40$

b) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \rightarrow d=\sqrt{(1.3+4.5)^2+(4.7+2.8)^2} \approx 9.48$

c) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \rightarrow d=\sqrt{(\frac{1}{4}+3)^2+(-2+\frac{1}{3})^2} \approx 3.65$

2. a) $M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \rightarrow M=\left(\frac{5-5}{2}, \frac{-2-2}{2}\right) = (0,-2)$

b) $M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \rightarrow M=\left(\frac{\frac{3}{5}+\frac{1}{2}}{2}, \frac{\frac{2}{3}-3}{2}\right) = \left(\frac{11}{20}, \frac{-7}{6}\right)$

c) $M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \rightarrow M=\left(\frac{a+c}{2}, \frac{-b+d}{2}\right)$

3. a) $M_x=\frac{x_1+x_2}{2} \rightarrow -2=\frac{x_1+6}{2} \rightarrow x_1=-10,$

$M_y=\frac{y_1+y_2}{2} \rightarrow 3=\frac{y_1-1}{2} \rightarrow y_1=7, \therefore A(-10,7)$

b) $M_x=\frac{x_1+x_2}{2} \rightarrow -3=\frac{x_1+6}{2} \rightarrow x_1=-12,$

$M_y=\frac{y_1+y_2}{2} \rightarrow -8=\frac{y_1+2}{2} \rightarrow y_1=-18, \therefore A(-12,-18)$

c) $M_x=\frac{x_1+x_2}{2} \rightarrow a=\frac{x_1+b}{2} \rightarrow x_1=2a-b,$

$M_y=\frac{y_1+y_2}{2} \rightarrow 0=\frac{y_1+c}{2} \rightarrow y_1=-c, \therefore A(2a-b,-c)$

4. a) Method 1: (using the distance formula)

Call a point on the line (x, y). Distance from $(-5, 3)$ to (x, y) call d_1

Distance from $(7, 2)$ to (x, y) call d_2

$$d_1=d_2$$
$$d_1^2=d_2^2$$
$$(x+5)^2+(y-3)^2=(x-7)^2+(y-2)^2$$
$$x^2+10x+25+y^2-6y+9=x^2-14x+49+y^2-4y+4$$
$$10x-6y+34=-14x-4y+53$$
$$24x-2y=19 \quad \text{or} \quad y=12x-\tfrac{19}{2}$$

4. a) Method 2: (using midpoint, slope of $\perp$, and equation of a line)

Find the $\perp$ bisector of the line that connects the points

midpoint $\left(\dfrac{-5+7}{2},\dfrac{3+2}{2}\right)=\left(1,\dfrac{5}{2}\right)$, slope $m=\dfrac{3-2}{-5-7}=-\dfrac{1}{12}$, $m_{\perp}=12$

$y-\frac{5}{2}=12(x-1) \rightarrow 2y-5=24x-24 \rightarrow 24x-2y=19$ or $y=12x-\frac{19}{2}$

b) Method 1: (using the distance formula)

Call a point on the line (x,y) Distance from $(-5,3)$ to (x,y) call d_1

Distance from $(7,2)$ to (x,y) call d_2

$$d_1=d_2$$
$$d_1^2=d_2^2$$
$$(x-2)^2+(y+6)^2=(x+8)^2+(y+2)^2$$
$$x^2-4x+4+y^2+12y+36=x^2+16x+64+y^2+4y+4$$
$$-4x+12y+40=16x+4y+68$$
$$20x-8y=-28 \rightarrow 5x-2y=-7 \quad \text{or} \quad y=\tfrac{5}{2}x+\tfrac{7}{2}$$

Method 2: (using midpoint, slope of $\perp$, and equation of a line)

Find the $\perp$ bisector of the line that connects the points

Midpoint $\left(\dfrac{2-8}{2},\dfrac{-6-2}{2}\right)=(-3,-4)$, slope $m=\dfrac{-6+2}{2+8}=-\dfrac{2}{5}$, $\therefore\ m_{\perp}=\dfrac{5}{2}$

$y+4=\frac{5}{2}(x+3) \rightarrow 2y+8=5x+15 \rightarrow 5x-2y=-7$ or $y=\frac{5}{2}x+\frac{7}{2}$

5. a) circle **b)** hyperbola **c)** ellipse **d)** parabola

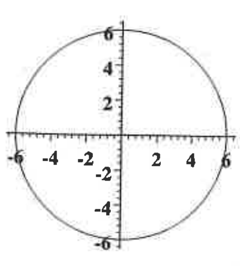
e) straight line (non-conic) **f)** hyperbola **g)** point (non-conic) **h)** two intersecting lines (non-conic)

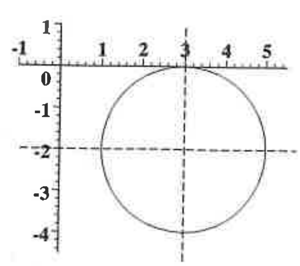
6. a) circle **b)** ellipse **c)** parabola **d)** hyperbola **e)** hyperbola

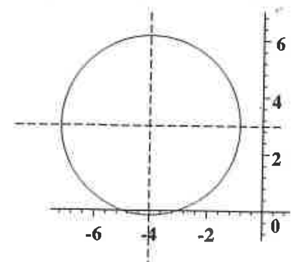
f) circle **g)** ellipse **h)** hyperbola **i)** parabola **j)** ellipse

7. a) $x^2+y^2=9$ **b)** $(x+3)^2+(y-4)^2=25$ **c)** $(x+4)^2+(y+2)^2=7$

8. a) $C(0,0)$, $r=6$ **b)** $C(3,-2)$, $r=2$ **c)** $C(-4,3)$, $r=\sqrt{10}$







9. a) $x^2+y^2-2y+6x+1=0$

$(x^2+6x+\_\_\_)+(y^2-2y+\_\_\_)=-1$

$(x^2+6x+9)+(y^2-2y+1)=-1+9+1$

$(x+3)^2+(y-1)^2=9$, Therefore, $C(-3,1)$, $r=3$

9. b) $9x^2+9y^2+6x-6y-142=0$ (*recognize this as a circle: divide by* 9)

$x^2+y^2+\frac{2}{3}x-\frac{2}{3}y-\frac{142}{9}=0$

$(x^2+\frac{2}{3}x+\_\_)+(y^2-\frac{2}{3}y+\_\_)=\frac{142}{9}$

$(x^2+\frac{2}{3}x+\frac{1}{9})+(y^2-\frac{2}{3}y+\frac{1}{9})=\frac{142}{9}+\frac{1}{9}+\frac{1}{9}$

$\left(x+\frac{1}{3}\right)^2+\left(y-\frac{1}{3}\right)^2=16$ Therefore, $C(-\frac{1}{3},\frac{1}{3})$, $r=4$

c) $4x^2-12x+4y^2-30=0$ (*recognize this as a circle: divide by* 4)

$x^2-3x+y^2-\frac{15}{2}=0$

$(x^2-3x+\_\_\_)+y^2=\frac{15}{2}$

$(x^2-3x+\frac{9}{4})+y^2=\frac{15}{2}+\frac{9}{4}$

$\left(x-\frac{3}{2}\right)^2+y^2=\frac{39}{4}$ Therefore, $C\left(\frac{3}{2},0\right)$, $r=\frac{\sqrt{39}}{2}$

10. a) $x^2+(y-2)^2=9$

$x^2+y^2-4y+4=9$

$x^2+y^2-4y-5=0$

b) $(x-1)^2+(y+3)^2=9$

$x^2-2x+1+y^2+6y+9=9$

$x^2+y^2-2x+6y+1=0$

c) $(x+2)^2+(y-4)^2=7$

$x^2+4x+4+y^2-8y+16=7$

$x^2+y^2+4x-8y+13=0$

11. New centre is $(-4,1)$ and new $r=5$ Therefore, $(x+4)^2+(y-1)^2=25$

12. a) centre = midpoint of diameter $\left(\dfrac{2-4}{2},\dfrac{-7+1}{2}\right)=(-1,-3)$

radius $r=\sqrt{(2-(-1))^2+(-7-(-3))^2}=\sqrt{9+16}=5$ Therefore, $(x+1)^2+(y+3)^2=25$

b) If $(3,-6)$ and $(7,2)$ are endpoints, then the midpoint or centre is $\left(\dfrac{3+7}{2},\dfrac{-6+2}{2}\right)=(5,-2)$

so, $r=\sqrt{(7-5)^2+(2-(-2))^2}=\sqrt{20}$ Therefore, $(x-5)^2+(y+2)^2=20$

13. $x^2+y^2-2x+4y+k=0$

$(x^2-2x+\_\_)+(y^2+4y+\_\_)=-k$

$(x^2-2x+1)+(y^2+4y+4)=-k+1+4$

$(x-1)^2+(y+2)^2=-k+5$ Therefore, $-k+5=16\rightarrow k=-11$

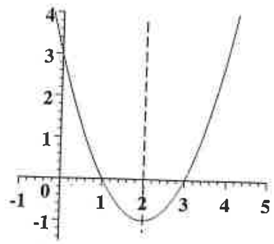
14. $(m+4)^2=9^2\rightarrow m+4=\pm9\rightarrow m=-4\pm9\rightarrow m=5$ or -13

15. Distance from centre $(0,y)$ to $(1,5)$ = distance from $(0,y)$ to $(7,4)$ (both are radii)
So $r_1=r_2\rightarrow r_1^2=r_2^2\rightarrow (0-1)^2+(y-5)^2=(0-7)^2+(y-4)^2\rightarrow$
$1+y^2-10y+25=49+y^2-8y+16\rightarrow -10y+26=65-8y\rightarrow y=-\frac{39}{2}$ Therefore, $C(0,-\frac{39}{2})$

16. Centre is $(2,6)$ so $(x-2)^2+(y-6)^2=r^2$. If it is tangent to $y=x+2$ then the slope of the tangent is $\perp$ to the radius, so its slope is -1 $\therefore$ $y-6=-1(x-2)$ is the equation of the radius, which simplifies to $y=-x+8$. This intersects $y=x+2$ at $-x+8=x+2\rightarrow x=3$ so the point is $(3,5)$. The distance to the centre $(2,6)$ is $r=\sqrt{(3-2)^2+(5-6)^2}=\sqrt{2}$ Therefore, circle is $(x-2)^2+(y-6)^2=2$

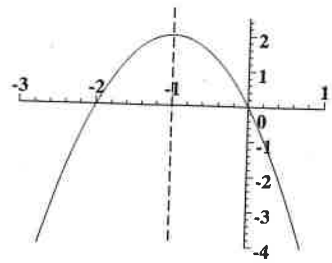
17. Take the $\perp$ bisector of $(-12, 0)$ and $(2, 0)$ which is $x = -5$
Take the $\perp$ bisector of $(0, -2)$ and $(0, 12)$ which is $y = 5$
The intersection point is $(-5, 5)$ which is the centre of the circle.
Find the distance from $(-5, 5)$ to any of the 4 points given, it will give the radius;
use $(2, 0)$ therefore, $r = \sqrt{(2-(-5))^2 + (5-0)^2} = \sqrt{74}$ so circle is $(x+5)^2 + (y-5)^2 = 74$

18. a) $y = (x-2)^2 - 1$



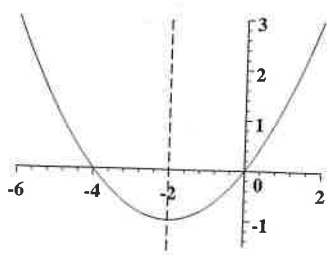
vertex $(2, -1)$
axis of symmetry $x = 2$
x – intercepts $1, 3$
y – intercept 3

b) $y = -2(x+1)^2 + 2$



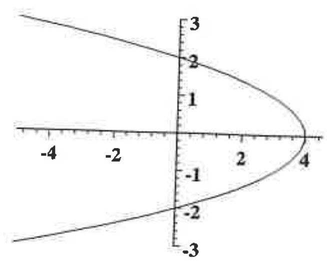
vertex $(-1, 2)$
axis of symmetry $x = -1$
x – intercepts $0, -2$
y – intercept 0

c) $y = \frac{1}{4}(x+2)^2 - 1$



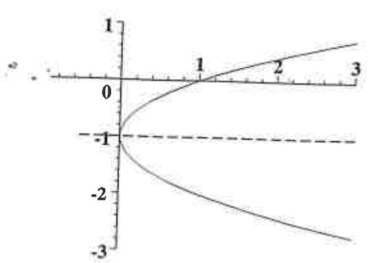
vertex $(-2, -1)$
axis of symmetry $x = -2$
x – intercepts $0, -4$
y – intercept 0

d) $x = -y^2 + 4$



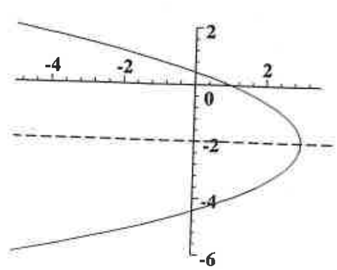
vertex $(4, 0)$
axis of symmetry $y = 0$
x – intercept 4
y – intercepts $-2, 2$

e) $x = (y+1)^2$



vertex $(0, -1)$
axis of symmetry $y = -1$
x – intercept 1
y – intercept -1

f) $x = -\frac{1}{2}(y+2)^2 + 3$



vertex $(3, -2)$
axis of symmetry $y = -2$
x – intercept 1
y – intercepts $-2 \pm \sqrt{6} \approx 0.45, -4.45$

19. a)

$y = -x^2 + 6x - 4$

$y + 4 = -(x^2 - 6x + \_\_\_)$

$y + 4 - 9 = -(x^2 - 6x + 9)$

$y - 5 = -(x-3)^2$

$y = -(x-3)^2 + 5$

Vertex $(3, 5)$

b)

$x = -3y^2 + 6y - 2$

$x + 2 = -3(y^2 - 2y + \_\_\_)$

$x + 2 - 3 = -3(y^2 - 2y + 1)$

$x - 1 = -3(y-1)^2$

$x = -3(y-1)^2 + 1$

Vertex $(1, 1)$

c)

$x + 3y^2 + 24y + 41 = 0$

$x + 41 = -3y^2 - 24y$

$x + 41 = -3(y^2 + 8y + \_\_\_)$

$x + 41 - 48 = -3(y^2 + 8y + 16)$

$x - 7 = -3(y+4)^2$

$x = -3(y+4)^2 + 7$

Vertex $(7, -4)$

d)

$\frac{1}{2}x^2 + 2x + 2y - 7 = 0$

$2y - 7 = -\frac{1}{2}x^2 - 2x$

$2y - 7 = -\frac{1}{2}(x^2 + 4x + \_\_\_)$

$2y - 7 - 2 = -\frac{1}{2}(x^2 + 4x + 4)$

$2y - 9 = -\frac{1}{2}(x+2)^2$

$2y = -\frac{1}{2}(x+2)^2 + 9$

$y = -\frac{1}{4}(x+2)^2 + \frac{9}{2}$

Vertex $(-2, \frac{9}{2})$

e)

$3x^2 + 2y + 6x - 1 = 0$

$2y - 1 = -3x^2 - 6x$

$2y - 1 = -3(x^2 + 2x + \_\_\_)$

$2y - 1 - 3 = -3(x^2 + 2x + 1)$

$2y - 4 = -3(x+1)^2$

$2y = -3(x+1)^2 + 4$

$y = -\frac{3}{2}(x+1)^2 + 2$

Vertex $(-1, 2)$

f)

$y + 4x = -2x^2 + 1$

$y - 1 = -2x^2 - 4x$

$y - 1 = -2(x^2 + 2x + \_\_\_)$

$y - 1 - 2 = -2(x^2 + 2x + 1)$

$y - 3 = -2(x+1)^2$

$y = -2(x+1)^2 + 3$

Vertex $(-1, 3)$

20. a)

$y = (x-2)^2 + 3$

$y = x^2 - 4x + 4 + 3$

$x^2 - 4x - y + 7 = 0$

b)

$x = -\frac{1}{2}(y+1)^2 - 2$

$x = -\frac{1}{2}(y^2 + 2y + 1) - 2$

$-2x = y^2 + 2y + 1 + 4$

$y^2 + 2x + 2y + 5 = 0$

c)

$y = 2(x+1)^2 - 3$

$y = 2(x^2 + 2x + 1) - 3$

$y = 2x^2 + 4x + 2 - 3$

$2x^2 + 4x - y - 1 = 0$

d)

$x = -2(y + \frac{1}{2})^2 - \frac{3}{2}$

$x = -2(y^2 + y + \frac{1}{4}) - \frac{3}{2}$

$x = -2y^2 - 2y - \frac{1}{2} - \frac{3}{2}$

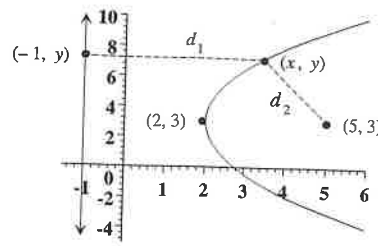
$x = -2y^2 - 2y - 2$

$2y^2 + x + 2y + 2 = 0$

21. This is a parabola going left or right, so if $(3, 4)$ is a point then $(3, -4)$ must also be a point.

22.a) Method 1

$$d_1 = d_2$$
$$d_1^{\,2} = d_2^{\,2}$$
$$(x+1)^2 + (y-y)^2 = (x-5)^2 + (y-3)^2$$
$$x^2 + 2x + 1 = x^2 - 10x + 25 + (y-3)^2$$
$$12x = (y-3)^2 + 24$$
$$x = \tfrac{1}{12}(y-3)^2 + 2$$



Method 2:

$x-h=\dfrac{1}{4p}(y-k)^2,\quad (h\ ,\ k)$ is the vertex

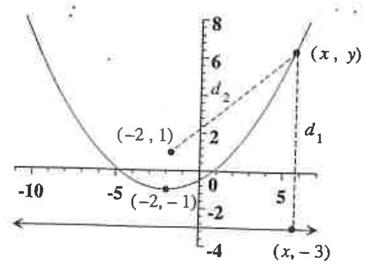
Vertex is halfway between focus $(5\ ,\ 3)$ and vertical line $x = -1$

p is the distance from the vertex $(2\ ,\ 3)$ to focus point $(5\ ,\ 3)$

$x-2=\dfrac{1}{4p}(y-3)^2\ ,\qquad p=3 \rightarrow x-2=\tfrac{1}{12}(y-3)^2 \rightarrow x=\tfrac{1}{12}(y-3)^2+2$

b) Method 1:

$$d_1 = d_2$$
$$d_1^{\,2} = d_2^{\,2}$$
$$(x-x)^2 + (y+3)^2 = (x+2)^2 + (y-1)^2$$
$$y^2 + 6y + 9 = (x+2)^2 + y^2 - 2y + 1$$
$$8y = (x+2)^2 - 8$$
$$y = \tfrac{1}{8}(x+2)^2 - 1$$



Method 2:

$y-k=\dfrac{1}{4p}(x-h)^2,\quad (h\ ,\ k)$ is the vertex

Vertex is halfway between focus $(-2\ ,\ 1)$ and horizontal line $y = -3$

p is the distance from the vertex $(-2\ ,-1)$ to focus point $(-2\ ,\ 1)$

$y+1=\dfrac{1}{4p}(x+2)^2\ ,\qquad p=2 \rightarrow y+1=\tfrac{1}{8}(x+2)^2 \rightarrow y=\tfrac{1}{8}(x+2)^2-1$

23. a) $y=x^2$, flip over x-axis is $y=-x^2$, slide horizontally left 4 is $y=-(x+4)^2$, slide up 2 is $y=-(x+4)^2+2$

b) $y=-(x-3)^2+4$, flip over $y=4 \rightarrow y=(x-3)^2+4$ side down 2 units $\rightarrow\ y=(x-3)^2+2$

c) $y=x^2+1$ has vertex $(0,1)$, $y=(x+4c)^2+1$ has vertex $(-4c\,,1)$ $\ \therefore\ -4c=-3 \rightarrow c=\tfrac{3}{4}$

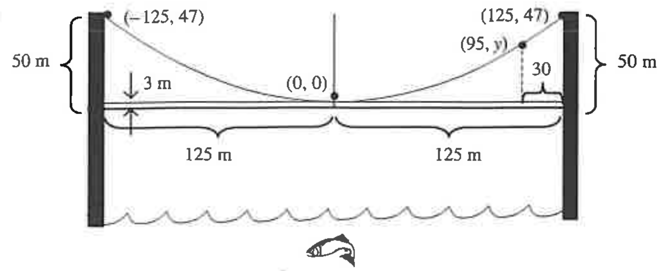
24. $y=a\,(x-h)^2+k$ because of a vertical axis of symmetry $\rightarrow\ y=a\,(x+2)^2-3$ and $(1,1)$ is a point so $1=a\,(1+2)^2-3 \rightarrow a=\tfrac{4}{9}\ \ \therefore\ y=\tfrac{4}{9}(x+2)^2-3$

25. a) $y=a\,(x-3)^2+4$ but $(2,0)$ is a point so $0=a\,(2-3)^2+4 \rightarrow a=-4\ \therefore\ y=-4\,(x-3)^2+4$

b) $x=ay^2 \rightarrow -4=a\,(1) \rightarrow a=-4 \rightarrow x=-4y^2$

26. Let the vertex of the parabola cable be (0, 0), then a point on $y = ax^2$ must be (125, 47) or (– 125, 47)

Therefore, $y = ax^2 \rightarrow 47 = a(125)^2 \rightarrow$
$a = 0.003008 \rightarrow y = 0.003008x^2$

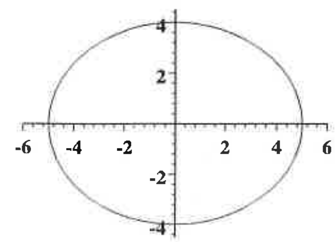


But 30 metres from the end of parabola is 95 metres horizontally from the vertex

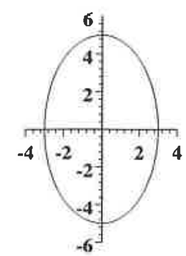
Therefore, $y = 0.003008(95)^2$
$= 27.1472$ metres above vertex

Height of cable 30 metres from the tower is 27.1472 + 3 = 30.1472 metres above the road

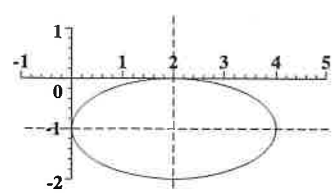
27. a) $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$
Centre (0 , 0)
Vertices (5 , 0) , (– 5 , 0)
Minor axis intercepts (0 , 4) , (0 , – 4)
Major axis length 10
Minor axis length 8



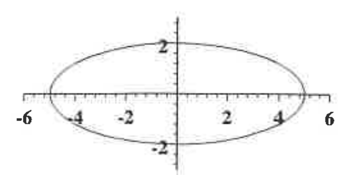
b) $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$
Centre (0 , 0)
Vertices (0 , 5) , (0 , – 5)
Minor axis intercepts (3 , 0) , (– 3 , 0)
Major axis length 10
Minor axis length 6



c) $\dfrac{(x-2)^2}{4} + (y+1)^2 = 1$
Centre (2 , – 1)
Vertices $(2 \pm \sqrt{4}\ , -1) = (4\ ,\ -1)\ ,\ (0\ ,\ -1)$
Minor axis intercepts $(2\ , -1 \pm \sqrt{1}\ ) = (2\ , 0)\ ,\ (2\ ,\ -2)$
Major axis length 4
Minor axis length 2



d) $4x^2 + 25y^2 = 100$
$\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$
Centre (0 , 0)
Vertices (5 , 0) , (– 5 , 0)
Minor axis intercepts (0 , 2) , (0 , – 2)
Major axis length 10
Minor axis length 4



27. e) $4(x+3)^2+(y-2)^2=16$

$$\frac{(x+3)^2}{4}+\frac{(y-2)^2}{16}=1$$

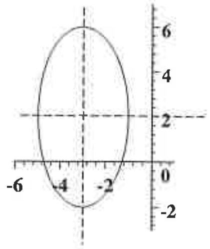
Centre $(-3\ ,2)$

Vertices $(-3\ ,2\pm\sqrt{16}\ )=(-3\ ,6)\ ,\ (-3\ ,\ -2)$

Minor axis intercepts $(-3\ \pm\sqrt{4}\ ,2\ )=(-5\ ,2)\ ,\ (-1\ ,2)$

Major axis length 8

Minor axis length 4



f) $25(x+1)^2+16(y-1)^2=400$

$$\frac{(x+1)^2}{16}+\frac{(y-1)^2}{25}=1$$

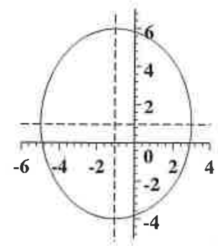
Centre $(-1\ ,1)$

Vertices $(-1\ ,1\pm\sqrt{25}\ )=\ (-1\ ,6)\ ,(-1\ ,\ -4)$

Minor axis intercepts $(-1\pm\sqrt{16}\ ,1)\ =\ (-5\ ,1)\ ,\ (3\ ,\ 1)$

Major axis length 10

Minor axis length 8



g) $25x^2+9y^2=1$

$$\frac{x^2}{\frac{1}{25}}+\frac{y^2}{\frac{1}{9}}=1$$

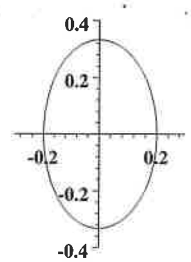
Centre $(0\ ,0)$

Vertices $(0\ ,\frac{1}{3})\ ,\ (0,\ \ -\frac{1}{3})$

Minor axis intercepts $(\frac{1}{5}\ ,0)\ ,\ (-\frac{1}{5}\ ,\ 0)$

Major axis length $\frac{2}{3}$

Minor axis length $\frac{2}{5}$



h) $(x+4)^2+\frac{y^2}{9}=1$

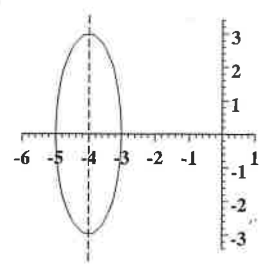
Centre $(-4\ ,0)$

Vertices $(-4\ ,0\pm\sqrt{9}\ )\ =\ (-4\ ,3)\ ,\ (-4\ ,\ -3)$

Minor axis intercepts $(-4\pm\sqrt{1}\ ,0)=\ (-5\ ,0)\ ,\ (-3\ ,\ 0)$

Major axis length 6

Minor axis length 2



28. a) $16x^2+4y^2+96x-8y+84=0$

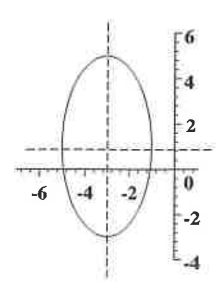
$$16(x^2+6x+\_\_\_)\ +\ 4(y^2-2y+\_\_\_)\ =-84$$

$$16(x^2+6x+9)\ +\ 4(y^2-2y+1)\ =-84+144+4$$

$$16(x+3)^2\ +\ 4(y-1)^2=64$$

$$\frac{(x+3)^2}{4}+\frac{(y-1)^2}{16}=1$$

Centre $(-3\ ,1)$



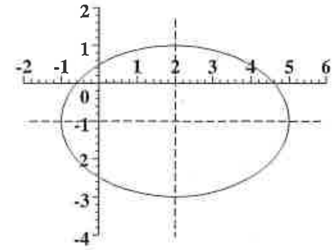
28. b) $4x^2+9y^2-16x+18y-11=0$

$4(x^2-4x+\_\_\_)+9(y^2+2y+\_\_\_)=11$

$4(x^2-4x+4)+9(y^2+2y+1)=11+16+9$

$4(x-2)^2+9(y+1)^2=36$

$\dfrac{(x-2)^2}{9}+\dfrac{(y+1)^2}{4}=1,$ Centre $(2\,,\,-1)$



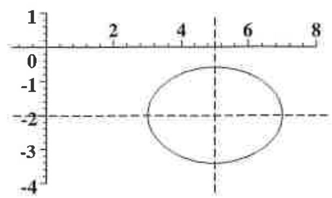
c) $x^2+2y^2-10x+8y+29=0$

$(x^2-10x+\_\_\_)+2(y^2+4y+\_\_\_)=-29$

$(x^2-10x+25)+2(y^2+4y+4)=-29+25+8$

$(x-5)^2+2(y+2)^2=4$

$\dfrac{(x-5)^2}{4}+\dfrac{(y+2)^2}{2}=1,$ Centre $(5\,,\,-2)$



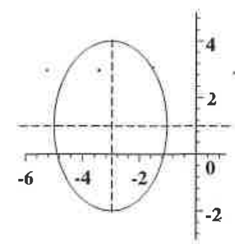
d) $9x^2+4y^2+54x-8y+49=0$

$9(x^2+6x+\_\_\_)+4(y^2-2y+\_\_\_)=-49$

$9(x^2+6x+9)+4(y^2-2y+1)=-49+81+4$

$9(x+3)^2+4(y+1)^7=36$

$\dfrac{(x+3)^2}{4}+\dfrac{(y-1)^2}{9}=1,$ Centre $(-3\,,\,1)$



2

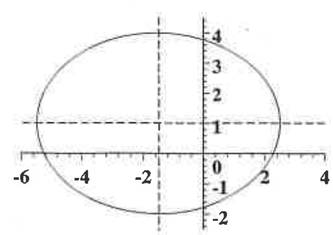
e) $36x^2+64y^2+108x-128y-431=0$

$36(x^2+3x+\_\_\_)+64(y^2-2y+\_\_\_)=431$

$36(x^2+3x+\frac{9}{4})+64(y^2-2y+1)=431+81+64=576$

$\dfrac{36(x+\frac{3}{2})^2}{576}+\dfrac{64(y-1)^2}{576}=\dfrac{576}{576}$

$\dfrac{(x+\frac{3}{2})^2}{16}+\dfrac{(y-1)^2}{9}=1,$ Centre $(-\frac{3}{2}\,,\,1)$



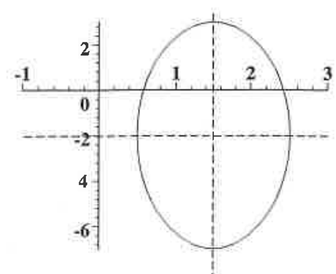
f) $100x^2+4y^2-300x+16y+141=0$

$100(x^2-3x+\_\_\_)+4(y^2+4y+\_\_\_)=-141$

$100(x^2-3x+\frac{9}{4})+4(y^2+4y+4)=-141+225+16$

$100(x-\frac{3}{2})^2+4(y+2)^2=100$

$\dfrac{(x-\frac{3}{2})^2}{1}+\dfrac{(y+2)^2}{25}=1,$ Centre $(\frac{3}{2},\,-2)$



29. a) $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

$36\left(\dfrac{x^2}{9}+\dfrac{y^2}{4}=1\right)$

$4x^2+9y^2=36$

$4x^2+9y^2-36=0$

b) $\dfrac{(x-1)^2}{16}+\dfrac{y^2}{25}=1$

$400\left[\dfrac{(x-1)^2}{16}+\dfrac{y^2}{25}=1\right]$

$25(x-1)^2+16y^2=400$

$25(x^2-2x+1)+16y^2=400$

$25x^2-50x+25+16y^2=400$

$25x^2+16y^2-50x-375=0$

c) $x^2+\dfrac{(y-1)^2}{9}=1$

$9\left[x^2+\dfrac{(y-1)^2}{9}=1\right]$

$9x^2+(y-1)^2=9$

$9x^2+y^2-2y+1=9$

$9x^2+y^2-2y-8=0$

d) $\dfrac{(x+2)^2}{4}+\dfrac{(y-1)^2}{16}=1$

$16\left[\dfrac{(x+2)^2}{4}+\dfrac{(y-1)^2}{16}=1\right]$

$4(x+2)^2+(y-1)^2=16$

$4(x^2+4x+4)+(y^2-2y+1)=16$

$4x^2+16x+16+y^2-2y+1=16$

$4x^2+y^2+16x-2y+1=0$

e) $\dfrac{(x-3)^2}{64}+\dfrac{(y+1)^2}{32}=1$

$64\left[\dfrac{(x-3)^2}{64}+\dfrac{(y+1)^2}{32}=1\right]$

$(x-3)^2+2(y+1)^2=64$

$(x^2-6x+9)+2(y^2+2y+1)=64$

$x^2-6x+9+2y^2+4y+2=64$

$x^2+2y^2-6x+4y-53=0$

30. When the ellipse crosses the x-axis, $y=0$

Therefore, $\dfrac{x^2}{49}+\dfrac{0}{25}=1\rightarrow x^2=49\rightarrow x=\pm7\rightarrow(7,0)$ or $(-7,0)$

31. If $\dfrac{(x-3)^2}{25}+\dfrac{(y+2)^2}{81}=1$ then $a=9$ therefore, major axis is 18

and $b=5$ therefore, minor axis is 10

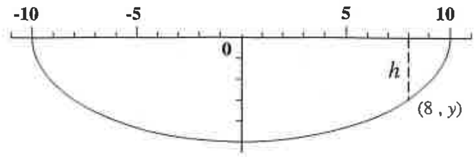
32. a) $\dfrac{(x-3)^2}{4}+\dfrac{(y+1)^2}{25}=1$

b) $\dfrac{(x-2)^2}{9}+(y+5)^2=1$

33. $\frac{x^2}{100}+\frac{y^2}{25}=1 \rightarrow \frac{64}{100}+\frac{y^2}{25}=1 \rightarrow \frac{y^2}{25}=\frac{36}{100} \rightarrow$

$y^2=\frac{25\cdot 36}{100} \rightarrow y=\pm\frac{5\cdot 6}{10}=\pm 3$

Therefore, $h=3$ m



34. $16x^2+9y^2=144$ is a square with centre $(0,0)$ then $x=y$

so $16x^2+9x^2=144 \rightarrow 25x^2=144 \rightarrow x^2=\frac{144}{25} \rightarrow x=\frac{12}{5}$ and $y=\frac{12}{5}$

but the area of the square must be $(2x)(2y)=2\cdot\frac{12}{5}\cdot 2\cdot\frac{12}{5}=\frac{576}{25}$ *units*^{2}

35. If end points are $(8,4)$ and $(-4,4)$ the centre is $\left(\frac{8-4}{2},4\right)=(2,4)$

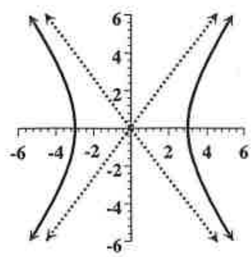
The ellipse is $\frac{(x-2)^2}{a^2}+\frac{(y-4)^2}{b^2}=1$ and the semi major axis is $8-2=6$

Therefore, $\frac{(x-2)^2}{36}+\frac{(y-4)^2}{b^2}=1$, but graph goes through origin $(0,0)$

so $\frac{(0-2)^2}{36}+\frac{(0-4)^2}{b^2}=1 \rightarrow \frac{16}{b^2}=1-\frac{4}{36} \rightarrow b^2=18$ Therefore, $\frac{(x-2)^2}{36}+\frac{(y-4)^2}{18}=1$

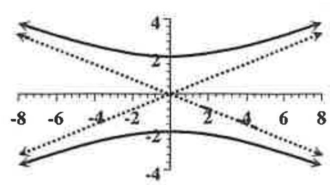
36. a) $\frac{x^2}{9}-\frac{y^2}{16}=1$

Centre $(0,0)$
Vertices $(3,0),(-3,0)$
Asymptotes $y=\pm\frac{4}{3}x$



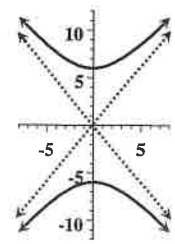
b) $\frac{x^2}{25}-\frac{y^2}{4}=-1 \rightarrow \frac{y^2}{4}-\frac{x^2}{25}=1$

Centre $(0,0)$
Vertices $(0,2),(0,-2)$
Asymptotes $y=\pm\frac{2}{5}x$



c) $\frac{y^2}{36}-\frac{x^2}{25}=1$

Centre $(0,0)$
Vertices $(0,6),(0,-6)$
Asymptotes $y=\pm\frac{6}{5}x$



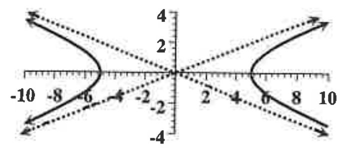
36. d) $4x^2 - 25y^2 = 100$

$\frac{x^2}{25} - \frac{y^2}{4} = 1$

Centre $(0, 0)$

Vertices $(5, 0)$, $(-5, 0)$

Asymptotes $y = \pm\frac{2}{5}x$



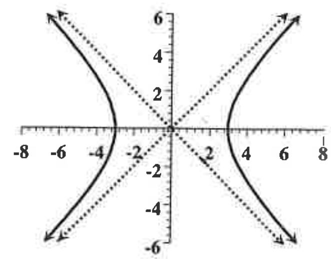
e) $x^2 - y^2 = 9 \rightarrow \frac{x^2}{9} - \frac{y^2}{9} = 1$

Centre $(0, 0)$

Vertices $(3, 0)$, $(-3, 0)$

Asymptotes $y = \pm x$

Note: This is called a rectangular hyperbola when slope of asymptotes are ± 1



f) $(x-1)^2 - \frac{(y+2)^2}{4} = 1$

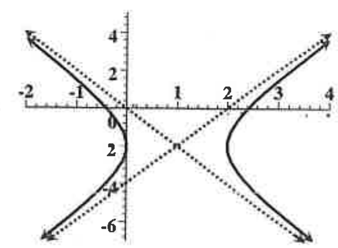
Centre $(1, -2)$

Vertices $(1 \pm\sqrt{1}, -2) = (0, -2)$, $(2, -2)$

Asymptotes $y = \pm 2x + b$, through centre $(1, -2)$

$-2 = 2(1) + b \rightarrow b = -4 \rightarrow y = 2x - 4$

$-2 = -2(1) + b \rightarrow b = 0 \rightarrow y = -2x$



g) $\frac{(x+2)^2}{9} - \frac{(y+1)^2}{16} = 1$

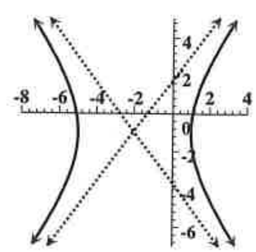
Centre $(-2, -1)$

Vertices $(-2 \pm\sqrt{9}, -1) = (1, -1)$, $(-5, -1)$

Asymptotes $y = \pm\frac{4}{3}x + b$, through center $(-2, -1)$

$-1 = \frac{4}{3}(-2) + b \rightarrow b = \frac{5}{3} \rightarrow y = \frac{4}{3}x + \frac{5}{3}$

$-1 = -\frac{4}{3}(-2) + b \rightarrow b = -\frac{11}{3} \rightarrow y = -\frac{4}{3}x - \frac{11}{3}$



h) $\frac{(x-2)^2}{25} - \frac{(y-1)^2}{16} = -1 \rightarrow \frac{(y-1)^2}{16} - \frac{(x-2)^2}{25} = 1$

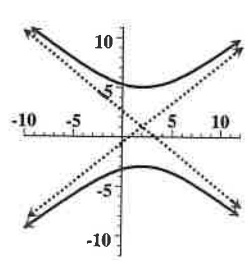
Centre $(2, 1)$

Vertices $(2, 1 \pm\sqrt{16}) = (2, 5)$, $(2, -3)$

Asymptotes $y = \pm\frac{4}{5}x + b$, through centre $(2, 1)$

$1 = \frac{4}{5}(2) + b \rightarrow b = -\frac{3}{5} \rightarrow y = \frac{4}{5}x - \frac{3}{5}$

$1 = -\frac{4}{5}(2) + b \rightarrow b = \frac{13}{5} \rightarrow y = -\frac{4}{5}x + \frac{13}{5}$



36. i) $(x+1)^2-(y-2)^2=-1 \rightarrow (y-2)^2-(x+1)^2=1$

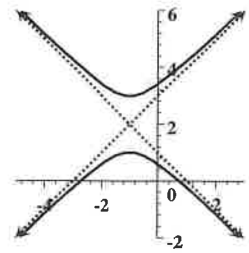
Centre $(-1, 2)$

Vertices $(-1, 2\pm\sqrt{1}) = (-1, 1), (-1, 3)$

Asymptotes $y = \pm x + b$, through centre $(-1, 2)$

$2=-1+b \rightarrow b=3 \rightarrow y=x+3$

$2=1+b \rightarrow b=1 \rightarrow y=-x+1$



37. a) $4x^2-y^2+8x-4y-4=0$

$4(x^2+2x+\_\_\_)-(y^2+4y+\_\_\_)=4$

$4(x^2+2x+1)-(y^2+4y+4)=4+4-4$

$4(x+1)^2-(y+2)^2=4$

$(x+1)^2-\dfrac{(y+2)^2}{4}=1$

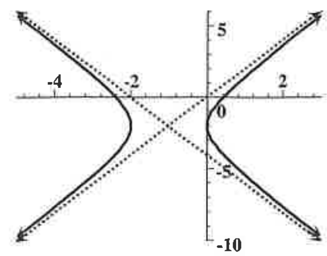
Centre $(-1,-2)$

Vertices $(-1\pm\sqrt{1}, -2) = (-2,-2), (0,-2)$

Asymptotes $y = \pm 2x+b$, through centre $(-1,-2)$

$-2=2(-1)+b \rightarrow b=0 \rightarrow y=2x$

$-2=-2(-1)+b \rightarrow b=-4 \rightarrow y=-2x-4$



b) $x^2-y^2-2x-4y-2=0$

$(x^2-2x+\_\_\_)-(y^2+4y+\_\_\_)=2$

$(x^2-2x+1)-(y^2+4y+4)=2+1-4$

$(x-1)^2-(y+2)^2=-1 \rightarrow (y+2)^2-(x-1)^2=1$

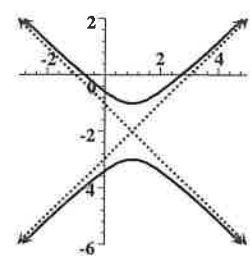
Centre $(1,-2)$

Vertices $(1, -2\pm\sqrt{1}) = (1,-1), (1,-3)$

Asymptotes $y = \pm x+b$

$-2=1+b \rightarrow b=-3 \rightarrow y=x-3$

$-2=-1+b \rightarrow b=-1 \rightarrow y=-x-1$



c) $36x^2-y^2-24x+6y-41=0$

$36(x^2-\frac{2}{3}x+\_\_\_)-(y^2-6y+\_\_\_)=41$

$36(x^2-\frac{2}{3}x+\frac{1}{9})-(y^2-6y+9)=41+4-9$

$36(x-\frac{1}{3})^2-(y-3)^2=36$

$(x-\frac{1}{3})^2-\dfrac{(y-3)^2}{36}=1$

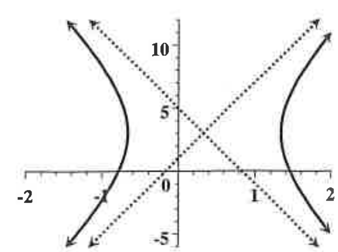
Centre $(\frac{1}{3}, 3)$

Vertices $(\frac{1}{3}\pm\sqrt{1}, 3) = (\frac{4}{3}, 3), (-\frac{2}{3}, 3)$

Asymptotes $y = \pm 6x+b$, through centre $(\frac{1}{3}, 3)$

$3=6(\frac{1}{3})+b \rightarrow b=1 \rightarrow y=6x+1$

$3=-6(\frac{1}{3})+b \rightarrow b=5 \rightarrow y=-6x+5$



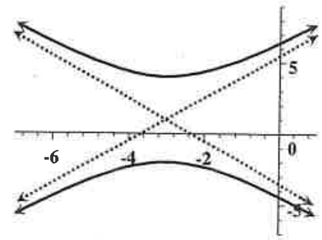
37. d) $9x^2-4y^2+54x+8y+113=0$

$9(x^2+6x+\_\_\_) \;-\; 4(y^2-2y+\_\_\_) \;=\; -113$

$9(x^2+6x+9) \;-\; 4(y^2-2y+1) \;=\; -113+81-4$

$9(x+3)^2 \;-\; 4(y-1)^2=-36$

$\frac{(x+3)^2}{4}-\frac{(y-1)^2}{9}=-1 \rightarrow \frac{(y-1)^2}{9}-\frac{(x+3)^2}{4}=1$



Centre $(-3\ ,\ 1)$, Vertices $(-3\ ,1\pm\sqrt{9}\ ) \;=\; (-3\ ,4)\ ,\ (-3,\ -2)$

Asymptotes $y \;=\; \pm\frac{3}{2}x+b \rightarrow$ $1=\frac{3}{2}(-3)+b \rightarrow b=\frac{11}{2} \rightarrow y=\frac{3}{2}x+\frac{11}{2}$

through centre $(-3\ ,1)$ $\quad 1=-\frac{3}{2}(-3)+b \rightarrow b=-\frac{7}{2} \rightarrow y=-\frac{3}{2}x-\frac{7}{2}$

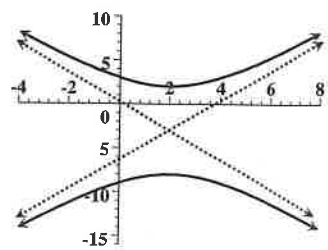
e) $25x^2-9y^2-100x-54y+244=0$

$25\,(x^2-4x+\_\_\_)-9\,(y^2+6y+\_\_\_)=-244$

$25\,(x^2-4x+4)-9\,(y^2+6y+9)=-244+100-81=-225$

$\frac{25\,(x-2)^2}{-225}-\frac{9\,(y+3)^2}{-225}=\frac{-225}{-225}$

$\frac{(y+3)^2}{25}-\frac{(x-2)^2}{9}=1$



Centre $(2\ ,-3\ )$, Vertices $(2\ ,\ -3\pm\sqrt{25}\ )=(2\ ,-8\ )\ ,\ (2\ ,\ 2)$

Asymptotes $y \;=\; +\frac{5}{3}x+b \rightarrow$ $-3=\frac{5}{3}(2)+b \rightarrow b=-\frac{19}{3} \rightarrow y=\frac{5}{3}x-\frac{19}{3}$

through centre $(2,-3)$ $\quad -3=-\frac{5}{3}(2)+b \rightarrow b=\frac{1}{3} \rightarrow y=-\frac{5}{3}x+\frac{1}{3}$

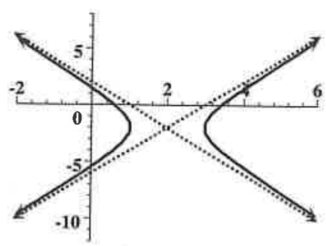
f) $4x^2-y^2-16x-4y+8=0$

$4\,(x^2-4x+\_\_\_)-(y^2+4y+\_\_\_)=-8$

$4\,(x^2-4x+4)-(y^2+4y+4)=-8+16-4$

$4\,(x-2)^2-(y+2)^2=4$

$(x-2)^2-\frac{(y+2)^2}{4}=1$



Centre $(2,-2)$, Vertices $(\,2\pm\sqrt{1}\ ,\ -2)=\ (1,-2)\ ,\ (3,-2)$

Asymptotes $y \;=\; \pm 2x+b \rightarrow$ $-2=2(2)+b \rightarrow b=-6 \rightarrow y=2x-6$

through centre $(2\ ,-2)$ $\quad -2=-2(2)+b \rightarrow b=2 \rightarrow y=-2x+2$

38. a) $\frac{x^2}{4}-\frac{y^2}{9}=1$

$36\left(\frac{x^2}{4}-\frac{y^2}{9}=1\right)$

$9x^2-4y^2=36$

$9x^2-4y^2-36=0$

b) $\frac{(x-2)^2}{16}-\frac{y^2}{25}=1$

$400\left[\frac{(x-2)^2}{16}-\frac{y^2}{25}=1\right]$

$25(x-2)^2-16y^2=400$

$25(x^2-4x+4)-16y^2=400$

$25x^2-100x+100-16y^2=400$

$25x^2-16y^2-100x-300=0$

c) $\frac{(x+2)^2}{4}-\frac{(y-1)^2}{16}=1$

$16\left[\frac{(x+2)^2}{4}-\frac{(y-1)^2}{16}=1\right]$

$4(x+2)^2-(y-1)^2=16$

$4(x^2+4x+4)-(y^2-2y+1)=16$

$4x^2+16x+16-y^2+2y-1=16$

$4x^2-y^2+16x+2y-1=0$

38. d) $x^2-\dfrac{(y-1)^2}{9}=-1$

$9\left[x^2-\dfrac{(y-1)^2}{9}=-1\right]$

$9x^2-(y-1)^2=-9$

$9x^2-(y^2-2y+1)=-9$

$9x^2-y^2+2y-1=-9$

$9x^2-y^2+2y+8=0$

e) $\dfrac{(y-1)^2}{12}-\dfrac{(x+2)^2}{18}=1$

$36\left[\dfrac{(y-1)^2}{12}-\dfrac{(x+2)^2}{18}=1\right]$

$3(y-1)^2-2(x+2)^2=36$

$3(y^2-2y+1)-2(x^2+4x+4)=36$

$3y^2-6y+3-2x^2-8x-8=36$

$2x^2-3y^2+8x+6y+41=0$

39. If $9(y-1)^2-4x^2=36\rightarrow\dfrac{(y-1)^2}{4}-\dfrac{x^2}{9}=1$, then centre is $(0,1)$ and slope of asymptotes are $\pm\frac{2}{3}$

Then, by equation of line: $y-1=\pm\frac{2}{3}(x-0)\ \rightarrow\ y=\pm\frac{2}{3}x+1$

40. If $\dfrac{y^2}{4}-\dfrac{x^2}{b^2}=1$, then point (3 , 2) gives asymptotes $y=\pm\dfrac{2}{3}x$, by definition $y=\pm\dfrac{a}{b}\cdot x=\pm\dfrac{2}{b}x$

Therefore $b=3$, so the equation is $\dfrac{y^2}{4}-\dfrac{x^2}{9}=1$

41. a) $y=\frac{1}{2}x$ and $y=-\frac{1}{2}x+2$ intersect when $\frac{1}{2}x=-\frac{1}{2}x+2\rightarrow x=2$, therefore centre is $(2,1)$

So given point $(2,5)$ must be a vertex

Then distance from $(2,1)$ to $(2,5)$ is 4, so $\dfrac{(y-1)^2}{16}-\dfrac{(x-2)^2}{b^2}=1$

By asymptote definition $y=\pm\dfrac{4}{b}x+(y\text{-intercept})\ \therefore\ \dfrac{4}{b}=\dfrac{1}{2}\rightarrow b=8$, so $\dfrac{(y-1)^2}{16}-\dfrac{(x-2)^2}{64}=1$

b) If the asymptotes are $y=\pm\dfrac{4}{3}x$, the centre is $(0,0)$ with y-intercept $(0,\pm 8)\rightarrow\dfrac{y^2}{64}-\dfrac{x^2}{b^2}=1$

The slope of asymptotes are $\pm\dfrac{8}{b}$ but we are given asymptotes slopes of $\pm\dfrac{4}{3}$

so $\dfrac{8}{b}=\dfrac{4}{3}\rightarrow b=6$ Therefore, $\dfrac{y^2}{64}-\dfrac{x^2}{36}=1$

c) Asymptotes $y=-x$ and $y=x+2$ intersect when $-x=x+2\rightarrow x=-1$

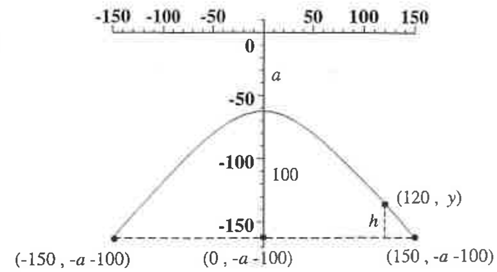
So the intersection at $(-1,1)$ is the centre of the hyperbola.

It has a vertical axis, so $\dfrac{(y-1)^2}{a^2}-\dfrac{(x+1)^2}{b^2}=1$

If the length of transverse axis is 4, then $a=2$ and $\dfrac{(y-1)^2}{4}-\dfrac{(x+1)^2}{b^2}=1$

This gives a slope of the asymptotes of $\pm\dfrac{2}{b}$, but the given slopes are ± 1

Therefore, $\dfrac{1}{1}=\dfrac{2}{b}\rightarrow b=2$ so $\dfrac{(y-1)^2}{4}-\dfrac{(x+1)^2}{4}=1$ or $(y-1)^2-(x+1)^2=4$



42. $y^2 - x^2 = a^2 \rightarrow$ at point $(150\,, -a-100)$

so $(-a-100)^2 - 150^2 = a^2$

$a^2 + 200a + 10\,000 - 22\,500 = a^2 \rightarrow a = 62.5$

so $y^2 - x^2 = 62.5^2 = 3\,906.25$

at $(120\,, y) \rightarrow y^2 - 120^2 = 3\,906.25 \rightarrow y = -135.3$

height above base $= 162.5 - 135.3 = 27.2$ m

43. a) To be a circle in quadrant II, $\boxed{A = C > 0,\ D > 0,\ E < 0}$ with F being a value that makes the radius positive. Completing the square of $Ax^2 + Cy^2 + Dx + Ey + F = 0 \rightarrow$

$$\left(x^2 + \frac{D}{A}x + \_\_\_\right) + \left(y^2 + \frac{E}{C}y + \_\_\_\right) = -F \rightarrow$$

$$\left(x^2 + \frac{D}{A}x + \left(\frac{D}{2A}\right)^2\right) + \left(y^2 + \frac{E}{C}y + \left(\frac{E}{2C}\right)^2\right) = -F + \left(\frac{D}{2A}\right)^2 + \left(\frac{E}{2C}\right)^2, \rightarrow$$

$$\left(x + \frac{D}{2A}\right)^2 + \left(y + \frac{E}{2C}\right)^2 = -F + \left(\frac{D}{2A}\right)^2 + \left(\frac{E}{2C}\right)^2$$

therefore, $-F + \left(\dfrac{D}{2A}\right)^2 + \left(\dfrac{E}{2C}\right)^2 > 0 \rightarrow F < \left(\dfrac{D}{2A}\right)^2 + \left(\dfrac{E}{2C}\right)^2$

or $A = C < 0,\ D < 0,\ E > 0$ with $F > -\left(\dfrac{D}{2A}\right)^2 - \left(\dfrac{E}{2C}\right)^2$

b) To be an ellipse with major axis on x-axis is $\boxed{E = 0 \text{ with } C > A > 0.}$

To calculate F, $Ax^2 + Cy^2 + Dx + F = 0 \rightarrow A\left(x^2 + \dfrac{D}{A}x + \_\_\_\right) + Cy^2 = -F \rightarrow$

$$A\left(x^2 + \frac{D}{A}x + \left(\frac{D}{2A}\right)^2\right) + Cy^2 = -F + \frac{D^2}{4A}, \rightarrow$$

$A\left(x + \dfrac{D}{2A}\right)^2 + Cy^2 = -F + \dfrac{D^2}{4A}$, therefore, $-F + \dfrac{D^2}{4A} > 0 \rightarrow F < \dfrac{D^2}{4A}$

or $E = 0$ with $C < A < 0$ then $F > \dfrac{-D^2}{4A}$.

c) To open up or down $\boxed{A > 0,\ C = 0,\ E > 0}$; open down must have $AE > 0$; to have axis of symmetry not on y-axis, $\boxed{D \neq 0}$ no restriction on F.

d) To be hyperbola, $AC < 0$; to have transverse axis on y-axis $\boxed{A < 0,\ C > 0,\ D = 0}$ no restrictions on E. To calculate F, $Ax^2 + Cy^2 + Ey + F = 0 \rightarrow Ax^2 + C\left(y^2 + \dfrac{E}{C}y + \_\_\_\right) + F = 0 \rightarrow$

$$Ax^2 + C\left(y^2 + \frac{E}{C}y + \left(\frac{E}{2C}\right)^2\right) + F = \frac{E^2}{4C} \rightarrow Ax^2 + C\left(y + \frac{E}{2C}\right)^2 + F = \frac{E^2}{4C}, \text{ with } F < \frac{E^2}{4C}$$

or $AC < 0,\ A > 0,\ C < 0,\ D = 0,\ F > \dfrac{-E^2}{4C}$

CONICS - MULTIPLE-CHOICE SOLUTIONS

ANSWERS

1. d	**6.** b	**11.** a	**16.** d	**21.** d	**26.** c	**31.** c	**36.** a	**41.** d	**46.** a
2. c	**7.** d	**12.** b	**17.** b	**22.** c	**27.** b	**32.** a	**37.** c	**42.** b	**47.** c
3. b	**8.** c	**13.** c	**18.** a	**23.** d	**28.** a	**33.** b	**38.** b	**43.** c	**48.** b
4. d	**9.** b	**14.** a	**19.** a	**24.** c	**29.** b	**34.** a	**39.** d	**44.** c	**49.** d
5. a	**10.** d	**15.** d	**20.** d	**25.** b	**30.** a	**35.** a	**40.** c	**45.** b	**50.** c

SOLUTIONS

1. $x-2=0 \rightarrow x=2$, $y+4=0 \rightarrow y=-4$, answer is d.

2. $b^2=16 \rightarrow b=4$ then multiply by $2=8$, answer is c.

3. $x-2=0 \rightarrow x=2$, answer is b.

4. $y+1=0 \rightarrow y=-1$, answer is d.

5. $4x^2-9y^2=36 \rightarrow \dfrac{x^2}{9}-\dfrac{y^2}{4}=1$; asymptote $y=\pm\dfrac{b}{a}x \rightarrow y=\pm\dfrac{\sqrt{4}}{\sqrt{9}}x \rightarrow y=\pm\dfrac{2}{3}x$, answer is a.

6. Circle: $(x-h)^2+(y-k)^2=r^2 \rightarrow (x-0)^2+(y+2)^2=4^2 \rightarrow x^2+(y+2)^2=16$, answer is b.

7. $\dfrac{(x-2)^2}{9}+\dfrac{(y+3)^2}{25}=1$, centre $(2, -3)$, $b^2=9 \rightarrow b=\pm3$, therefore, $2-3=-1$ and $2+3=5$, domain is $-1\leq x\leq 5$, answer is d.

8. $16(x-2)^2+25(y+3)^2=400 \rightarrow \dfrac{(x-2)^2}{25}+\dfrac{(y+3)^2}{16}=1$, centre is $(2, -3)$, $b^2=16 \rightarrow b=\pm4$, therefore, $-3-4=-7$, and $-3+4=1$, range is $-7\leq y\leq 1$, answer is c.

9. The y-intercepts have $x=0$, therefore, $16(0)^2+9y^2=144 \rightarrow y^2=\dfrac{144}{9} \rightarrow y=\pm\dfrac{12}{3}=\pm4$, answer is b.

10. If both squared terms are positive and the coefficients are equal, equation is a circle, answer is d.

11. This is a hyperbola going up and down, therefore, vertices are $(0, \pm 2)$, answer is a.

12. Equation of asymptote is $y=\pm\dfrac{a}{b}x \rightarrow y=\pm\dfrac{\sqrt{9}}{\sqrt{4}}x \rightarrow y=\pm\dfrac{3}{2}x$, therefore, slope is $\pm\dfrac{3}{2}$, answer is b.

13. Conjugate axis is perpendicular to the transverse axis, therefore, $b^2 = 16 \rightarrow b = \pm 4$, length is $4 \times 2 = 8$, answer is c.

14. Centre $(-1,\ 2)$ with vertices $\pm\sqrt{9} = \pm 3$ units from centre or $(-1,\ -1), (-1,\ 5)$, therefore, line is $x = -1$, answer is a.

15. A must be a positive value not equal to 3, answer is d.

16. If $A = -C$, $A \neq 0$, then this is a rectangular hyperbola, answer is d.

17. $x^2 + y^2 + 4x - 2y = 4 \rightarrow (x^2 + 4x + \_\_\_) + (y^2 - 2y + \_\_\_) = 4 \rightarrow$
$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1 \rightarrow (x+2)^2 + (y-1)^2 = 9$, $r = 3$, answer is b.

18. $x = a(y+1)^2 + 2$ with $(0,\ 1) \rightarrow 0 = a(1+1)^2 + 2 \rightarrow 4a = -2 \rightarrow a = -\dfrac{1}{2}$, therefore, $x = -\frac{1}{2}(y+1)^2 + 2$, answer is a.

19. $2y^2 + 3x + 4y + 14 = 0 \rightarrow 3x + 14 = -2y^2 - 4y \rightarrow 3x + 14 = -2(y^2 + 2y + \_\_\_) \rightarrow$
$3x + 14 - 2 = -2(y^2 + 2y + 1) \rightarrow 3x = -2(y+1)^2 - 12 \rightarrow x = \dfrac{-2}{3}(y+1)^2 - 4$
Vertex $(-4,\ -1)$, answer is a.

20. $y = 2x^2 - 12x + 20 \rightarrow y - 20 = 2(x^2 - 6x + \_\_\_) \rightarrow y - 20 + 18 = 2(x^2 - 6x + 9) \rightarrow y = 2(x-3)^2 + 2$, vertex $(3,\ 2)$, answer is d.

21. $(y-1)^2 - (x+3)^2 = a^2$, $a = 4 \rightarrow (y-1)^2 - (x+3)^2 = 16$, answer is d.

22. $y = \pm\dfrac{b}{a}x + c \rightarrow y = \pm\dfrac{4}{3}x + c \rightarrow$ centre $(0,\ -1)$, $-1 = \pm\dfrac{4}{3}(0) + c \rightarrow c = -1 \rightarrow y = \pm\dfrac{4}{3}x - 1$, answer is c.

23. Distance from $(3,\ -1)$ to $(-1,\ 2)$ is $d = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{25} = 5$, therefore, $(x-3)^2 + (y+1)^2 = 5^2$, answer is d.

24. $Ax^2 - By^2 = 9 \rightarrow \dfrac{x^2}{\frac{9}{A}} - \dfrac{y^2}{\frac{9}{B}} = 1$; transverse axis length $2\sqrt{\dfrac{9}{A}} = \dfrac{6}{\sqrt{A}}$, answer is c.

25. $4x^2 + 9y^2 = A \rightarrow \dfrac{x^2}{\frac{A}{4}} - \dfrac{y^2}{\frac{A}{9}} = 1$; $\dfrac{A}{4}$ is larger, therefore, major axis is $2\sqrt{\dfrac{A}{4}} = \sqrt{A}$, answer is b.

26. If major axis is horizontal, then $A < B$; to be on x-axis $C = 0$, answer is c.

27. To be a hyperbola $A > 0$, $B < 0$ or $A < 0$, $B > 0$. To be on x-axis $A > 0$, answer is b.

28. To have a horizontal axis of symmetry $A = 0$, $C \neq 0$, $D \neq 0$, answer is a.

29. To be a parabola opening up or down $A \neq 0$, $B = 0$, $E \neq 0$. To open down $AE > 0$, answer is b.

30. To be a parabola opening up or down $A \neq 0$, $B = 0$, $E \neq 0$, vertex on y-axis $D = 0$, opens down $AE > 0$, answer is a.

31. $4y^2 - 9x^2 - 54x - 8y - 113 = 0 \to 4(y^2 - 2y + \_\_\_) - 9(x^2 + 6x + \_\_\_) = 113 \to$
$4(y^2 - 2y + 1) - 9(x^2 + 6x + 9) = 113 + 4 - 81 \to 4(y-1)^2 - 9(x+3)^2 = 36 \to$
$\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1$, answer is c.

32. $2y^2 + x + 4y + 5 = 0 \to x + 5 = -2y^2 - 4y \to x + 5 = -2(y^2 + 2y + \_\_\_) \to$
$x + 5 - 2 = -2(y^2 + 2y + 1) \to x = -2(y+1)^2 - 3$, answer is a.

33. $144\left[\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1\right] \to 9(x^2 + 4x + 4) - 16(y^2 - 2y + 1) = 144 \to$
$9x^2 + 36x + 36 - 16y^2 + 32y - 16 = 144 \to 9x^2 - 16y^2 + 36x + 32y - 124 = 0$, answer is b.

34. $400\left[\frac{(x+3)^2}{25} + \frac{(y-4)^2}{16} = 1\right] \to 16(x^2 + 6x + 9) + 25(y^2 - 8y + 16) = 400 \to$
$16x^2 + 96x + 144 + 25y^2 - 200y + 400 = 400 \to 16x^2 + 25y^2 + 96x - 200y + 144 = 0$, answer is a.

35. These basic definitions are very important to understand and learn, answer is a.

36. If $(2, 2)$ and $(2, -6)$ are vertices, then $(2, -2)$ is the centre with $a = 4$, therefore,
$\frac{(x-2)^2}{b^2} + \frac{(y+2)^2}{16} = 1$, substitute $\frac{(0-2)^2}{b^2} + \frac{(-2+2)^2}{16} = 1 \to b^2 = 4$, answer is a.

37. $4x^2 + 2y^2 + 8x - 12y + F = 0 \to 4(x^2 + 2x + \_\_) + 2(y^2 - 6y + \_\_) + F = 0 \to$
$4(x^2 + 2x + 1) + 2(y^2 - 6y + 9) + F = 4 + 18 \to 4(x+1)^2 + 2(y-3)^2 + F = 22$.
The right side must have a positive value to be an ellipse, therefore, $F < 22$, answer is c.

38. $y = \pm\frac{\sqrt{4}}{\sqrt{9}}x + b \to y = \pm\frac{2}{3}x + b$, centre $(-3, 1) \to 1 = \frac{2}{3}(-3) + b \to b = 3$, $y = \frac{2}{3}x + 3$, answer is b.

39. $x^2 + y^2 - 6x + 4y - 12 = 0 \to (x^2 - 6x + \_\_\_) + (y^2 + 4y + \_\_\_) = 12 \to$
$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4 \to (x-3)^2 + (y+2)^2 = 25$, centre $(3, -2)$ with radius 5,
5 units up for $(3, -2)$ is $(3, 3)$, answer is d.

40. In a rectangular hyperbola going up and down the asymptotes are $y = \pm x$, therefore, the point must be between 45° and 135° or 225° and 315°, the only point that fits is $(2, -5)$, answer is c.

41. $\frac{x^2}{60^2} + \frac{20^2}{40^2} = 1 \to x^2 = 2700 \to x = 30\sqrt{3}$, length is $2x = 60\sqrt{3}$, answer is d.

42. If $y^2 - x^2 = a^2$ has point $(3,\ 6)$, then $36 - 9 = a^2 \rightarrow a^2 = 27 \rightarrow y^2 - x^2 = 27$ with point $(-5,\ k) \rightarrow$ $k^2 - (-5)^2 = 27 \rightarrow k^2 = 52 \rightarrow k = \sqrt{52} = 7.21$, answer is b.

43. If endpoints are $(-4,\ -2)$ and $(8,\ -2)$, then centre is $(2,\ -2)$ with $a = 6$, therefore, $\dfrac{(x-2)^2}{36} - \dfrac{(y+2)^2}{b^2} = 1$,

asymptote is $y = \dfrac{b}{6}x + c$, but slope is $\dfrac{4}{3}$, therefore, $\dfrac{b}{6} = \dfrac{4}{3} \rightarrow b = 8$, so $\dfrac{(x-2)^2}{36} - \dfrac{(y+2)^2}{64} = 1$, answer is c.

44. The length of the major axis of $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ is $2\sqrt{9} = 6$, so $\dfrac{x^2}{k} - y^2 = 1$ must have a transverse axis length of 8, therefore, $2\sqrt{k} = 8 \rightarrow k = 16$, answer is c.

45. $x = a(y+2)^2 + 8 \rightarrow 0 = -a(0+2)^2 + 8 \rightarrow 4a = -8 \rightarrow a = -2$, $x = -2(y+2)^2 + 8$, answer is b.

46. If parabola has vertex $(-3,\ 1)$ and passes through points $(-7,\ -6)$ and $(1,\ -6)$, then the parabola must open down. $y = a(x+3)^2 + 1$, use either point to solve for a, $-6 = a(1+3)^2 + 1 \rightarrow 16a = -7 \rightarrow$ $a = \dfrac{-7}{16}$, answer is a.

47. Open to the right gives $x = a(y+1)^2 + 3$, substitute $7 = a(2+1)^2 + 3 \rightarrow 9a = 4 \rightarrow a = \dfrac{4}{9}$, answer is c.

48. If vertices are $(2,\ 3)$ and $(2,\ -5)$, then centre is $(2,\ -1)$ with $a = 4$, therefore, $\dfrac{(y+1)^2}{16} - \dfrac{(x-2)^2}{b^2} = 1$,

with asymptote slope $\dfrac{4}{b}$ but give slope is $\dfrac{2}{3}$, therefore, $\dfrac{4}{b} = \dfrac{2}{3} \rightarrow b = 6$, $\dfrac{(y+1)^2}{16} - \dfrac{(x-2)^2}{36} = 1$, answer is b.

49. The $\perp$ bisector line from $(2,\ 0)$ and $(8,\ 0)$ is line $x = 5$
The tangent to the axis of point $(0,\ 4)$ is line $y = 4$
Therefore, centre of circle is $(5,\ 4)$ with radius 5, so $(x-5)^2 + (y-4)^2 = 25$, answer is d.

50. If vertices are $(-3,\ -1)$ and $(-3,\ 5)$, then centre is $(-3,\ 2)$, with $a = 3$, therefore, $(y-2)^2 - (x+3)^2 = 9$, answer is c.