Ch. 5.1 - Defining Polynomials

A <u>polynomial</u> is a monomial or the sum and/or difference of monomials.

A <u>monomial</u> is a <u>number</u>, a <u>variable</u>, or the product of <u>numbers and variables</u>.

(mono- = <u>ONE</u> , poly- = <u>MANY</u>)

Important definitions:

<u>VARIABLE</u> - a letter (or symbol) whose value is an <u>unknown real number</u>.

* most commonly used : x, y, z.

others: θ, n, a, b, c

$\boxed{\text{TERM}}$ - a term is a number, or the product of a number and variable(s) raised to a power.

eg: $5, 2x, -3x^2, -2xy, 0$.

COEFFICIENT - the coefficient of a term is the **numerical** factor.

eg:

Term	Coefficient
$5x$	5
$\frac{x^2}{3}$	$\frac{1}{3}$
$-1.3xy$	-1.3
$-y$	-1
3	3

also referred to as a **CONSTANT** (ie. just a number, no variable)

$$\underbrace{2x^3}_{\text{TERM}}$$

coefficient → 2; exponent → 3; Variable → x

note: $4x^0 = \boxed{4}$

MONOMIAL - an expression of the form ax^n, where a is a real number coefficient, x is a variable, and n is a non-negative integer.

eg: $\frac{3}{4}y^2,\ -4,\ 2x,\ x^0$ non-eg: $\frac{1}{x},\ \sqrt{x},\ x^{\frac{2}{3}},\ x^{-2}$

- every monomial is a term, but every term may not be a monomial.

POLYNOMIAL - a polynomial is a monomial, or a combination of sums and/or differences of monomials.

eg: $3x$, $2x-1$, $4x^2+x+2$, $4x^2+y^2$

Degree and Leading Term of a Polynomial

The degree of a $\boxed{\text{term}}$ is the sum of the exponents of its variables.

The leading term of a polynomial is the term possessing the highest degree.

Thus, the degree of a polynomial is the same as the degree of its leading term.

eg1: For each of the following polynomials, list each term and each term's degree, then list the leading term and the degree of the polynomial:

a) $5x^2 - 3x + 4$

Three terms:	$5x^2$	$-3x$	4
degree:	2	1	0

over →

Leading term: $5x^2$

Degree of Polynomial: 2

b) $-5 + xy$

Two terms: -5 and xy

degree: 0 2

Leading term: xy

Degree of Polynomial: 2

c) $4x^2y - 5x^3y^4 + 3xy - 6x$

Four terms:	$4x^2y$	$-5x^3y^4$	$3xy$	$-6x$
degree:	3	7	2	1

Leading term: $-5x^3y^4$

Degree of Polynomial: 7

Note: A polynomial is usually written in descending order of powers.

eg: $2x + 4 - 3x^2$ is written as:

$$-3x^2 + 2x + 4.$$

Note: A polynomial with more than one variable is written in alphabetical order.

eg. $xy + y^2 - x^2$ is written as:

$$\boxed{-x^2 + xy + y^2}$$

More terminology:

Monomial - a polynomial of one term.

Binomial - a polynomial of two terms.

eg: $\boxed{x + 1 \ , \quad 3x - 2}$

Trinomial - a polynomial of three terms

eg: $\boxed{3x^2 + x - 2 \ , \quad 5y^2 - 2ay - a^2}$

(bi- = TWO , tri- = THREE)

Like Terms

Terms with the same variable(s) raised to exactly the same powers are called LIKE TERMS.

eg:

LIKE TERMS	UNLIKE TERMS
$5x, 3x$	$5x^2, 3x$
$2x^2, -x^2$	$3x^2, 3y^2$
$2xy, -4xy$	$2xy, 2xz$
$\underbrace{xy^2, y^2x}_{\text{ie. } xy = yx}$	xy^2, x^2y

- We are able to ADD/SUBTRACT <u>like</u> terms, but we cannot add/subtract <u>unlike</u> terms.

$4x + 3x$ is <u>NOT</u> a binomial!

Why not?

$\boxed{4x + 3x = 7x \quad \text{(MONOMIAL!)}}$

eg2: Simplify each of the following:

a) $4x^2 + 3x^2 - 2x^2$

$\boxed{= 5x^2}$

b) $4x + 3x^2 - 2x^2$

$= 4x + x^2$

$\boxed{= x^2 + 4x}$

c) $2xy^2 + 3x^2y - 4x^2y + xy^2$

$= 2xy^2 + xy^2 + 3x^2y - 4x^2y$

$= 3xy^2 - x^2y$

$\boxed{= -x^2y + 3xy^2}$

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#1–5, 8, 10

Ch. 5.2 - Adding and Subtracting Polynomials

Adding Polynomials Horizontally

- try to group like terms together before adding

eg.1: Add: $(-2x^2-4)+(3x^2-2x+2)$

$= -2x^2 - 4 + 3x^2 - 2x + 2$

$= -2x^2 + 3x^2 \quad -2x \quad -4+2$ grouping like terms

$\boxed{= x^2 - 2x - 2}$

Adding Polynomials Vertically

- place like terms in the same columns (in descending order of powers) → if there exists a term(s) with no match, leave a blank space (term with coefficient of 0).

same eg1: Add $(-2x^2-4)+(3x^2-2x+2)$

$$\begin{array}{rrrr} & -2x^2 & +\,0x & -4 \\ + & 3x^2 & -\,2x & +2 \\ \hline & \boxed{x^2} & \boxed{-2x} & \boxed{-2} \end{array}$$

eg2: Add vertically:

$$(6x^2 - 7x + 1) + (4x^2 + 6x)$$

$$\begin{array}{r} 6x^2 - 7x + 1 \\ + \quad 4x^2 + 6x + 0 \\ \hline \boxed{10x^2 - x + 1} \end{array}$$

Subtracting Polynomials Horizontally

- when subtracting an entire polynomial, change the sign of each term to its opposite (this is the equivalent of multiplying the polynomial by -1).

eg: $-(-2x^2 + 4x + 1) = 2x^2 - 4x - 1$

eg3: Subtract: $(2x^2 - 4x - 3) - (-x^2 + 2x - 1)$

$= 2x^2 - 4x - 3 + x^2 - 2x + 1$

$= 2x^2 + x^2 - 4x - 2x - 3 + 1$

$\boxed{= 3x^2 - 6x - 2}$

Subtracting Polynomials Vertically

- similar in set-up to adding vertically, but multiply negative sign outside the brackets into the polynomial, then add the result.

Same eg3: Subtract $(2x^2-4x-3)-(-x^2+2x-1)$

$$\begin{array}{r} 2x^2-4x-3 \\ -(-x^2+2x-1) \end{array}$$

$$\begin{array}{rr} & 2x^2-4x-3 \\ + & x^2-2x+1 \\ \hline & \boxed{3x^2-6x-2} \end{array}$$

eg4: Subtract: $-7x^2-6x+2-(4x-6x^2-5)$

$$\begin{array}{r} -7x^2-6x+2 \\ -(-6x^2+4x-5) \end{array}$$

$$\begin{array}{rr} & -7x^2-6x+2 \\ + & 6x^2-4x+5 \\ \hline & \boxed{-x^2-10x+7} \end{array}$$

p.169-176 1-7, 8a-e
(ignore algebra tiles)

Ch. 5.3 - Multiplying Polynomials

Recall Ch. 1.6: we multiplied 'constant monomials' by other 'constant monomials' $\Rightarrow 2^3 \times 2^4$

$$= 2^{3+4}$$
$$= 2^7$$

So... **MULTIPLYING MONOMIALS**:

To multiply two monomials, multiply the numerical (coefficient) factors, then multiply the variable factors.

eg: a) Multiply $(2x^2)(-3x)$

$$= (2)(-3)(x^2)(x)$$
$$= -6(x^{2+1}) = \boxed{-6x^3}$$

b) Multiply $(7x^2)(5x^3y)$

$$= (7)(5)(x^2)(x^3)(y)$$
$$= 35(x^{2+3})y$$
$$\boxed{= 35x^5y}$$

- to find the product of a monomial and a polynomial with more than one term, we use the <u>DISTRIBUTIVE PROPERTY</u>:

$$a(b+c) = \underline{ab + ac}$$

Check: $2(3+4) = (2)(3) + (2)(4)$

$= 6 + 8$

$= \underline{\underline{14}}$

<u>OR</u>

$2(3+4) = 2(7) = \underline{\underline{14}}$

So... <u>Multiplying Monomials by Polynomials</u>:

- use distributive property, then simplify.

<u>eg 2:</u> Multiply:

a) $-4(3-x)$

$= (-4)(3) + (-4)(-x)$

$= -12 + 4x$

$\boxed{= 4x - 12}$

b) $-4x(x^2 - 3x + 1)$

$= (-4x)(x^2) + (-4x)(-3x) + (-4x)(1)$

$\boxed{= -4x^3 + 12x^2 - 4x}$

$\boxed{\text{p. 181 – 186 \# 1–3, 8}}$

Ch. 5.4 - Dividing Polynomials

Dividing a Polynomial by a Constant

- when dividing a polynomial by a constant, be sure to divide each term by the constant!

ie: $\dfrac{a + b + c + \dots z}{\#} = \boxed{\dfrac{a}{\#} + \dfrac{b}{\#} + \dfrac{c}{\#} + \dots + \dfrac{z}{\#}}$

(# = CONSTANT) (where $\# \neq 0$)

eg1. Divide $9x^2 - 3x + 6$ by 3.

ie. $(9x^2 - 3x + 6) \div 3$

OR

$$\frac{9x^2 - 3x + 6}{3} = \frac{9x^2}{3} - \frac{3x}{3} + \frac{6}{3}$$

$$\boxed{= 3x^2 - x + 2}$$

eg2: Divide: $\dfrac{20x^2 + 6x - 4}{4}$

$$= \frac{20x^2}{4} + \frac{6x}{4} - \frac{4}{4}$$

$$\boxed{= 5x^2 + \frac{3}{2}x - 1}$$

Dividing a Polynomial by a Monomial

- when dividing a polynomial by a monomial, be sure to divide <u>each</u> <u>term</u> by the monomial!

ie: $$\frac{a + b + c + \dots + z}{☺} = \boxed{\frac{a}{☺} + \frac{b}{☺} + \frac{c}{☺} + \dots + \frac{z}{☺}}$$

(☺ = monomial) $\qquad$ (☺ $\neq 0$)

eg3: Divide: $$\frac{-9x^2 + 6xy}{3x}$$

$$= \frac{-9x^2}{3x} + \frac{6xy}{3x}$$

$$= -3x^{2-1} + 2x^{1-1}y$$

$$\boxed{= -3x + 2y}$$

* show "cross-out" method (canceling)

eg4. Divide: $$\frac{14x^4y + 18xy^2 - 6x^2y^3}{6xy}$$

$$= \frac{14x^4y}{6xy} + \frac{18xy^2}{6xy} - \frac{6x^2y^3}{6xy}$$

$$\boxed{= \frac{7}{3}x^3 + 3y - xy^2}$$

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Also: CHAPTER REVIEW!