4.1 – Estimating Roots

Name:
Date:

Goal: to explore decimal representations of different roots of numbers

Toolkit:
- Finding a square root
- Finding a cube root
- Multiplication
- Estimating

Main Ideas:

p. 206 # 2, 3, 4ab

Definitions:

Radical: an expression consisting of a radical sign, a radicand, and an index.

index $\rightarrow \sqrt[n]{x}$ ← radical
radicand

Perfect squares and cubes to memorize: $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{16} = 4$, $\sqrt{25} = 5$, $\sqrt{36} = 6$, $\sqrt{49} = 7$, $\sqrt{64} = 8$, $\sqrt{81} = 9$, $\sqrt[3]{8} = 2$, $\sqrt[3]{27} = 3$, $\sqrt[3]{64} = 4$, $\sqrt[3]{125} = 5$

Ex 1) Evaluate the following radicals, identify the radicand and index for each:

a) $\sqrt{16} = 4$

$(4 \cdot 4 = 16)$

Radicand: 16
Index: 2

b) $\sqrt[3]{64} = 4$

$(4 \cdot 4 \cdot 4 = 64)$

Radicand: 64
Index: 3

Estimating square roots

Ex 2) Estimate the value of $\sqrt{20}$ to one decimal place.

Step 1: Find the two perfect squares that are closest to the radicand you are looking for (one that is lower and one that is higher).

$$16 < 20 < 25$$
$$\sqrt{16} < \sqrt{20} < \sqrt{25}$$
$$4 < \sqrt{20} < 5$$

Step 2: Find which of the two perfect squares is closest to your radicand; this will determine the decimal point of your root.

$\sqrt{20}$ closer to $\sqrt{16}$ than $\sqrt{25}$

$\therefore \sqrt{20} = \sim 4.4$ $\quad (4.4)^2 = 19.36$

Evaluate $\sqrt{20}$, how close was your estimate?

$\sqrt{20} = 4.47$

Estimating cube roots	Ex 3) Estimate the value of $\sqrt[3]{22}$ **Step 1:** Find the two perfect cubes that are closest to the radicand you are looking for. $8 < 22 < 27$ $\sqrt[3]{8} < \sqrt[3]{22} < \sqrt[3]{27}$ $2 < \sqrt[3]{22} < 3$ **Step 2:** Find which of the two perfect cubes is closest to your radicand. $\sqrt[3]{22}$ closer to $\sqrt[3]{27}$ than $\sqrt[3]{8}$ $\sqrt[3]{22} = \sim 2.8$ $(2.8)^3 = 21.952$ Evaluate $\sqrt[3]{22}$, how close was your estimate? $= 2.802$
Why can you take the cube root of a negative number but not the square root of a negative number?	Ex 4) Estimate the value of $\sqrt[3]{-32}$ $-64 < -32 < -27$ $\sqrt[3]{-64} < \sqrt[3]{-32} < \sqrt[3]{-27}$ $-4 < \sqrt[3]{-32} < -3$ $\boxed{= \sim -3.2}$ Ex 5) Evaluate $\sqrt{0.64}$ $= \sqrt{\frac{64}{100}} = \frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10} = \boxed{0.8}$ Ex 6) Evaluate $\sqrt{0.0196}$ $= \sqrt{\frac{196}{10000}} = \frac{\sqrt{196}}{\sqrt{10000}} = \frac{14}{100} = \boxed{0.14}$ Ex 7) Write an equivalent form of 0.3 as a cube root. $(0.3)^3 = 0.027$ so $0.3 = \sqrt[3]{0.027}$

Reflection: How would you write 5 as a square root? A cube root? A fourth root?

4.2 – Irrational Numbers

Name:
Date:

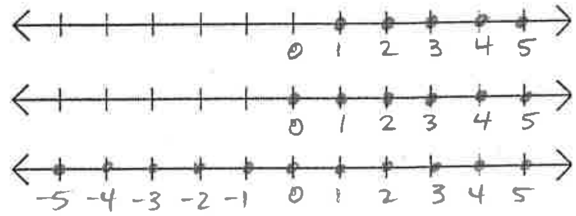
Goal: to classify real numbers, and to identify & order irrational numbers

Toolkit:

- Estimating roots
- Placing numbers on number lines
- Anything you remember about classifying Real Numbers

Looking back:

p.211 # 3-5, 8-9, 11, 14-17.



Natural Numbers ($\mathbb{N}$) 1, 2, 3, ...

Whole Numbers ($\mathbb{N}_0$) 0, 1, 2, 3, ...

Integers ($\mathbb{Z}$) ... -3, -2, -1, 0, 1, 2, 3 ...

Rational Numbers ($\mathbb{Q}$) - any repeating pattern decimal OR terminating decimal.
- any number that can be written as a fraction with the denominator $\neq 0$. eg: $0.\overline{3}$, $\frac{1}{4}$, -0.75, 2, etc...

Irrational Numbers ($\overline{\mathbb{Q}}$) - any decimal that is non-repeating and non-terminating
- cannot be written as a fraction. eg: $\sqrt{2}$, π

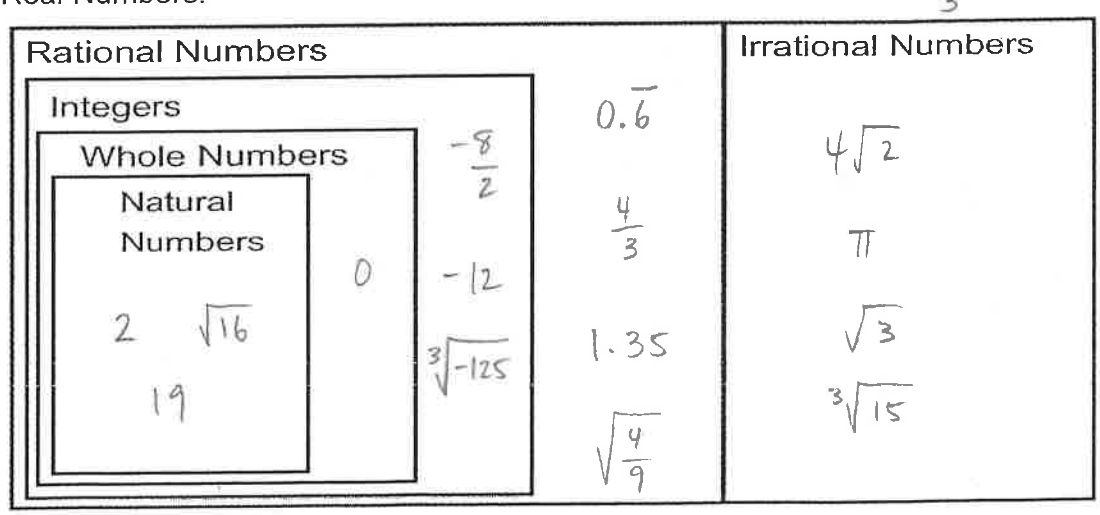
Classifying Real Numbers

Ex1) Where do these numbers belong in the diagram of Real numbers?

$2 \quad 0.\overline{6} \quad 4\sqrt{2} \quad \frac{4}{3} \quad \frac{-8}{2} = -4 \quad -12 \quad \pi \quad 0 \quad \sqrt{16} = 4$

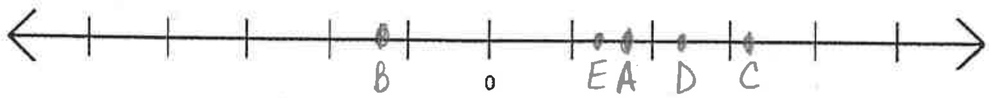
$1.35 \quad \sqrt[3]{-125} = -5 \quad \sqrt{3} \quad \sqrt[3]{15} \quad 19 \quad \sqrt{\frac{4}{9}} = \frac{2}{3}$

Real Numbers:



Ordering numbers on a number line	Ex2) Use a number line to order these numbers from least to greatest.

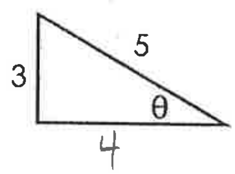
A	B	C	D	E
$\sqrt[3]{6}$	$\sqrt[3]{-2}$	$\sqrt{11}$	$\sqrt[4]{30}$	$\sqrt{2}$
$\sqrt[3]{1}$ $\sqrt[3]{8}$	$\sqrt[3]{-8}$ $\sqrt[3]{-1}$	~ 3.2	~ 2.3	~ 1.4
~ 1.8	~ -1.2			



Connect:

Ex3) Is the tangent ratio for θ in each right triangle rational or irrational?

a)



$a^2 + b^2 = c^2$

$3^2 + b^2 = 5^2$

$b^2 = 5^2 - 3^2$

$b^2 = 25 - 9$

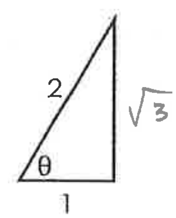
$b^2 = 16$

$b = 4$

$\tan\theta = \frac{3}{4}$

RATIONAL

b)



$a^2 + b^2 = c^2$

$1^2 + b^2 = 2^2$

$b^2 = 3$

$b = \sqrt{3}$

$\tan\theta = \sqrt{3}$ IRRATIONAL

Reflection: How could you order a set of irrational numbers if you do not have a calculator?

4.3A – From Entire to Mixed Radicals

Name:
Date:

Goal: to express an entire radical as a mixed radical

Toolkit:
- Understanding Radicals
- Identifying Factors of a Number

Main Ideas: p. 218 #3, 4, 6, 9–11, 14–17

Perfect Squares - 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, …..

Perfect Cubes - 1, 8, 27, 64, 125, 216, …..

What is an entire radical?

A radical symbol with a RADICAND. eg: $\sqrt{28}$, $\sqrt{x^5}$

What is a mixed radical?

A radical with a COEFFICIENT. eg: $2\sqrt{5}$, $-3\sqrt[3]{9}$

Equivalent Forms:

Ex 1)

a) $\sqrt{16 \cdot 9}$ is equivalent to $\sqrt{16} \cdot \sqrt{9}$ because:

$\sqrt{144} = 4 \cdot 3$

$12 = 12$

b) $\sqrt[3]{8 \cdot 27}$ is equivalent to $\sqrt[3]{8} \cdot \sqrt[3]{27}$ because:

$\sqrt[3]{216} = 2 \cdot 3$

$6 = 6$

What is the Multiplication Property of Radicals?

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, where n is a natural number, and a and b are real numbers

***We can use this property to simplify square roots and cube roots that are *not* perfect squares or perfect cubes, but have *factors* that are perfect squares or perfect cubes.**

Simplifying Square Roots

We can simplify $\sqrt{24}$ because 24 has a perfect square factor of 4.
(hint:look at list of perfect squares!)

- Re-write $\sqrt{24}$ as a product of two factors, with the first one being the perfect square:

$$\sqrt{24} = \sqrt{4 \cdot 6}$$
$$= \sqrt{4} \cdot \sqrt{6} = \boxed{2\sqrt{6}}$$

Simplifying Cube Roots

We can also simplify $\sqrt[3]{24}$ because 24 has a perfect cube factor of 8.
(hint:look at list of perfect cubes!)

- Re-write $\sqrt[3]{24}$ as a product of two factors, with the first one being the perfect cube:

$$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3}$$
$$= \sqrt[3]{8} \cdot \sqrt[3]{3}$$
$$= \boxed{2\sqrt[3]{3}}$$

Tip: If there is MORE than one perfect square or perfect cube factor, choose the LARGEST one!

Ex 2) Simplify each radical: (remember your list of perfect squares and perfect cubes!)

a) $\sqrt{80}$
$= \sqrt{16 \cdot 5}$
$= \sqrt{16} \cdot \sqrt{5}$
$= \boxed{4\sqrt{5}}$

b) $\sqrt{32}$
$= \sqrt{16 \cdot 2}$
$= \sqrt{16} \cdot \sqrt{2}$
$= \boxed{4\sqrt{2}}$

c) $\sqrt{98}$
$= \sqrt{49 \cdot 2}$
$= \sqrt{49} \cdot \sqrt{2}$
$= \boxed{7\sqrt{2}}$

d) $\sqrt[3]{162}$
$= \sqrt[3]{27 \cdot 6}$
$= \sqrt[3]{27} \cdot \sqrt[3]{6}$
$\boxed{= 3\sqrt[3]{6}}$

e) $\sqrt[3]{108}$
$= \sqrt[3]{27 \cdot 4}$
$= \sqrt[3]{27} \cdot \sqrt[3]{4}$
$= \boxed{3\sqrt[3]{4}}$

How do you simplify something with an index of 4? (a fourth root?)

think of 'perfect' fourth roots!
1, 16, 81, 256

Ex 3) Simplify $\sqrt[4]{162}$

$= \sqrt[4]{81 \cdot 2}$
$= \boxed{3\sqrt[4]{2}}$

Ex 4) Simplify $\sqrt[4]{48}$

$= \sqrt[4]{16 \cdot 3}$
$= \boxed{2\sqrt[4]{3}}$

Word Problem

Ex 5) A cube has a volume of $128cm^3$. Write the edge length of the cube in simplest radical form.

$\ell = \sqrt[3]{V} = \sqrt[3]{128}$
$= \sqrt[3]{64 \cdot 3}$
$= \boxed{4\sqrt[3]{3} \text{ cm}}$

Reflection: How do you use the index of a radical when you simplify a radical? Use an example.

4.3B – From Mixed to Entire Radicals

Name:
Date:

Goal: to express a mixed radical as an entire radical

Toolkit:
- List of Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100...
- List of Perfect Cubes: 1, 8, 27, 64, 125, 216, ...
- Multiplication Property of Radicals ($\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$)
- Mixed Radical....ex.
- Entire Radical......ex.

Main Ideas:

p. 218-219
#5, 12, 13, 18, 22

How do you write a mixed radical as an entire radical?

Write the mixed radical $4\sqrt{3}$ as an entire radical:

$4\sqrt{3}$
$= 4\cdot\sqrt{3}$ - Use the Multiplication Property of Radicals
$= \sqrt{16}\cdot\sqrt{3}$ (re-write 4 as a radical.....think .....4 = $\sqrt{?}$.....$\sqrt{16}$!)
$= \sqrt{16\cdot 3}$ - Combine these under the same radical sign and multiply
$= \sqrt{48}$ (***NOTICE...these are the *opposite* steps to writing an entire radical as a mixed radical)

Ex. 1) Write each as an entire radical:

a) $5\sqrt{2}$ $5^2 = 25$
$= \sqrt{25}\cdot\sqrt{2}$
$= \sqrt{25\cdot 2}$
$= \boxed{\sqrt{50}}$

b) $3\sqrt{3}$ $3^2 = 9$
$= \sqrt{9}\cdot\sqrt{3}$
$= \sqrt{9.3}$
$= \boxed{\sqrt{27}}$

c) $3\sqrt[3]{2}$ $3^3 = 27$
$= \sqrt[3]{27}\cdot\sqrt[3]{2}$
$= \sqrt[3]{27\cdot 2}$
$= \boxed{\sqrt[3]{54}}$

d) $2\sqrt[3]{6}$ $2^3 = 8$
$= \sqrt[3]{8}\cdot\sqrt[3]{6}$
$= \sqrt[3]{8\cdot 6}$
$= \boxed{\sqrt[3]{48}}$

What do you do if the index is 4 or 5 (or higher?)

Write $3\sqrt[5]{2}$ as an entire radical:

First, re-write 3 as $\sqrt[5]{?}$ .... $3 = \sqrt[5]{3\cdot3\cdot3\cdot3\cdot3} = \sqrt[5]{243}$

So now, $3\sqrt[5]{2}$
$= 3 \cdot \sqrt[5]{2}$
$= \sqrt[5]{243}\cdot\sqrt[5]{2}$ now, using the Multiplication Property of Radicals...
$= \sqrt[5]{243\cdot 2}$
$= \sqrt[5]{486}$

Ex. 2) Write each as an entire radical:

a) $2\sqrt[4]{5}$ $2^4 = 16$
$= \sqrt[4]{16}\cdot\sqrt[4]{5}$
$= \sqrt[4]{16.5}$
$= \boxed{\sqrt[4]{80}}$

b) $4\sqrt[5]{2}$ $4^5 =$
$= \sqrt[5]{1024}\cdot\sqrt[5]{2}$
$= \sqrt[5]{1024\cdot 2}$
$= \boxed{\sqrt[5]{2048}}$

How can entire radicals be used to help you order a set of mixed radicals with the same index?

Ex. 3) Arrange the following in order from greatest to least: $3\sqrt{5}, 2\sqrt{13}, 4\sqrt{3}, 2, 9\sqrt{2}$

Re-express all as an entire radical to compare!

$3\sqrt{5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{45}$

$2\sqrt{13} = \sqrt{4} \cdot \sqrt{13} = \sqrt{52}$

$4\sqrt{3} = \sqrt{16} \cdot \sqrt{3} = \sqrt{48}$

$2 = \sqrt{4}$

$9\sqrt{2} = \sqrt{81} \cdot \sqrt{2} = \sqrt{162}$

$\sqrt{162} > \sqrt{52} > \sqrt{48} > \sqrt{45} > \sqrt{4}$

$9\sqrt{2} > 2\sqrt{13} > 4\sqrt{3} > 3\sqrt{5} > 2$

Reflection: How do you use the index of a radical when you write a mixed radical as an entire radical? Use an example to help your explanation

4.4 – Fractional Exponents and Radicals

Name:
Date:

Goal: to relate rational exponents and radicals

Toolkit:

- Exponent Laws
- Taking square and cube roots
- Converting decimals to fractions
- Order of operations

Main Ideas:

p. 227 # 3–12, 15

Evaluating powers of the form $a^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When n is as natural number and x is a rational number,
$x^{\frac{1}{n}} = \sqrt[n]{x}$ ... for example... $16^{\frac{1}{2}} = \sqrt[2]{16} = 4$

Ex 1) Write each power as a radical then evaluate without using a calculator.

a) $1000^{\frac{1}{3}}$
$= \sqrt[3]{1000}$
$= \boxed{10}$

b) $0.25^{0.5}$
$= \sqrt{0.25}$
$= \sqrt{\frac{25}{100}}$
$= \frac{\sqrt{25}}{\sqrt{100}} = \frac{5}{10} = \boxed{0.5}$

c) $(-8)^{\frac{1}{3}}$
$= \sqrt[3]{-8}$
$= \boxed{-2}$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$
$= \sqrt[4]{\frac{16}{81}}$
$= \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \boxed{\frac{2}{3}}$

Rewriting powers in radical and exponent form

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,
$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$ ... ex) $25^{\frac{3}{2}} = \left(25^{\frac{1}{2}}\right)^3 = \left(\sqrt[2]{25}\right)^3 = (5)^3 = 125$

or

$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$ ... ex) $25^{\frac{3}{2}} = (25^3)^{\frac{1}{2}} = \sqrt[2]{25^3} = \sqrt{15625} = 125$

Ex 2) Write $26^{\frac{2}{5}}$ in radical form in two different ways.

$26^{\frac{2}{5}} = \left(26^2\right)^{\frac{1}{5}} = \sqrt[5]{26^2}$
$= \left(26^{\frac{1}{5}}\right)^2 = \left(\sqrt[5]{26}\right)^2$

Ex 3) Write the following in exponent form.

a) $\sqrt{6^5}$
$= \left(6^5\right)^{\frac{1}{2}}$
$= \boxed{6^{\frac{5}{2}}}$

b) $\left(\sqrt[4]{19}\right)^3$
$= \left(19^{\frac{1}{4}}\right)^3$
$= \boxed{19^{\frac{3}{4}}}$

Evaluating powers with rational exponents and rational bases

Ex 4) Evaluate the following:

a) $0.01^{\frac{3}{2}}$

$= \left(0.01^{\frac{1}{2}}\right)^3$

$= \left(\sqrt{0.01}\right)^3$

$= \left(\sqrt{\frac{1}{100}}\right)^3$

$= \left(\frac{1}{10}\right)^3$

$= \frac{1}{1000} = \boxed{0.001}$

b) $(-27)^{\frac{4}{3}}$

$= \left(-27^{\frac{1}{3}}\right)^4$

$= \left(\sqrt[3]{-27}\right)^4$

$= (-3)^4$

$= \boxed{81}$

c) $32^{0.4}$

$= 32^{\frac{4}{10}}$

$= 32^{\frac{2}{5}}$

$= \left(32^{\frac{1}{5}}\right)^2$

$= \left(\sqrt[5]{32}\right)^2$

$= 2^2$

$= \boxed{4}$

d) $16^{0.75}$

$= 16^{\frac{3}{4}}$

$= \left(16^{\frac{1}{4}}\right)^3$

$= \left(\sqrt[4]{16}\right)^3$

$= 2^3$

$= \boxed{8}$

Applying rational exponents

Ex 5) Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass, m kilograms. Use the formula to estimate the brain mass of each animal.

a) A moose with a body mass of 512kg

$b = 0.01(512)^{\frac{2}{3}}$

$b = 0.01\left(512^{\frac{1}{3}}\right)^2$

$b = 0.01\left(\sqrt[3]{512}\right)^2$

$b = 0.01(8)^2$

$= 0.01(64)$

$= \boxed{0.64\ kg}$

b) A cat with a body mass of 5kg

$b = 0.01(5)^{\frac{2}{3}}$

$= 0.01\left(5^{\frac{1}{3}}\right)^2$

$= 0.01\left(\sqrt[3]{5}\right)^2$

$= 0.01(2.9241) = \boxed{0.03\ kg}$

Reflection: In the power $x^{\frac{m}{n}}$, m and n are natural numbers and x is a rational number. What does the numerator m represent? What does the denominator n represent? Use an example to explain your answer.

4.5 – Negative Exponents and Reciprocals

Name:
Date:

Goal: To relate negative exponents to reciprocals

Toolkit:
- Simplifying and evaluating with rational exponents
- Multiplying fractions

Main Ideas:

p. 233 #4-10, 13

What is a reciprocal?

Two numbers with a product of **1** are reciprocals.

Ex. 1) Since $4 \cdot \frac{1}{4} = \mathbf{1}$, the numbers 4 and $\frac{1}{4}$ are <u>reciprocals</u>

Ex. 2) Since $\frac{2}{3} \cdot \frac{3}{2} = \mathbf{1}$, the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are <u>reciprocals</u>

Powers with Negative Exponents

> When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.
>
> That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$
>
> $x^n \cdot x^{-n} = x^{n-n} = x^0 = 1$

also note: $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$

Evaluate a power with a negative exponent

<u>Evaluate each power:</u>

Ex. 3)

a) 3^{-2}
$= \left(\frac{1}{3}\right)^2$
$= \frac{1^2}{3^2}$
$= \boxed{\frac{1}{9}}$

b) $(-5)^{-3}$
$= \left(-\frac{1}{5}\right)^3$
$= \frac{(-1)^3}{5^3}$
$= \boxed{\frac{-1}{125}}$

c) $(-\frac{3}{4})^{-3}$
$= \left(-\frac{4}{3}\right)^3$
$= \frac{(-4)^3}{3^3}$
$= \boxed{\frac{-64}{27}}$

d) $(\frac{10}{3})^{-2}$
$= \left(\frac{3}{10}\right)^2$
$= \frac{3^2}{10^2}$
$= \boxed{\frac{9}{100}}$

Evaluate a power with a negative *rational* exponent

<u>To evaluate a power with a negative *rational* (fraction) exponent:</u>

Ex. 4) Evaluate $8^{-\frac{2}{3}}$

$= \frac{1}{8^{\frac{2}{3}}}$ *write with a positive exponent*

$= \frac{1}{(\sqrt[3]{8})^2}$ *re-write into radical form, then work from inside out*

$= \frac{1}{(2)^2}$ *evaluate (write answer with NO exponents)*

$= \frac{1}{4}$

Ex. 5) Evaluate:

a) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{\frac{3}{2}}$

$= \frac{16^{\frac{3}{2}}}{9^{\frac{3}{2}}}$

$= \frac{(\sqrt{16})^3}{(\sqrt{9})^3}$

$= \boxed{\frac{64}{27}}$

b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

$= \left(\frac{36}{25}\right)^{\frac{1}{2}}$

$= \frac{\sqrt{36}}{\sqrt{25}}$

$= \boxed{\frac{6}{5}}$

c) $16^{-\frac{5}{4}}$

$= \left(\frac{1}{16}\right)^{\frac{5}{4}}$

$= \frac{(\sqrt[4]{1})^5}{(\sqrt[4]{16})^5}$

$= \boxed{\frac{1}{32}}$

d) $-25^{-1.5}$

(hint: change 1.5 to a fraction in lowest terms!)

$= -\frac{1}{25}^{\frac{3}{2}}$

$= -\left(\frac{(\sqrt{1})^3}{(\sqrt{25})^3}\right)$

$= \boxed{-\frac{1}{125}}$

Applying Negative Exponents (word problems)

Ex. 6) Use the formula $v = 0.155s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$ (answer is a speed in m/s)

Substitute values into the proper places in the formula

$V = 0.155(1.5)^{\frac{5}{3}}(0.3)^{-\frac{7}{6}}$

Evaluate, using your calculator

$V = 1.24$ m/s

Reflection:

4.6A – Simplifying with Exponent Laws

Name:
Date:

Goal: to apply all of the exponent laws to simplify expressions

Toolkit:
- Exponent Laws
- Fractional and negative exponents
- Operations with fractions, integers

Main Ideas:

p. 241-243 # 3-9, 11, 14, 16, 21

Exponent Laws

Product of powers: $x^m \cdot x^n = x^{m+n}$

Quotient of powers: $x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$ $\quad x \neq 0$

Power of a power: $(x^m)^n = x^{mn}$

Power of a product: $(xy)^m = x^m y^m$

Power of a quotient: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ $\quad y \neq 0$

Power of zero: $x^0 = 1$

Fractional exponents: $x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$

Negative exponents: $x^{-m} = \left(\frac{1}{x}\right)^m$, $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$

Note: write all powers with POSITIVE EXPONENTS.

Which law(s) did you use?

Ex 1) Simplify by writing as a single power.

a) $0.6^2 \cdot 0.6^{-6}$ b) $x^{-4} \cdot x^7$ c) $m^7 \div m^{-2}$ d) $\frac{0.4^3}{0.4^4}$ e) $(n^2)^{-4}$

a) $= (0.6)^{2+(-6)}$
$= 0.6^{-4}$
$= \left(\frac{3}{5}\right)^{-4}$
$= \left(\frac{5}{3}\right)^4$

b) $= x^{-4+7}$
$\boxed{= x^3}$

c) $= m^{7-(-2)}$
$= \boxed{m^9}$

d) $= 0.4^{3-4}$
$= \boxed{0.4^{-1}}$
$= \left(\frac{2}{5}\right)^{-1}$
$= \left(\frac{5}{2}\right)^1 = \boxed{\frac{5}{2}}$

e) $= n^{-8}$
$= \left(\frac{1}{n}\right)^8$
$= \frac{1}{n^8}$

Note: write all powers with POSITIVE EXPONENTS.

Ex 2) Simplify by writing as a single power.

a) $\left[\left(-\frac{4}{7}\right)^2\right]^{-3} \div \left[\left(-\frac{4}{7}\right)^4\right]^{-5}$

$=\left(-\frac{4}{7}\right)^{-6} \div \left(-\frac{4}{7}\right)^{-20}$

$=\left(-\frac{4}{7}\right)^{-6-(-20)}$

$=\boxed{\left(\frac{-4}{7}\right)^{14}}$

b) $\frac{\left(2.3^{-3}\right)^{-5}}{2.3^5}$

$=\frac{2.3^{15}}{2.3^5}$

$=\boxed{2.3^{10}}$

c) $\frac{8^{\frac{5}{4}} \cdot 8^{-\frac{1}{4}}}{8^{\frac{3}{4}}}$

$=\frac{8^{\frac{5}{4}+\left(-\frac{1}{4}\right)}}{8^{\frac{3}{4}}}$

$=\frac{8}{8^{\frac{3}{4}}} = 8^{1-\frac{3}{4}}$

$\boxed{=8^{\frac{1}{4}}}$

Ex 3) Simplify.

a) $(x^4y^{-2})(x^2y^3)$

$=x^{4+2}y^{-2+3}$

$=x^6y$

b) $(27x^6y^9)^{\frac{1}{3}}$

$=\left(27^{\frac{1}{3}}\right)\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}$

$=\boxed{3x^2y^3}$

c) $\left(\frac{6a^4b^{-3}}{14a^{-2}b^2}\right)^{-2}$

$=\left(\frac{3}{7}a^{4-(-2)}b^{-3-2}\right)^{-2}$

$=\left(\frac{3}{7}a^6b^{-5}\right)^{-2}$

$=\left(\frac{3a^6}{7b^5}\right)^{-2}$

$=\left(\frac{7b^5}{3a^6}\right)^2$

$=\boxed{\frac{49b^{10}}{9a^{12}}}$

d) $\left(\frac{50m^2n^4}{2m^4n^2}\right)^{\frac{1}{2}}$

$=\left(25m^{-2}n^2\right)^{\frac{1}{2}}$

$=\left(\frac{25n^2}{m^2}\right)^{\frac{1}{2}}$

$=\sqrt{\frac{25n^2}{m^2}}$

$=\frac{\sqrt{25n^2}}{\sqrt{m^2}}$

$=\boxed{\frac{5n}{m}}$

e) $\frac{\left(2x^{\frac{3}{2}}y^2\right)\left(3x^{\frac{1}{2}}y^{-1}\right)}{(4x^3y^{-1})}$

$=\frac{6x^2y}{4x^3y^{-1}}$

$=\frac{3}{2}x^{-1}y^2$

$=\boxed{\frac{3y^2}{2x}}$

Reflection: How would you simplify the expression $\left(\frac{x^a}{x^3}\right)^2$ and how is it similar/different compared to the other problems we've done?

4.6B – Evaluating with Exponent Laws

Name:
Date:

Goal: to apply all of the exponent laws to evaluate expressions

Toolkit:
- Exponent Laws, incl. fractional /negative
- Operations with fractions, integers
- Substitution, BEDMAS

Main Ideas:

p. 241 # 10, 15
Ch. Review p. 246-248 # 1, 3, 6, 7, 11, 12, 15, 17-19, 24, 28-30.

What is the difference between "simplifying" and "evaluating"?

Simplify:

Ex 1) Simplify $x^{\frac{5}{3}} \cdot x^{\frac{1}{3}}$

$= x^{\frac{5}{3}+\frac{1}{3}}$

$= x^{\frac{6}{3}}$

$= \boxed{x^2}$

Evaluate:

Ex 2) Evaluate $1.5^{\frac{5}{3}} \cdot 1.5^{\frac{1}{3}}$

$= 1.5^{\frac{5}{3}+\frac{1}{3}}$

$= 1.5^{\frac{6}{3}}$

$= 1.5^2 = \boxed{2.25}$

Ex 3) Evaluate each expression for $m = -1$ and $n = 2$

Step 1: Simplify the expression
Step 2: Substitute → replace letters with numeric values
Step 3: Evaluate

a) $(m^2n^3)(m^3n^2)$

$= m^{2+3}n^{3+2}$

$= m^5n^5$

$= (-1)^5(2)^5$

$= \boxed{-32}$

b) $\left(\frac{m^{-5}n^5}{m^{-2}n^6}\right)^{-3}$

$= (m^{-3}n^{-1})^{-3}$

$= m^9n^3$

$= (-1)^9(2)^3$

$= \boxed{-8}$

c) $\frac{(m^n)^2}{m^3}$

$= \frac{m^{2n}}{m^3}$

$= \frac{(-1)^{2(2)}}{(-1)^3}$

$= \frac{(-1)^4}{(-1)^3} = \frac{1}{-1} = \boxed{-1}$

Solving Problems using the Exponent Laws

Ex 4) A sphere has volume 600m^3.
a) Write an expression for the radius in exponent form
b) What is the radius of the sphere to the nearest tenth of a metre?

Reflection: Why is it important to simplify BEFORE evaluating?