

## Ch. 1.1 - Arithmetic Sequences / Sequences

- a sequence is simply a list of numbers.
- there are two types:
  - i) \_\_\_\_\_ - eg: 1, 4, 7, 10
  - ii) \_\_\_\_\_ - eg: 1, 4, 7, 10, 13, ...
- each number represents a term in the sequence.

eg: 1, 4, 7, 10, 13, ...  
    ↑ ↑ ↑ etc...  
    term term term etc...  
    1     2     3

- the term number represents the domain of the sequence (function), whereas the actual numbers in the list represent the range.

eg: 1, 4, 7, 10, 13, ...

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

- thus, the domain of a sequence is a set of Natural numbers (positive integers).

- even though a sequence is a function, function notation is not used to represent a sequence; \_\_\_\_\_ notation is.

eg: \_\_\_\_\_

- the subscript is the \_\_\_\_\_ of the sequence (\_\_\_\_).
- $a_1, a_2, \dots$  etc. represent the \_\_\_\_\_.

$a_1 = 1^{\text{st}}$  term     $a_3 = 3^{\text{rd}}$  term     $a_n = n^{\text{th}}$  term

The entire sequence is denoted by  $\{a_n\}$ .

So...

A finite sequence possesses a domain which is a subset of natural numbers.

i.e. Domain of finite sequence:  $x = \{1, 2, \dots, n\}$

An infinite sequence possesses a domain which is the entire (infinite) set of natural numbers.

i.e. Domain of infinite sequence:  $x = \{1, 2, 3, \dots\}$

Note: the range of any sequence may include values  $\leq 0$ , along with rational numbers.

eg1: Write the first four terms of each sequence:

a)  $a_n = \frac{n+1}{n}$

b)  $b_n = 2n - 3$

c)  $t_n = 2^n$

## Recursive Sequence

- a sequence where each term is defined from \_\_\_\_\_ in the sequence. It requires knowledge of this term or terms.

eg2: Write the next four terms of the recursive formula:  $a_1 = 3$   $a_n = \frac{a_{n-1}}{n}$

## Sigma Notation

- here and there, it is important to find the sum of a sequence.

$$\text{ie. } a_1 + a_2 + a_3 + \dots + a_n$$

- more often than not, sigma notation (utilizing the greek letter sigma ( $\Sigma$ )) is used to represent these situations.

$$\text{eg: } \underbrace{a_1 + a_2 + \dots + a_n}_{\text{expanded notation}} = \underbrace{\Sigma}_{\text{sigma notation}}$$

note: k is the \_\_\_\_\_ of the sum, showing where the summation begins. n shows where the summation ends.

another note: any summation possesses

\_\_\_\_\_ terms, or,

\_\_\_\_\_ terms.

eg3: Find the sum of each sequence:

a)  $\sum_{k=1}^4 (2k + 1)$

b)  $\sum_{k=1}^5 (k^2 + 1)$

c)  $\sum_{k=1}^3 (k^3 - k)$

Eg4: Re-write each summation in sigma notation:

a)  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{12}{12+1}$

b)  $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots + \left(\frac{2}{3}\right)^n$

### Arithmetic Sequence

- a sequence is deemed "Arithmetic" when the difference between successive terms is \_\_\_\_\_.

Eg:

This arithmetic sequence has a COMMON DIFFERENCE ( ) of \_\_\_\_.  
ie.  $d = \underline{\hspace{2cm}}$

So:

$$1^{\text{st}} \text{ term } (a_1) = a_1$$

$$2^{\text{nd}} \text{ term } (a_2) = a_1 + d$$

$$\begin{aligned} 3^{\text{rd}} \text{ term } (a_3) &= a_2 + d = (a_1 + d) + d \\ &= a_1 + 2d \end{aligned}$$

$$\begin{aligned} 4^{\text{th}} \text{ term } (a_4) &= a_3 + d = (a_1 + 2d) + d \\ &= a_1 + 3d \end{aligned}$$

$$n^{\text{th}} \text{ term } (a_n) = a_1 + (n-1)d$$

for any integer  $n \geq 1$

Again, for an arithmetic sequence  $\{a_n\}$ ,  
whose first term is  $a_1$ , with common  
difference  $d$ :

---

eg5: What is  $d$  for  $11, 8, 5, 2, \dots ?$

eg6: Determine if the sequence  
 $\{t_n\} = \{3 - 2n\}$  is arithmetic.

eg7: Find the 14<sup>th</sup> term of the arithmetic sequence 3, 7, 11, ...

eg8: Which term in the arithmetic sequence 4, 7, 10, ... has a value of 439?

eg9:  $t_7 = 78$ ;  $t_{18} = 45$ . The sequence  
is arithmetic. Find  $t_1$ .

q10: Find  $x$  so that  $3x+2$ ,  $2x-3$ , and  $2-4x$  are consecutive terms in an arithmetic sequence.

p. 9 - 12

#1-17 (18, 19 for 'fun')

## Ch. 1.2 - Arithmetic Series

- a series is the \_\_\_\_\_ of the terms of a sequence.

- a series can be finite or infinite  
↓  
\* this section \*

### Deriving the Sum Formula for Finite Arithmetic Series

Let  $\{a_n\} = a_1, a_2, a_3, \dots, a_n$  be a finite arithmetic sequence:

Then  $a_1 + a_2 + a_3 + \dots + a_n$  is a finite arithmetic series.

Let  $d$  = common difference

$S_n$  = sum of the series

$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$\text{eq'n 1: } S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$$

Let  $l = a + (n-1)d \rightarrow$  the last term

$$\text{eq'n 2: } S_n = l + (l-d) + (l-2d) + \dots + a \quad (\text{SUM IN REVERSE ORDER})$$

Add eq'n 1 and 2:

$$S_n = a + (a+d) + (a+2d) + \dots + l$$

$$\textcircled{+} \quad S_n = l + (l-d) + (l-2d) + \dots + a$$

$$\underline{2S_n = (a+l) + (a+l) + (a+l) + \dots + (a+l)}$$

$$2S_n = n(a + l)$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{n}{2}(a + (a + (n-1)d))$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

### SUM OF A FINITE ARITHMETIC SERIES:

The sum of the first  $n$  terms of an arithmetic series is given by:

OR

where  $a = a_1$  = the \_\_\_\_\_ term

$l = a + (n-1)d$  = the \_\_\_\_\_ ( $n^{\text{th}}$ ) term

$d$  = common difference

q1: Find the sum of the even integers from 2 to 60 inclusive.

q2: Find the sum of the first 25 terms of the series,  $11 + 15 + 19 + \dots$

eg3: Find the sum of the series

$$7 + 10 + 13 + \dots + 100$$

### Summation (Sigma Notation)

- if the summation expression is LINEAR,  
then the summation is an arithmetic series.

eg:  $\sum_{k=1}^{10} (2k+1) \rightarrow \underline{\hspace{2cm}}$ , since  $2k+1$  is linear

$$\sum_{k=1}^{10} (k^2+1) \rightarrow \underline{\hspace{2cm}}, \text{ since } k^2+1 \text{ is not linear}$$

Eg4: Evaluate  $\sum_{k=1}^{100} (2k + 1)$

Eg5: Write  $5 + 9 + 13 + \dots + 137$  in summation (sigma) notation.

~~eg6:~~ The sum of the first  $n$  terms of an arithmetic series is  $S_n = 5n^2 - 3n$ .  
Find  $d$ .

~~eg7:~~ Find the sum of all multiples of 6 between 100 and 1000.

~~eg8:~~ Find two arithmetic means between  
8 and 29.

P. 16-19

# 1-6, 8-12 \* try 7, 13, 14 for fun.

## Ch. 1.3 - Geometric Sequences

- recall that arithmetic sequences possess terms that differ by a common difference,  $d$ . (ie. a constant number is ADDED to each term to get the next term).
- in a GEOMETRIC sequence, each term is \_\_\_\_\_ by a constant number to get the next term. (ie. geometric sequences possess terms that differ by a common \_\_\_\_\_ (or factor of), \_\_\_\_\_.)

eg: 2, 4, 8, 16, 32, etc...

### Geometric Sequence

A sequence is GEOMETRIC if the ratio of consecutive terms is constant.

i.e.  $a_1, a_2, a_3, \dots, a_n$  is geometric if there is a number  $r$  ( $r \neq 0$ ), such that:

again,

$r = \text{COMMON RATIO}$

Eg 1: For each geometric sequence, find the common ratio:

a) 2, 6, 18, 54, ...

b) 3, -6, 12, -24, ...

c) -8, -4, -2, -1, ...

The  $n^{\text{th}}$  term of a Geometric Sequence:

\_\_\_\_\_, for any integer  $n \geq 1$ .

$a$  = first term,  $t_1$

Derivation:

Let  $a = a_1$  = the first term

Let  $r$  = common ratio.

$$a_1 = a$$

$$a_2 = ar$$

$$a_3 = a_2r = ar(r) = ar^2$$

$$a_4 = a_3r = ar^2(r) = ar^3$$

⋮



\* another view: the exponent in each term  
is one        than the subscript of the term.

\* also: \_\_\_\_\_

q2: Find the 8<sup>th</sup> term of the geometric  
sequence: 3, 12, 48, 192, ...

Q3: The 4<sup>th</sup> term of a geometric sequence is 125. The 9<sup>th</sup> term of the same sequence is  $\frac{125}{32}$ . Find the 13<sup>th</sup> term.

Ex 4: If  $x, 2x+2, 3x+3, \dots$  represents a geometric sequence, then what is the value of  $x$ ? What is  $r$ ?

- p. 22-25 #1-5, 7-19. (\*<sub>6, 20</sub> for fun)  
NOTE: #17 should read: "after 12 SUCCESSIVE swings." #15  $\rightarrow$  after 9 cycles.

## Ch. 1.4 - Geometric Series

A geometric series is the \_\_\_\_\_ of all of the terms of a geometric sequence.

### Deriving the Sum Formula for Finite Geometric Series

Let  $S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$  be a geometric series with common ratio,  $r$ .

$S_n$  can be re-expressed by multiplying  $S_n$  by  $-r$ , then adding the result to  $S_n$ :

$$-rS_n = (-r)(a) + (-r)(ar) + (-r)(ar^2) + \dots + (-r)(ar^{n-1})$$

$$-rS_n = -ar - ar^2 - ar^3 - \dots - ar^{n-1} - ar^n$$

$$+ S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

$$S_n - rS_n = a - ar^n$$

Note:  $t_n = ar^{n-1} = l$

$$S_n(1-r) = a - ar^n$$

$$S_n = \frac{a - ar^n}{1-r} \quad \text{OR}$$

$$S_n = \frac{a - ar^n}{1-r}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{a - ar^{n-1} \cdot r}{1-r}$$

$$S_n = \frac{a - rl}{1-r}$$

The sum of the first  $n$  terms of a geometric series with first term,  $a$ , and last term,  $l$  (or  $ar^{n-1}$ ), is given by:

OR

} note:  $r \neq$

OR

Eg/: Find the sum of the geometric series  
 $2 + 6 + 18 + 54 + \dots 1458.$

eg2: Find the sum of the first eight terms of the geometric series  $3 + 6 + 12 + \dots$

eg3: Find the sum of the geometric series  $\sum_{k=1}^{10} 3(-2)^{k-1}$

Note: in sigma (summation) notation,  
 an ARITHMETIC series has its variable  
 in the \_\_\_\_\_. A GEOMETRIC series has  
 its variable in the \_\_\_\_\_.

eg: ARITHMETIC :

$$\sum_{k=1}^5 (2k - 3) \quad d = \underline{\hspace{2cm}}$$

GEOMETRIC :

$$\sum_{k=1}^5 2 \cdot 3^k \quad r = \underline{\hspace{2cm}}$$

NEITHER :

$$\sum_{k=1}^5 (2 + 3^k)$$

Why?  $k=1 \rightarrow \underline{\hspace{2cm}}$        $k=3 \rightarrow \underline{\hspace{2cm}}$   
 $k=2 \rightarrow \underline{\hspace{2cm}}$        $k=4 \rightarrow \underline{\hspace{2cm}}$   
 no \_\_\_\_\_.

eg 4: Write the geometric series, 6 + 18 + 54 +  
 162 + 486, using sigma notation with  
 index  $k = 1$ .

p.28-31  
 #1-19 (omit 14, 20)  
 TYPO #17 answer:  $t_2 = -8$

TYPO #12 answer: using  $S_6 \rightarrow a = \underline{\hspace{2cm}}, t_6 = \underline{\hspace{2cm}}$

## Ch. 1.5 - Infinite Geometric Series

$$\underbrace{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots, \left(\frac{1}{2}\right)^{n-1}}_{n \in \{\text{NATURAL numbers}\}}$$

an example of an INFINITE geometric sequence.

- as  $n$  gets larger,  $\left(\frac{1}{2}\right)^{n-1}$  gets closer and closer to  $\underline{\quad}$ .

e.g.  $n = 51 \rightarrow$

$n = 101 \rightarrow$

i.e.  $\lim_{n \rightarrow \infty} \left(\frac{1}{2}\right)^n = 0 \rightarrow$  as  $n$  approaches an infinitely large number,  $\left(\frac{1}{2}\right)^n$  approaches 0.

- this is an example of a \_\_\_\_\_ geometric sequence.

i.e. as the terms of an infinite geometric sequence get closer and closer to some real number, the sequence is said to be CONVERGENT.

$$1, 2, 4, 8, 16, 32, \dots, 2^{n-1} \quad n \in \{\text{NATURAL numbers}\}$$

- this sequence does NOT converge to some real number.

- a sequence that does not converge is said to be a \_\_\_\_\_ geometric sequence.

Convergent geometric sequences possess an r-value between \_\_\_\_\_ (\_\_\_\_\_) or \_\_\_\_\_.

Divergent geometric sequences possess an r-value \_\_\_\_\_ or \_\_\_\_\_ (\_\_\_\_\_ or \_\_\_\_\_).

The Sum of an Infinite Geometric Sequence is given by:

$$\begin{aligned} & |r| < 1 \\ & (-1 < r < 1) \end{aligned}$$

Proof :

$$S_n = \frac{a(1-r^n)}{1-r}$$

as  $n \rightarrow \infty$ ,  $r^n \rightarrow 0$  (when  $|r| < 1$ )

$$S_n = S_\infty = \frac{a(1-0)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

e.g.: Find the sum of each infinite geometric series:

a)  $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

b)  $\frac{2}{5} - \frac{4}{15} + \frac{8}{45} - \dots$

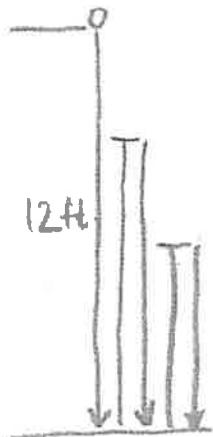
### SUMMATION (SIGMA) FORM

An infinite geometric series with first term  $a$  and common ratio  $r$  ( $|r| < 1$ ) is given by :

eg2: Find the sum of  $\sum_{k=2}^{\infty} 8 \left(-\frac{1}{2}\right)^{k-1}$

q3: Initially, a pendulum swings through an arc of 25 cm. On each successive swing, friction lessens the length of the pendulum swing by 5%. When the pendulum eventually stops, what total length will it have swung?

q4: A ball is dropped from 12 ft. and rebounds  $\frac{2}{3}$  the distance from which it fell. Find the total distance the ball traveled.



eg 5: Write  $0.\overline{24}$  as a fraction.

Eg6: Solve for  $x$ :

$$\sum_{k=1}^{\infty} (\tan x)^{k-1} = 1 \quad 0^\circ \leq x < 45^\circ$$

p. 35 - 39

# 1 - 5, 7 - 20.

+

Chapter Review