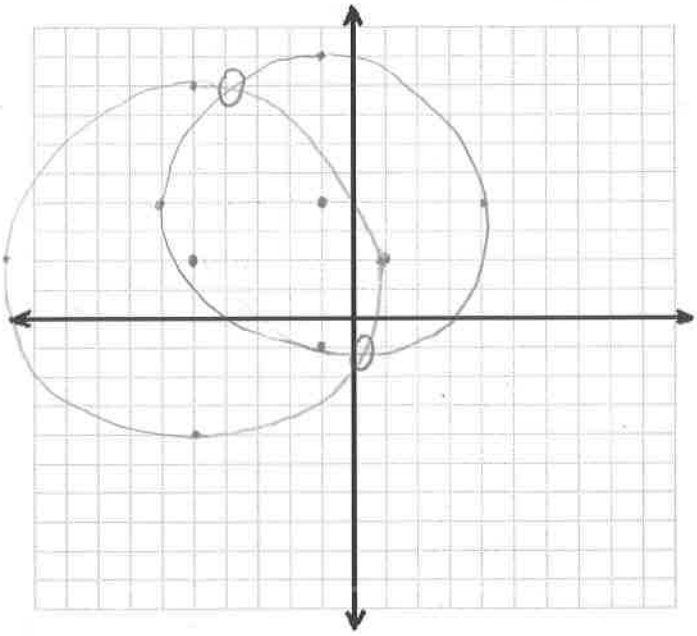
Conic Applications/'Thinking' Questions

1. Eduardo is flying his drone 30m above the ground in a path described by the equation: $(x+5)^2+(y-2)^2=36$. Shakira is also flying a drone at a height of 30m, yet her path is described by the equation: $(x+1)^2+(y-4)^2=25$. Do the paths of the two planes intersect? If so, at how many points? (You do not need to solve this algebraically)



Yes, at 2 points!

2. The points (1, 0) and (-1, -2) lie on a circle. The centre of the circle lies somewhere upon the line described by the equation: $y=-2x$. What is the equation of the circle?

Point on line: $(x, -2x)$

Distance (d) from $(1,0)$ to $(x,-2x)$ equals d $(-1,-2)$ to $(x,-2x)$

$$\sqrt{(x-1)^2+(-2x-0)^2} = \sqrt{(x-(-1))^2+(-2x-(-2))^2}$$

$$\left(\sqrt{(x-1)^2+(-2x)^2}\right)^2 = \left(\sqrt{(x+1)^2+(-2x+2)^2}\right)^2$$

$$x^2-2x+1+4x^2 = x^2+2x+1+4x^2-8x+4$$

$$5x^2-2x+1 = 5x^2-6x+5$$

$$-2x+1 = -6x+5$$

$$4x = 4$$

$$x=1 \quad y=-2x=-2(1)=-2$$

Centre: $(1,-2)$

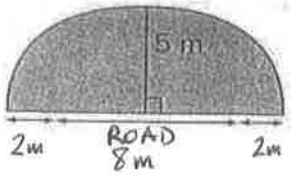
radius (r) = d from centre to $(1,0)$ or $(-1,-2)$

$$r=\sqrt{(1-1)^2+(-2-0)^2}$$

$$r=\sqrt{4}=2$$

ANSWER: $(x-1)^2+(y+2)^2=4$

3. A semi-elliptical covering is to be built over an 8m-wide road and the 2m-wide sidewalks on either side of it. If there is a maximum clearance of 5m over the road, what will be the minimum clearance over the road? What is the height of the tallest person that will be able to walk down the middle of the sidewalks? Are these numbers realistic?



Major axis = $12 = 2a$

$a = 6$

$a^2 = 36$

Minor axis = $10 = 2b$

$b = 5$

$b^2 = 25$

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

i) Find y-value (height) when $x = 4$:

$$\frac{4^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{16}{36} + \frac{y^2}{25} = 1$$

$$\frac{4}{9} + \frac{y^2}{25} = 1$$

$$\frac{y^2}{25} = \frac{5}{9}$$

$$9y^2 = 125$$

$$y^2 = \frac{125}{9}$$

$$y = \frac{5\sqrt{5}}{3} \text{ m}$$

$(3.73 \text{ m} \Rightarrow 147 \text{ in.} = 12'3'')$

REALISTIC

ii) Find y-value (height) when $x = 5$:

$$\frac{5^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{25}{36} + \frac{y^2}{25} = 1$$

$$\frac{y^2}{25} = \frac{11}{36}$$

$$36y^2 = 275$$

$$y^2 = \frac{275}{36}$$

$$y = \frac{5\sqrt{11}}{6} \text{ m}$$

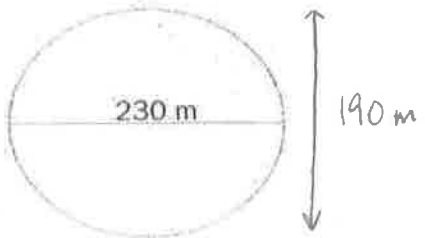
REALISTIC

$(2.76 \text{ m} \Rightarrow 109 \text{ in.} \Rightarrow 9'1'')$

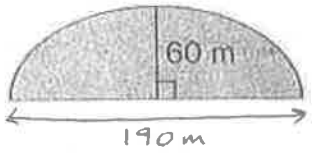
4. BC Place Stadium has an air-fabric domed roof that forms the shape of an ellipse when viewed from above. Its maximum length is approximately 230m, its maximum width is approximately 190m, and its maximum height is approximately 60m.

a. Find an equation for the ellipse formed by the base of the roof.

$$\frac{x^2}{115^2} + \frac{y^2}{95^2} = 1$$



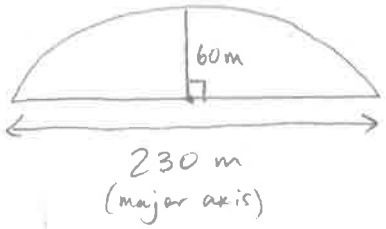
b. Taking a cross-section of the roof at its greatest width results in a semi-ellipse. Find an equation for this semi-ellipse.



$$\frac{x^2}{95^2} + \frac{y^2}{60^2} = 1$$

$(y \geq 0)$

c. The promoters of a Red Hot Chili Peppers concert plan to send fireworks up from a point on the stage that is 30m lower than the centre of the ellipse in part b), and 40m along the major axis of this ellipse from its centre. How far is that point on the stage from the roof? Answer to the nearest tenth of a meter.



$$\frac{x^2}{115^2} + \frac{y^2}{60^2} = 1$$

Find y when $x = 40$, then add 30m:

$$\frac{40^2}{115^2} + \frac{y^2}{60^2} = 1$$

$$\frac{y^2}{3600} = 0.879$$

$$y = 56.3 \text{ m}$$

Answer = 56.3 + 30

= 86.3 m

5. A ship is monitoring the movement of a pod of Orca with its radarscope. The radarscope screen can be modeled as a coordinate grid with the ship at the centre (0, 0). The pod appears to be moving along a curve such that the absolute value of the difference of its distances from (2, 7) and (2, -3) is always 6. What equation describes the path of the Orca pod?

$F_1 = (2, 7)$
$F_2 = (2, -3)$

$2a = 6$
$a = 3$
$a^2 = 9$

$7 - (-3) = 10$

$2c = 10$
$c = 5$
$c^2 = 25$

Centre: $(2, 7-5)$
$\boxed{(2, 2)}$

Pythagorean relationship for hyperbolas: $a^2 + b^2 = c^2$

$9 + b^2 = 25$
$b^2 = 16$

$$\boxed{\frac{(y-2)^2}{9} - \frac{(x-2)^2}{16} = 1}$$

6. Two hyperbolas are centred at (1, 2). One has a transverse axis parallel to the *x*-axis and the other has a transverse axis parallel to the *y*-axis. They share the same pair of asymptotes. If the equation of one hyperbola is: $\frac{(y-1)^2}{25} - \frac{(x-2)^2}{9} = 1$, what are the lengths of the conjugate and transverse axes of the other hyperbola?

Hyperbola 1: Horizontal

Slopes of asymptotes same as hyperbola 2 $\Rightarrow \pm\frac{5}{3}$

Slopes for horizontal $= \frac{b}{a}$

$b = 5$
$a = 3$

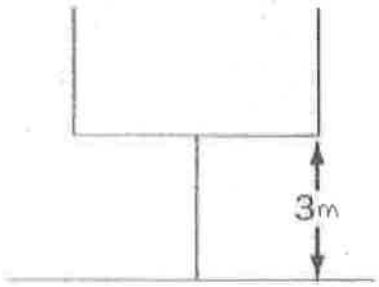
$\boxed{\text{transverse} = 6 \quad \text{conjugate} = 10}$

Hyperbola 2: Vertical

Slopes of asymptotes $= \frac{a}{b}$
$= \pm\frac{5}{3}$

7. One of the best known kickers in CFL (Canadian Football League) history is Lui Passaglia. During his first 20 seasons with the BC Lions, Passaglia scored 3160 points. Consider this situation. Passaglia kicks the ball from the 42-yard line (38m). Suppose the football follows the path:

$10y = -x^2 + 39x$, and is kicked directly at one of the field's goalposts, the bottom of which is 3m above the ground.



Find vertex:

$10y = -(x^2 - 39x)$

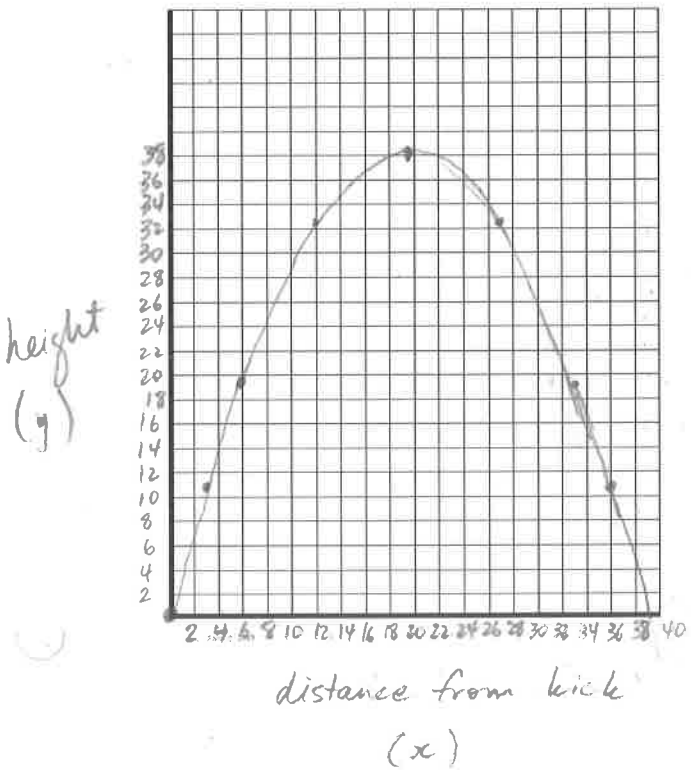
$10y - 380.25 = -(x^2 - 39x + 380.25)$

$10y = -(x - 19.5)^2 + 380.25$

$y = -\frac{1}{10}(x - 19.5)^2 + 38.025$

Vertex $(19.5, 38.025)$

a. Sketch the path of the football (you will have to define your scale).



x	y
0	0
19.5	38.025
6	19.8
12	32.4
33	19.8
27	32.4
3	10.8
36	10.8
39	0

b. Will Passaglia score the field goal?

$y = -\frac{1}{10}(x-19.5)^2 + 38.025$ set $x = 38$

$y = -\frac{1}{10}(38-19.5)^2 + 38.025$

$y = 3.8\text{ m}$ (higher than 3m) YES

c. If he was kicking from the 45-yard line (41.15m), would he have scored the field goal?

$y = -\frac{1}{10}(x-19.5)^2 + 38.025$ set $x = 41.15\text{ m}$

$y = -\frac{1}{10}(41.15-19.5)^2 + 38.025$

$y = -8.8\text{ m}$ (ie. ball on ground) NO

8. In 1992, the Toronto Blue Jays became the first Canadian team to win the World Series. In 1993, the Jays repeated as champs thanks to a walk-off 3-run home run by Joe Carter. Consider this situation. Carter hits the baseball from a point that is 110m from the 2m-high left-field wall. Let the point where Carter hit the ball be modeled as the origin with the unit interval of length 1m. If the ball travels in the path described by: $x^2 - 115x + 95y = 0$, then determine:

a. whether or not the baseball will clear the wall;

b. if it does clear the wall, whether a player standing at the wall could jump and catch it.

a) Find y when $x = 110$

$110^2 - 115(110) + 95y = 0$

$95y = 12650 - 12100$

$95y = 550$

$y = 5.8\text{ m}$ (higher than 2m) YES, it clears.

b) $5.8\text{ m} = 228.5\text{ inches} = 19'\frac{1}{2}''$

no one is jumping that high