

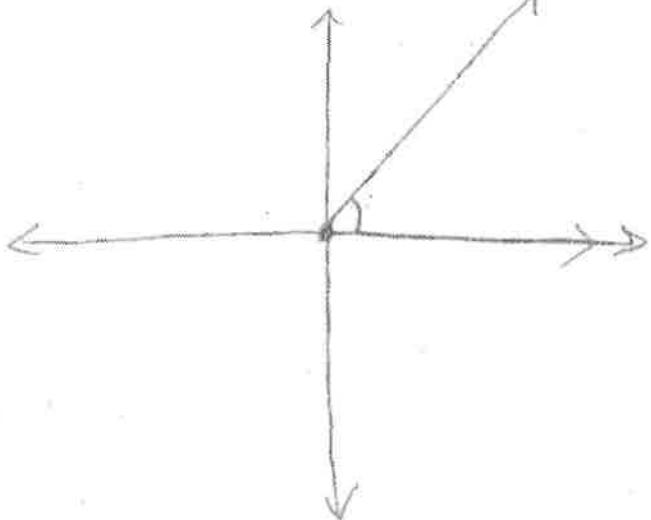
## Ch. 6.1 - Trig. Functions - Angles and their Measures

Angles on a Coordinate Plane in STANDARD POSITION:

- Standard position implies that the angle's \_\_\_\_\_ rests along the positive \_\_\_\_\_ of the coordinate plane, with its vertex at the \_\_\_\_\_.

It further implies that the initial arm is rotated about the origin (vertex) which creates \_\_\_\_\_.

The final 'resting place' of the arm is referred to as the angle's \_\_\_\_\_.



Note:

- a counter-clockwise rotation of the terminal arm creates a \_\_\_\_\_  $\angle \theta$ .
- a clockwise rotation of the terminal arm creates a \_\_\_\_\_  $\angle \theta$ .

- the measure of  $\angle \theta$ , in standard position, is governed by the \_\_\_\_\_ of the rotation and the \_\_\_\_\_ of the rotation.
- amount of rotation often measured in \_\_\_\_\_.  
(where one COMPLETE rotation is equivalent to \_\_\_\_\_).

One DEGREE ( $1^\circ$ ) = \_\_\_\_\_ of a complete rotation.

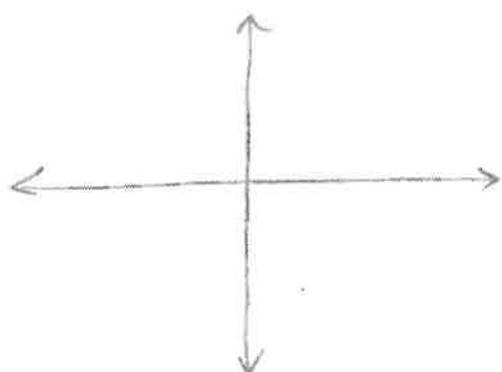
e.g.: Describe, in words, the direction and magnitude of each of the following angle measures:

a)  $2^\circ$

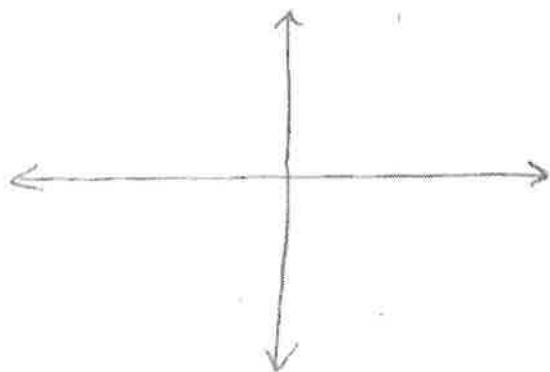
b)  $-90^\circ$  =

Eg 2: Sketch each of the following in standard position:

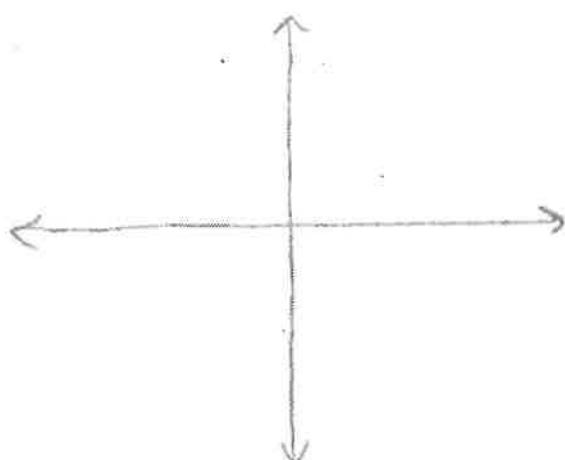
a)  $\theta = 135^\circ$



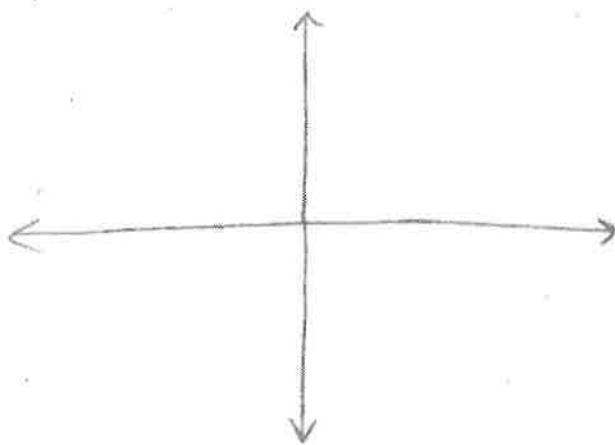
b)  $\theta = -70^\circ$



c)  $\theta = 405^\circ$



d)  $\theta = -570^\circ$



### Classes of Angles

Acute L:

Reflex L:

Right L:

Quadrantal Ls:

Obtuse L:

Straight L:

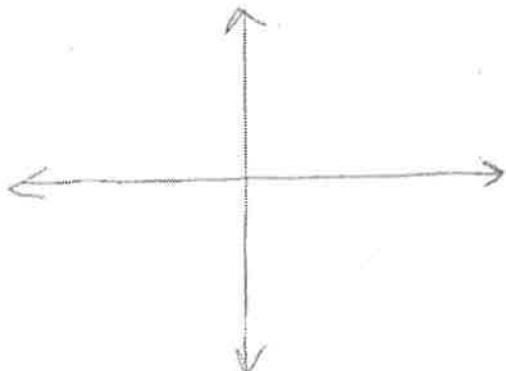
## Co-terminal Angles

Co-terminal angles are angles that share the same \_\_\_\_\_.

To find a co-terminal angle, add or subtract \_\_\_\_\_ to a given  $\angle \theta$ .

note: there exist infinitely many co-terminal angles.

eg3: On the grid provided, sketch  $\theta = 150^\circ$  and  $\gamma = -210^\circ$ . What conclusion can you draw?



Conclusion:

eg4: Given  $\theta = 465^\circ$ , determine the two smallest positive and the two 'smallest' negative co-terminal angles.

i) Positive:

ii) Negative:

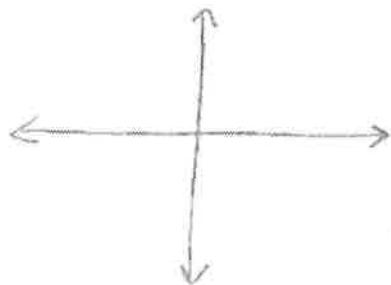
## Radian Measure and Conversion

- aside from degrees, \_\_\_\_\_ are another unit used to quantify an angle's magnitude.

Why another unit?

- radians are more suited to scientific/engineering applications since a radian also serves to directly define the length of an \_\_\_\_\_ of a circle 'created' by angle  $\theta$ . (note: this arc length is proportional to the radius of the circle).
- a UNIT CIRCLE (a circle with a radius of 1 unit) is used to define a radian:

An angle measuring 1 radian is a standard-position angle (counter-clockwise) that 'creates' an arc length of 1 on a UNIT circle.



Circumference (circle) = \_\_\_\_\_

Circumference (unit circle) = \_\_\_\_\_

So...  $360^\circ$  = \_\_\_\_\_

$\therefore 180^\circ$  = \_\_\_\_\_

note: there is no symbol for radians.

eg5: Convert each given degree value to radians:

a)  $240^\circ$

b)  $72^\circ$

eg6: Convert each given radian value to degrees:

a)  $\frac{3\pi}{4}$

b)  $2.13$

Key conversions to know:

$$\frac{\pi}{2} =$$

$$\frac{\pi}{4} =$$

$$\frac{\pi}{6} =$$

$$\frac{\pi}{3} =$$

$$\frac{3\pi}{2} =$$

### Arc length

The length of an arc of a circle ( $s$ ) is directly proportional to the size of  $\angle \theta$  (in radians) and the radius of the circle.

e.g. What is the arc length of an arc on a circle with  $r = 5 \text{ cm}$  and  $\theta = 60^\circ$ ?

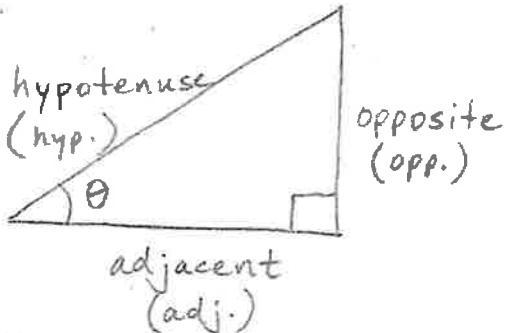
eg 8: Find  $\theta$  if a circle has a diameter of 12 cm and has an arc (defined by  $\theta$ ) of 18 cm? Round answer to nearest degree.

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## Ch. 6.2 - Trigonometric Functions of Acute Angles

- a trigonometric function is a ratio of two side lengths of a right triangle.



### The 6 Trig. Functions

For an acute angle  $\theta$  in a right triangle:

$$\text{SINE } \theta = \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{COSECANT } \theta = \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{COSINE } \theta = \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{SECANT } \theta = \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{TANGENT } \theta = \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{COTANGENT } \theta = \cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

Note:  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

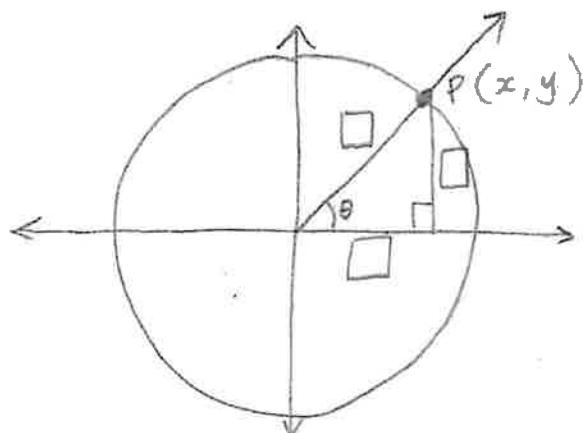
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

Consider angle  $\theta$  in standard position with point  $P(x, y)$  on the terminal side of  $\theta$ :



CONSTRUCT A RIGHT  $\Delta$   
BY CONNECTING P TO  
THE NEAREST POINT ON  
THE X-AXIS.

Using the Pythagorean Theorem:

$$x^2 + y^2 = r^2$$

$$r = \underline{\hspace{2cm}}$$

Note:  $r$  is always positive.

We can use  $x$ ,  $y$ , and  $r$  to define the 6 trig. ratios of any  $\angle\theta$  (even non-acute)

$$\sin\theta = \boxed{\phantom{00}}$$

$$\csc\theta = \boxed{\phantom{00}}$$

$$\cos\theta = \boxed{\phantom{00}}$$

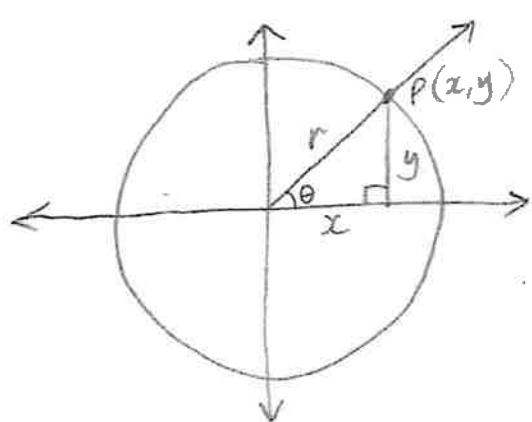
$$\sec\theta = \boxed{\phantom{00}}$$

$$\tan\theta = \boxed{\phantom{00}}$$

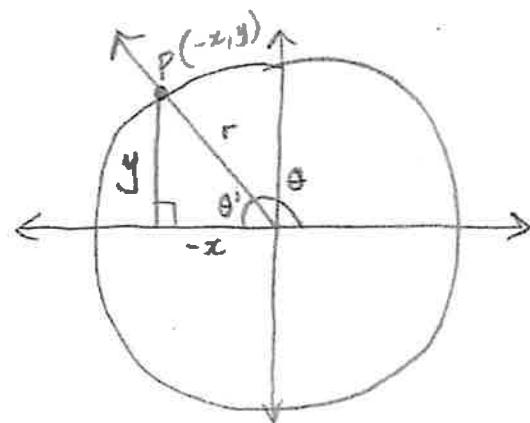
$$\cot\theta = \boxed{\phantom{00}}$$

$\theta$  may exist in any quadrant!

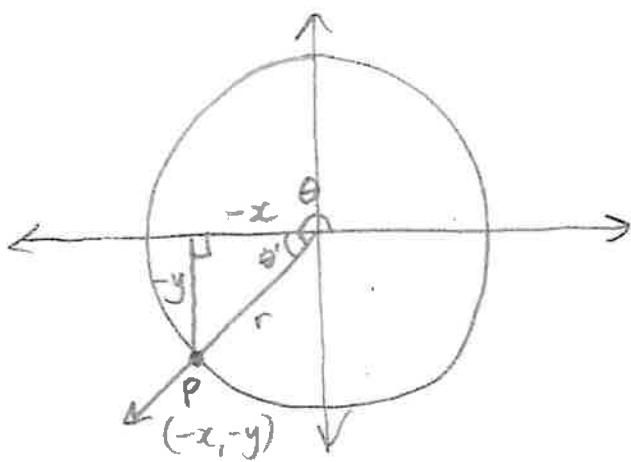
i)  $\theta$  in Quadrant I:



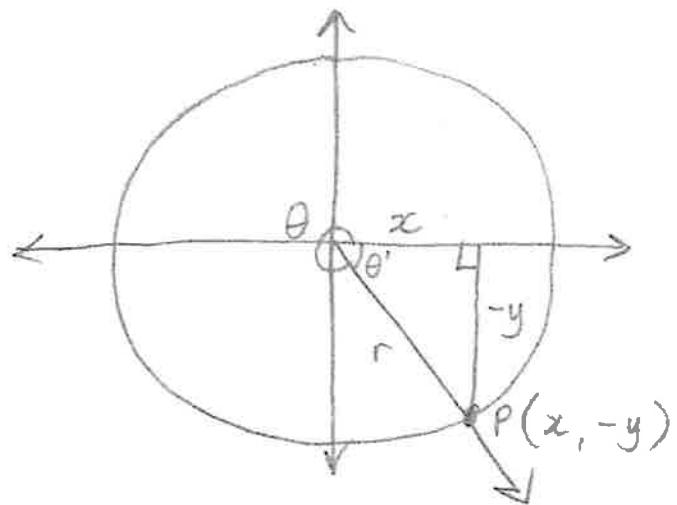
ii)  $\theta$  in Quadrant II:



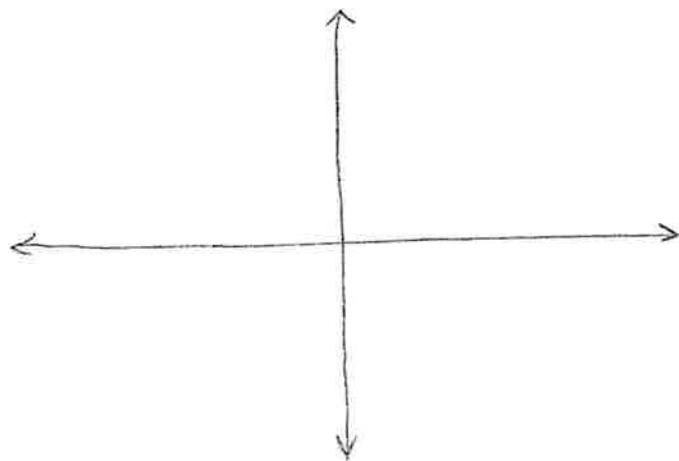
iii)  $\theta$  in Quadrant III:



iv)  $\theta$  in Quadrant IV:



Summary:



- \* if  $\sin \theta$  is positive, then so too is its reciprocal function,  $\csc \theta$ , and so on for  $\cos \theta$  and  $\tan \theta$ .

eg1: Which quadrant does  $\theta$  exist in standard position if  $\sin \theta < 0$  and  $\tan \theta > 0$ ?

eg2: Determine  $\cos \theta$  if  $\csc \theta = -\frac{3}{2}$  and  $\tan \theta < 0$ .

Eg 3: Determine  $\cot \theta$  if  $\sin \theta = \frac{2}{5}$ .

Eg 4: Given the point  $P(-2, -1)$  on the terminal side of  $\angle \theta$  in standard position, determine the value of all 6 trig. functions.

eg5: Determine  $\sin \theta$  and  $\cos \theta$  if  $\theta$  is an angle in standard position whose terminal side is the line  $2x + 3y = 0$  ( $x \leq 0$ ).

eg6: If  $\sin \theta = \frac{2}{3}$ , find  $\cos(90^\circ - \theta)$ .

Note:  $\sin \theta =$  \_\_\_\_\_ and,

$\cos \theta =$  \_\_\_\_\_  $\tan \theta =$  \_\_\_\_\_

$\sec \theta =$  \_\_\_\_\_  $\cot \theta =$  \_\_\_\_\_

$\csc \theta =$  \_\_\_\_\_

~~Eg~~ 7: Find the smallest positive angle  $\theta$  such that  $\sin \theta = \cos \theta$ .

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(14, 15 for 'fun').

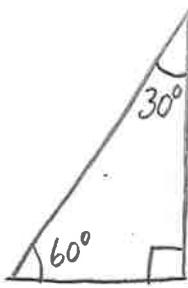
# Ch. 6.3 - Trig. Functions - General and Special Angles

## Special Angles

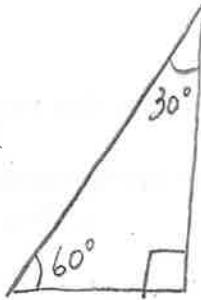
→  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  OR

- derived from special triangles:

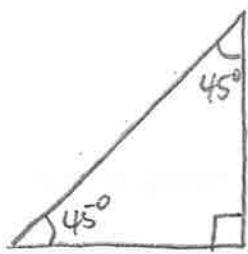
$30^\circ - 60^\circ - 90^\circ \Delta$



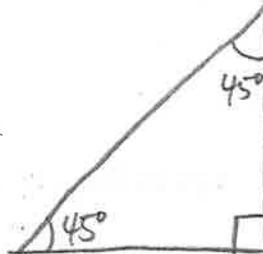
OR



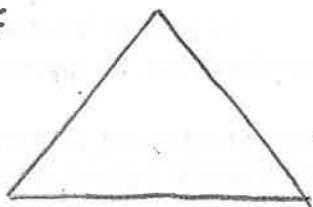
$45^\circ - 45^\circ - 90^\circ \Delta$



OR



Since:

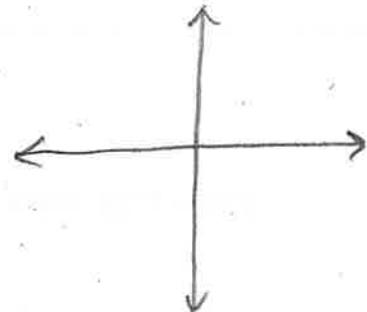
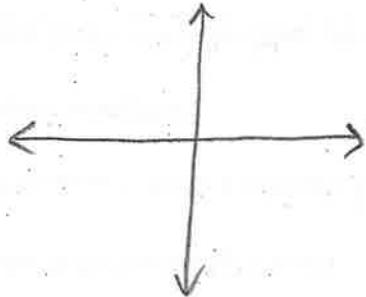
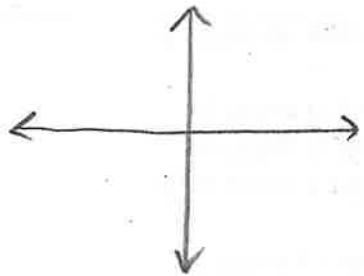


e.g.: Evaluate the following (include a diagram of the angle in standard position):

a)  $\sin 60^\circ$

b)  $\sec \frac{\pi}{4}$

c)  $\tan \frac{\pi}{6}$



Using your diagrams from example 1, fill in the following table:

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
$30^\circ$ or $\frac{\pi}{6}$						
$45^\circ$ or $\frac{\pi}{4}$						
$60^\circ = \frac{\pi}{3}$						

\* notice that all trig. ratios in the table are \_\_\_\_\_ since all three  $\theta$  values are in QI.

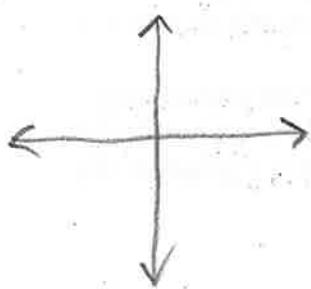
## Reference Angles

Def'n: For angle  $\theta$  in standard position, the REFERENCE ANGLE is the \_\_\_\_\_, \_\_\_\_\_ angle  $\theta'$  that is formed between the terminal side of  $\theta$  and the \_\_\_\_\_.

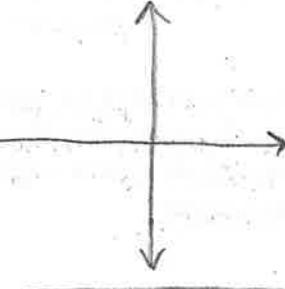
$$\underline{\angle \theta'} \quad \text{or} \quad \underline{\angle \theta'}$$

- a reference angle 'represents'  $\theta$  when  $\theta$  is too large to fit inside a right triangle.

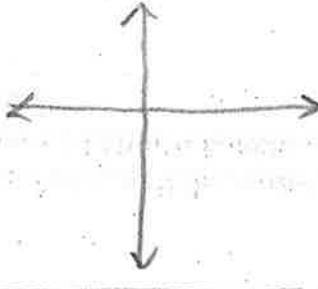
QI:



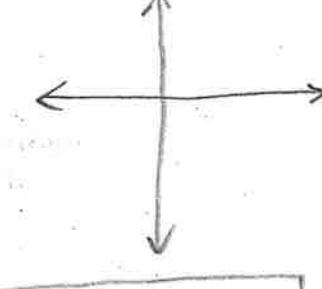
QII:



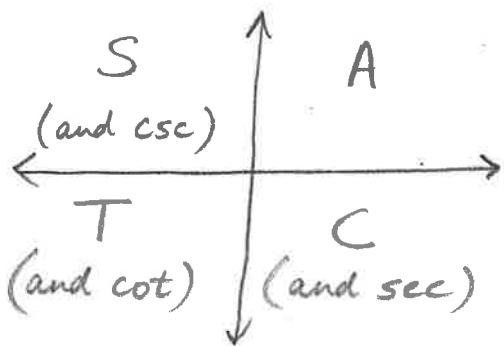
QIII:



QIV:



Also, recall



eg2 Determine the exact value of the following: (diagram required)

a)  $\sin 240^\circ$

b)  $\sec \frac{7\pi}{4}$

c)  $\tan \left(-\frac{19\pi}{6}\right)$

## Quadrantal Angles

→  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$  or \_\_\_\_\_

- requires use of the \_\_\_\_\_ (a circle with a radius = 1).

- also recall:

$$\sin \theta =$$

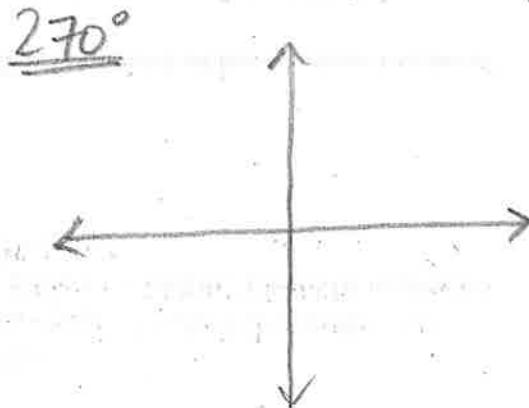
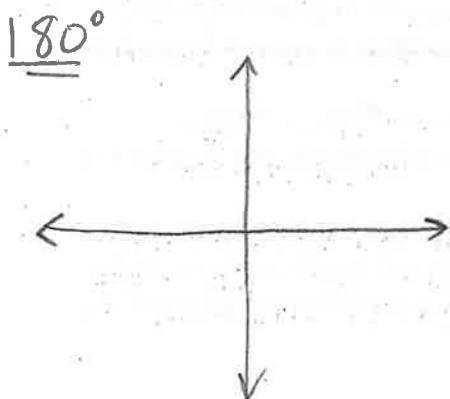
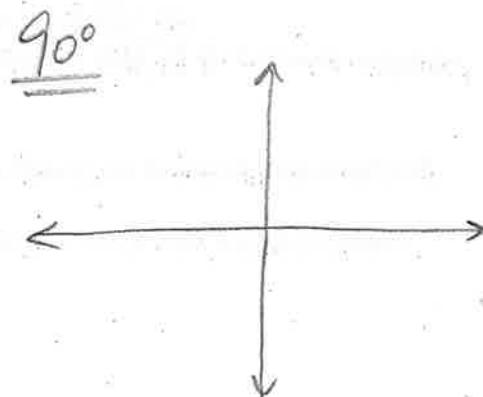
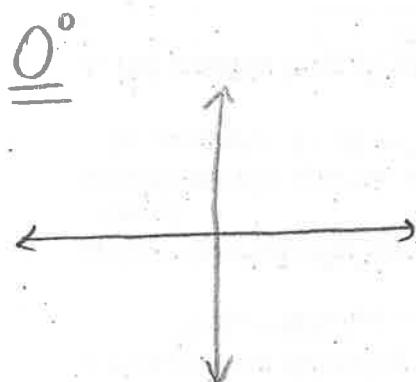
$$\csc \theta =$$

$$\cos \theta =$$

$$\sec \theta =$$

$$\tan \theta =$$

$$\cot \theta =$$



eg3: Using the diagrams on the previous page,  
find:

a)  $\cos 0^\circ$

b)  $\tan 90^\circ$

c)  $\sin \pi$

d)  $\csc \frac{3\pi}{2}$

e)  $\sec(-2\pi)$

Using the same diagrams, fill in the following  
table:

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
$0^\circ$ or $0$						
$90^\circ$ or $\frac{\pi}{2}$						
$180^\circ$ or $\pi$						
$270^\circ$ or $\frac{3\pi}{2}$						

eg4: Evaluate by showing ratio:

a)  $\cos 540^\circ$

b)  $\cot \frac{9\pi}{2}$

Finding  $\theta$

- to find  $\theta$  for special or quadrantal angles,  
reverse the reference angle or unit circle processes.

eg5: Find the smallest positive  $\theta$  (in deg. and rad.) such that:

a)  $\sin \theta = -\frac{\sqrt{3}}{2}$

b)  $\sec \theta = -\sqrt{2}$

c)  $\tan \theta = \text{undefined}$

eg6: Find exactly all  $\theta$ ,  $0^\circ \leq \theta < 360^\circ$ , such that :

a)  $\tan \theta = \frac{1}{\sqrt{3}}$

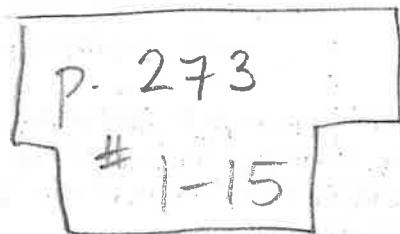
b)  $\cos \theta = -\frac{1}{2}$

eg7: Find exactly all  $x$ ,  $0 \leq x < 2\pi$ , such that:

a)  $\csc x = \frac{-2}{\sqrt{3}}$

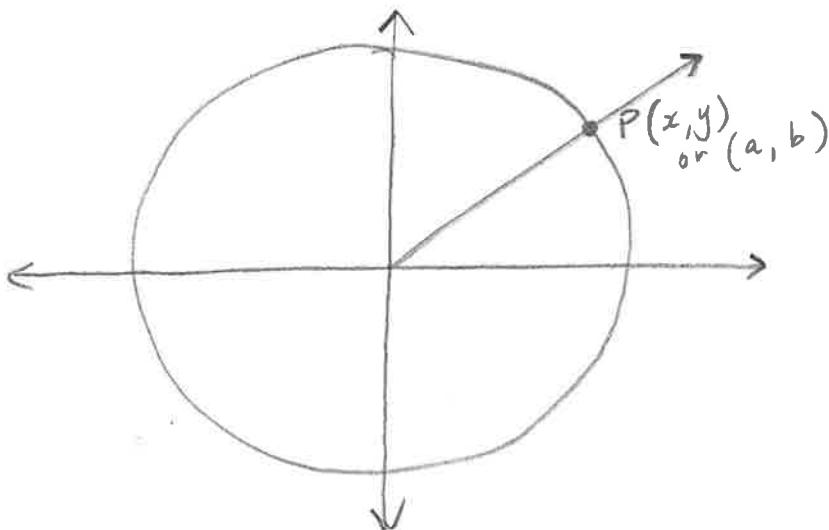
b)  $\sec x = -\sqrt{2}$

\* glance at p. 272.



## Ch. 6.4 - Graphing Basic Trig. Functions

- consider, once again, a UNIT CIRCLE:



As the terminal arm rotates:

$$\sin 0 = b =$$

$$\sin \frac{\pi}{2} = b =$$

$$\sin \pi = b =$$

$$\sin \frac{3\pi}{2} = b =$$

$$\cos 0 = a =$$

$$\cos \frac{\pi}{2} = a =$$

$$\cos \pi = a =$$

$$\cos \frac{3\pi}{2} = a =$$

Consider the functions, then :

$$f(x) = \sin x \quad \text{and} \quad f(x) = \cos x$$

( $y = \sin x$ )    ( $y = \cos x$ )

As $x$ varies from: (ie. as the terminal arm rotates from:)	$y = \sin x$ varies from:	$y = \cos x$ varies from:
$0$ to $\frac{\pi}{2}$		
$\frac{\pi}{2}$ to $\pi$		
$\pi$ to $\frac{3\pi}{2}$		
$\frac{3\pi}{2}$ to $2\pi$		

Also, note that :

$$\sin \frac{\pi}{6} = \underline{\hspace{2cm}}$$

$$\sin \frac{\pi}{4} = \underline{\hspace{2cm}}$$

$$\sin \frac{\pi}{3} = \underline{\hspace{2cm}}$$

again,  $\sin 0 = \underline{\hspace{2cm}}$  and  $\sin \frac{\pi}{2} = \underline{\hspace{2cm}}$

Therefore,  $y = \sin x$  is                 !

Further,

$$\cos \frac{\pi}{6} = \underline{\hspace{2cm}}$$

$$\cos \frac{\pi}{4} = \underline{\hspace{2cm}}$$

$$\cos \frac{\pi}{3} = \underline{\hspace{2cm}}$$

again,  $\cos 0 = \underline{\hspace{2cm}}$  and  $\cos \frac{\pi}{2} = \underline{\hspace{2cm}}$

Therefore,  $y = \cos x$  is                 !

eg1: Graph  $y = \sin x$  ( $0 \leq x \leq 4\pi$ )

\* note: this is the basic sine function.

It can be transformed:

---

eg2: Graph  $y = \cos x$  ( $0 \leq x \leq 4\pi$ )

\* note: this is the basic cosine function.

It can be transformed:

---

- notice that these graphs repeat in successive intervals; they are \_\_\_\_\_ in nature.
  - this, due to the existence of \_\_\_\_\_ angles.

- each repeated interval is called a

What is the period for:  $y = \sin x$ ? \_\_\_\_\_

$y = \cos x$ ? \_\_\_\_\_

In general: A function,  $f$ , is periodic if there exists a value  $c$  such that  $f(x \pm c) = f(x)$  for all  $x$  in  $f$ 's domain.

Notice from graphs on previous page:

$$\sin x =$$

$$\sin x =$$

$$\cos x =$$

## Transformations of $y = \sin x$ and $y = \cos x$

General forms:

$$y = f(x) = \underline{\hspace{10cm}}$$

and

$$y = f(x) = \underline{\hspace{10cm}}$$

}  $a \neq 0$   
 $b > 0$   
 (in Math 12)

\* BEWARE! If given:

you must factor out the  $b$  to find  
the true horizontal shift!

### Terminology:

$$|a| = \underline{\hspace{10cm}}$$

} Let  $M$  represent the  
minimum  $y$ -value and let  
 $m$  represent the maximum.  
Amplitude =

$$\frac{2\pi}{|b|} = \frac{2\pi}{b} = \underline{\hspace{10cm}}$$

(since  $b > 0$ )

Horizontal Translation : relies upon  $c$ ;

(aka Phase Shift) if  $c > 0$ , shift

if  $c < 0$ , shift

Vertical Translation : relies upon  $d$ ;

(aka Vert. Displacement) if  $d > 0$ , shift

[\*also:  $y = d$  is the ] if  $d < 0$ , shift

## Altering the Amplitude

- if 'a' is negative, reflect the function over the

Thus, there are 4 graph themes:

④ sin:



⊖ sin:



⊕ cos:



⊖ cos:



eg1: Graph one period of  $y = 2 \sin x$ .

eg2: Graph one period of  $y = -\frac{1}{2} \cos x$ .

## Altering the Period

- in Math 12,  $b > 0$ .
- remember, the period of sine and cosine functions equals  $\boxed{\phantom{00}}$ .

eg 3: Graph one period of  $y = \sin 2x$ .

eg 4: Graph one period of  $y = \cos \frac{1}{3}x$ .

## Altering the Vertical Displacement

- relies upon  $d$ :

- i) if  $d > 0$ , shift \_\_\_\_\_
- ii) if  $d < 0$ , shift \_\_\_\_\_

eg5: Graph one period of  $y = \sin x + 3$ .

eg6: Graph one period of  $y = \cos x - 1$ .

## Altering the Phase Shift

- relies upon c:

i) if  $c > 0$ , shift       

ii) if  $c < 0$ , shift       

eg7: Graph one period of  $y = \sin(x - \frac{\pi}{2})$ .

eg8: Graph one period of  $y = \cos(x + \frac{2\pi}{3})$ .

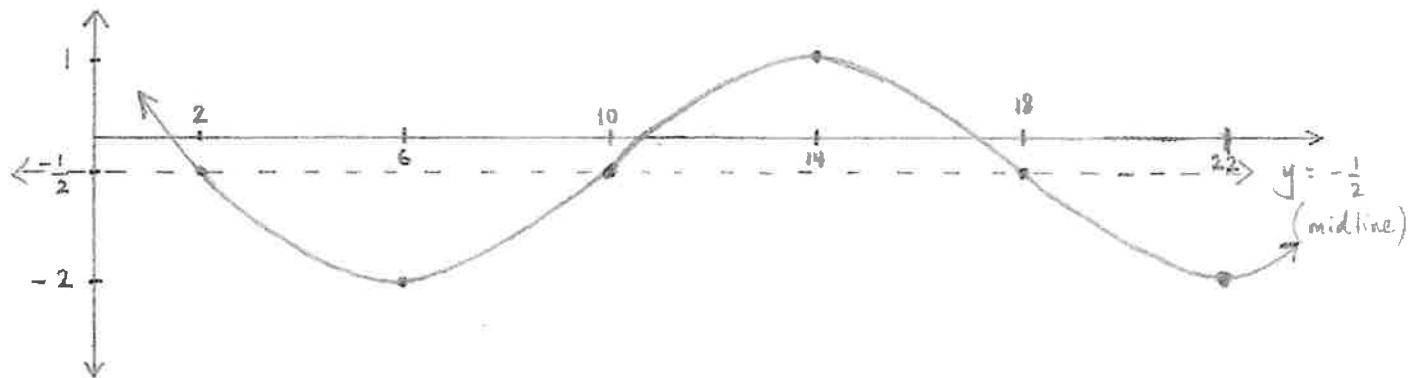
Eg 9: Graph one period of each of the following:

a)  $y = -3 \sin\left(\frac{3}{2}x + \frac{\pi}{2}\right) + 1$

b)  $y = 2 \cos\left(\frac{\pi}{6}(x-2)\right) - 3$

c)  $y = -\cos \frac{\pi}{4}(x+3) + 1$

~~q10:~~ Given the following graph, write an equation in terms of both sine and cosine.



## Graphing $y = \tan x$

Recall,  $y = \tan x$  is undefined when

$x = \underline{\hspace{2cm}}$  and when  $x = \underline{\hspace{2cm}}$   
along with all  $\underline{\hspace{2cm}}$  to these.

These values are depicted on a tan graph  
with  $\underline{\hspace{2cm}}$ .

Also recall,  $y = \tan x = 0$  when

$x = \underline{\hspace{2cm}}$  and when  $x = \underline{\hspace{2cm}}$   
along with all co-terminal angles to these

Of course, these values are depicted on a  
tan graph as  $\underline{\hspace{2cm}}$ .

$y = \tan x$  is the 'basic' tan function:

e.g.: Graph  $y = \tan x$  (graph 3 periods)

note:  $\frac{\pi}{2} =$  Period =

$\tan 1.57 =$  Domain:

$\tan 1.58 =$

Range :

$y = \tan x$  can be transformed:

$y =$  \_\_\_\_\_

Period = [ ] Note: In Math 12, \_\_\_\_\_  
\_\_\_\_\_

$y =$  \_\_\_\_\_

- to calculate the first vertical asymptote, set                         . Add/subtract the                          to find others
- to calculate the first  $x$ -intercept, set                         . Add/subtract the                          to find others

e.g. 2. Graph 3 periods of each:

a)  $y = \tan 2x$

$$b) \quad y = -\tan \frac{1}{2}x$$

$$c) \quad y = \tan \left( 3 \left( x - \frac{\pi}{6} \right) \right)$$

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## Graphing Trig Functions

On a separate sheet of paper, graph one period of each of the following:  
(be sure to label horiz. and vert. axes)

- ①  $y = -\sin \frac{1}{2}x$     ②  $y = \frac{1}{2} \cos 2x$   
③  $y = 2 \sin(2(x - \frac{\pi}{4}))$     ④  $y = -\cos(\frac{1}{2}(x + \frac{2\pi}{3}))$   
⑤  $y = \sin(3x + \frac{3\pi}{4}) + 1$     ⑥  $y = \cos(\frac{\pi}{2}x - \frac{\pi}{2}) - 3$   
⑦  $y = 4 \sin 3(x + \frac{\pi}{6}) - 2$     ⑧  $y = -2 \cos(\frac{1}{3}(x - \pi)) + 2$   
⑨  $y = -3 \sin(2x + \frac{\pi}{4}) - 2$     ⑩  $y = \frac{1}{3} \cos(2x - \frac{\pi}{2}) + 1$   
\* see below!  
⑪  $y = \tan x$     ⑫  $y = -\tan x$   
⑬  $y = \tan 3x$     ⑭  $y = -\tan \frac{1}{2}x$

\* two periods for 11-14

Answers online!

## Ch. 6.5 - Applications of Trig Functions

- a motion that involves a pattern repeating itself over fixed time intervals is called harmonic motion.

↳ eg: pendulums, a Ferris Wheel, the amount of daylight during a year, tide patterns, etc.

Note: Period =

Frequency =

Period =

Frequency =

eg: A car wheel takes  $\frac{1}{4}$  s to turn around once. What is its frequency?

q1: In a vacuum chamber, a weight is attached to a spring and set in motion by stretching the spring and then releasing it. The distance (in cm) that the spring is from its rest position at time  $t$  (in seconds) is given by:  $d = -5 \cos 4\pi t$ .

- How many cycles per second does the spring make?
- Graph 2 periods.

Eg 2: The voltage,  $E$ , of an electrical circuit has an amplitude of 220 volts and a frequency of 60 cycles per second. When  $t = 0$ ,  $E = 450$  volts (max. value).

Write a periodic equation for this situation in terms of both sine and cosine.

q3: The monthly sales ( $S$ ) of a seasonal product are approximated by:

$$S = 480 \cos \frac{\pi}{6}t + 760$$

Graph the function and using the graph, find the months where 1000 units of sales were made. ( $t = \text{time in months} \rightarrow t=1 \text{ (January)}$ )

eg4: A Ferris wheel has a radius of 20m and rotates once every 60s. A rider enters the seat at the lowest point, 3m above the ground, and the clock is started.

Graph the function and write an equation describing the function.

Then, calculate the amount of time the rider spends above 30m.

Let  $y = 30$

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