

Ch. 7.3 - Trigonometric Equations

When solving trig. equations, there are two types of solutions:

- i) Conditional solutions → usually _____;
- ii) General form solutions → often _____

{ - there are _____ general form solutions if a conditional solution exists.

~~e.g.:~~ Solve each of the following in two ways:

- i) $0 \leq x < 2\pi$
- ii) General form

a) $2 \sin x - 1 = 0$

$$b) \cos x + \sqrt{2} = -\cos x$$

$$c) \sqrt{3} \tan x + 1 = 0$$

$$d) \sin x \tan x = 2 \tan x$$

$$e) \sec^2 x - \sec x - 2 = 0$$

$$f) \quad 2\cos^2x + 3\sin x - 3 = 0$$

$$g) \cos x + 1 = \sin x$$

$$h) 4 \tan \frac{x}{2} + 4 = 0$$

$$i) \csc^2 x - 2 \cot x - 4 = 0$$

$$j) \cos^2 x - 3 \cos x - 2 = 0$$

$$k) \quad 6 \sin^2 2x - \sin 2x - 1 = 0$$

$$l) \sin^2 2x - \sin 2x - 2 = 0$$

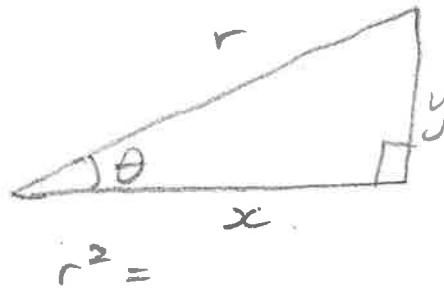
eg2: Solve $2\sin 3\theta + 1 = 0$ i) $0^\circ \leq \theta < 360^\circ$
ii) General form

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1-5 (+try #6)

Ch 7.1 - Trigonometric Identities and Equations

- an EQUATION is true for _____ values of the variable \rightarrow eg: $\sin x - 1 = 0$
- an IDENTITY is true for _____ values of the variable \rightarrow eg: $2x = 3x - x$
- there exist infinitely many trig. identities, however, we will only focus on a number of BASIC identities.

Recall: If θ is an angle in standard position with $P(x, y)$ on θ 's terminal side, then the six trig. ratios are:



$$\sin \theta =$$

$$\csc \theta =$$

$$r^2 =$$

$$\cos \theta =$$

$$\sec \theta =$$

$$\tan \theta =$$

$$\cot \theta =$$

Notice, then, that:

$$\sin \theta \cdot \csc \theta =$$

$$\cos \theta \cdot \sec \theta =$$

$$\tan \theta \cdot \cot \theta =$$



The Reciprocal Identities

$$1. \csc \theta =$$

$$2. \sec \theta =$$

$$3. \cot \theta =$$

* these are all IDENTITIES because they are always true for all allowable (within domain) values of the variable.

Also, notice :

$$\frac{\sin \theta}{\cos \theta} =$$

$$\frac{\cos \theta}{\sin \theta} =$$

The Quotient Identities

$$1. \tan \theta =$$

$$2. \cot \theta =$$

Next: examine $\sin^2 \theta + \cos^2 \theta$

NOTE: $\sin^2 \theta = \underline{\hspace{2cm}}$

i) $\sin^2 \theta + \cos^2 \theta =$

ii) start with $\sin^2 \theta + \cos^2 \theta = 1$, and divide each term by $\cos^2 \theta$:

$$\sin^2 \theta + \cos^2 \theta = 1$$

iii) start with $\sin^2 \theta + \cos^2 \theta = 1$, and divide each term by $\sin^2 \theta$:

$$\sin^2 \theta + \cos^2 \theta = 1$$

The Pythagorean Identities

1. $\sin^2 \theta + \cos^2 \theta = \underline{\hspace{2cm}}$

2. $1 + \tan^2 \theta = \underline{\hspace{2cm}}$

3. $1 + \cot^2 \theta = \underline{\hspace{2cm}}$

Summary: Fundamental Trig. Identities:

①

②

③

④

⑤

⑥

⑦

⑧

Strategies for Simplifying a Trig. Expression

1) Find and convert to a common denominator:

$$\text{eg: } \sin x + \frac{\sin x}{\cos x}$$

2) Factor:

$$\text{eg: } \frac{\sin x \cos x + \sin x}{\cos x}$$

$$\text{eg: } 1 - \sin^2 x$$

3) Change all terms to sine and cosine:

$$\text{eg: } \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$$

$$\text{eg: } \frac{\tan x}{\sec x}$$

4) Conjugate (multiply by the complement):

$$\text{eg: } \frac{1}{1-\cos x}$$

* recall:

$$\frac{\sqrt{2}}{\sqrt{5}-1}$$

q1: Simplify the following:

a) $(\sec^2 x - 1)(\cot^2 x)$ b) $\frac{2 \cos x}{1 - \sin^2 x}$

q2: Simplify the following:

a) $\frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x}$

$$b) \frac{1 + \sin x}{\cos x} - \frac{\cos x}{1 - \sin x}$$

eg3: Simplify the following:

$$a) \frac{\sin x \cos x + \sin x}{\cos x + \cos^2 x}$$

$$b) \frac{\cos x \cot x + \cos x}{\cot x + \cot^2 x}$$

eg 4. Determine the restriction(s) upon x in the expression $\tan x + \csc x$ where $0 \leq x < 2\pi$.

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Ch. 7.2 - Verifying Trigonometric Identities

- the key to proving/verifying identities is to utilize the eight fundamental identities along with basic algebra rules to rewrite trig. expressions.

Def'n's:

EXPRESSION - has no _____ sign. It is merely the sum/difference/product/quotient of functions.

EQUATION - a statement that is true for a set of specific values (ie. a _____ statement).

IDENTITY - an equation that is true for _____ real values.

- NOTE: proving an identity is quite different from solving an equation.

Helpful Rules for Proving / Verifying Identities

- no particular order
↓
there may be
≥ 1 way
to proving an identity
- { ① Change all trig. values to sin and/or cos;
② Write an expression with a common denominator;
③ Remember the conjugate step and how to factor;
④ Work with one side of an equation at a time, starting with the more 'complicated' side.

* Proven when ① column statement = ② column statement

eg1: Prove the identity : $\frac{\csc^2 \theta - 1}{\csc^2 \theta} = \cos^2 \theta$

eg2: Prove the identity: $\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = 2\csc^2\alpha$

eg3: Prove the identity: $\tan x + \cot x = \sec x \csc x$

eg4: Prove the identity: $\csc x + \cot x = \frac{\sin x}{1 - \cos x}$

eg5: Prove the identity: $\frac{\tan^2 x}{1 + \sec x} = \frac{1 - \cos x}{\cos x}$

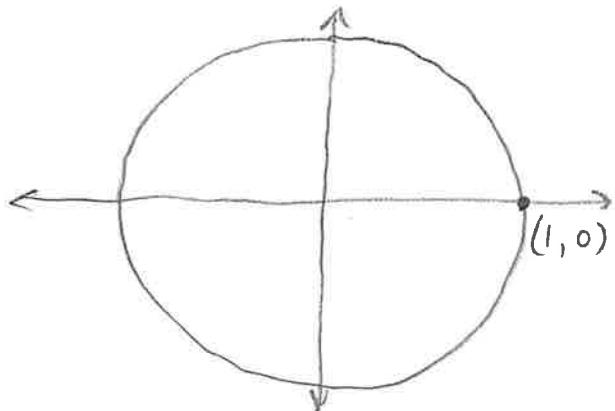
p. 311 # 1-26
typo: p. 314

* 23: $\tan x (\csc x + 1)$

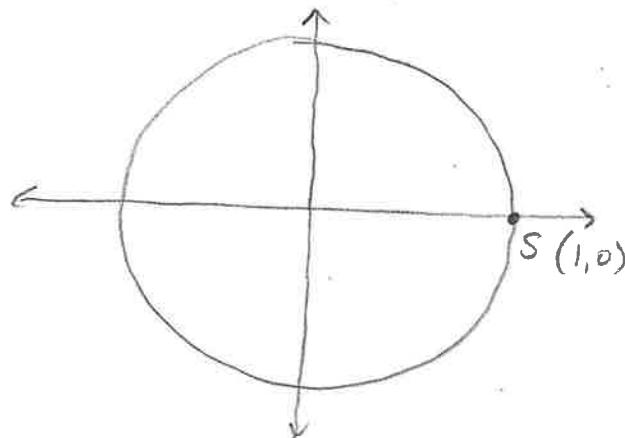
Ch. 7.4 - Sum and Difference Identities

Start with: $\cos(A-B) = \cos A \cos B + \sin A \sin B$

Proof: Unit Circle



Re-draw with $\angle(A-B)$
in standard position



Recall: DISTANCE FORMULA:

$$d =$$

Recall: $\cos(-x) = \underline{\hspace{2cm}}$ and $\sin(-x) = \underline{\hspace{2cm}}$

so, if $\cos(A-B) = \cos A \cos B + \sin A \sin B$,
then $\cos(A-(-B)) =$

Recall: $\sin x = \underline{\hspace{2cm}}$ and $\cos x = \underline{\hspace{2cm}}$

so, $\sin(A+B) =$

$\tan(A+B) =$

again, if $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$,

then:

$$\tan(-x) =$$

Sum and Difference Identities Summarized:

$$\sin(A+B) =$$

$$\sin(A-B) =$$

$$\cos(A+B) =$$

$$\cos(A-B) =$$

$$\tan(A+B) =$$

$$\tan(A-B) =$$

Even-Odd and Cofunction Identities Summarized:

$$\sin(-A) = \underline{\hspace{2cm}}$$

$$\cos(-A) = \underline{\hspace{2cm}}$$

$$\tan(-A) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{\pi}{2} - A\right) = \underline{\hspace{2cm}}$$

$$\cos\left(\frac{\pi}{2} - A\right) = \underline{\hspace{2cm}}$$

$$\csc\left(\frac{\pi}{2} - A\right) = \underline{\hspace{2cm}}$$

$$\sec\left(\frac{\pi}{2} - A\right) = \underline{\hspace{2cm}}$$

$$\tan\left(\frac{\pi}{2} - A\right) = \underline{\hspace{2cm}} \Rightarrow$$

$$\cot\left(\frac{\pi}{2} - A\right) = \underline{\hspace{2cm}}$$

- the sum and difference identities, along with the even-odd and cofunction identities, can be used to solve a wide variety of trig. problems:

e.g.: Find the exact value of $\cos 105^\circ$.

eg2: Simplify
$$\frac{\tan \frac{2\pi}{5} - \tan \frac{3\pi}{20}}{1 + \tan \frac{2\pi}{5} \tan \frac{3\pi}{20}}$$

eg3: Given $\sin A = -\frac{3}{5}$, with A in QIII, and
 $\cos B = \frac{5}{13}$, with B in QIV, find $\sin(A+B)$.

eg4: Solve: $\sin(x + \frac{\pi}{4}) + \sin(x - \frac{\pi}{4}) = -1$ ($0 \leq x < 2\pi$)

eg5: Prove the identity:
 $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

q6: Find the general form of the solution to :

$$2 \tan x + \tan(\pi - x) = \sqrt{3}$$

q7: Determine the amplitude, period, and phase shift of:

$$f(x) = 3\sqrt{2} \sin 2x \cos \frac{\pi}{4} + 3\sqrt{2} \cos 2x \sin \frac{\pi}{4}$$

eg 8: Simplify:

$$\csc(90^\circ - \theta) \sec(360^\circ - \theta) - \tan(720^\circ + \theta) \cot(450^\circ - \theta)$$

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1-9 omit 5b

Ch. 7.5 - Double-Angle Identities

- the sum and difference identities from Ch. 6.4 may be used to generate even more trig. identities.

Start with: $\sin 2A = 2 \sin A \cos A$

the double-angle formula for SINE

Derivation: $\sin(A+B) =$

Similarly: $\cos(A+B) =$

Again, similarly:

$$\tan(A + B) =$$

Summary: Double-Angle Identities.

$$\sin 2A =$$

$$\cos 2A =$$

$$\tan 2A =$$

e.g.: Solve: $\cos 2x = 2\sin^2 x \quad (0 \leq x < 2\pi)$

eg2: Use the double-angle formulas to simplify:

a) $12 \sin 4x \cos 4x$

b) $4 - 8 \cos^2 6x$

c) $\frac{4 \tan 3x}{1 - \tan^2 3x}$

eg3: Prove the identity: $\frac{\sin 6x}{1 + \cos 6x} = \tan 3x$

eg4: Given $\sin x = -\frac{12}{13}$ in QIII, find $\tan 2x$.

Power - Reducing Identities

- these are the double-angle identities written in a different way:

$$\cos 2x = 1 - 2 \sin^2 x \quad (\text{solve for } \sin^2 x)$$

$$\cos 2x = 2 \cos^2 x - 1 \quad (\text{solve for } \cos^2 x)$$

$$\tan^2 x =$$

Summary:

$$\sin^2 A =$$

$$\cos^2 A =$$

$$\tan^2 A =$$

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1-15 (9, 10, 11)

+ Ch. Rev. for fun

* typo:
p 343 * 5b
cos 2x

(15-16 require
graphing calc.)