Ch. 1.1: Number Systems / Sets

Natural Numbers : symbol → $\boxed{\mathbb{N}_1}$

$\{1, 2, 3, 4, \ldots\}$

Whole Numbers : symbol → $\boxed{\mathbb{N}_0}$

$\{0, 1, 2, 3, 4, \ldots\}$

Integers : symbol → $\boxed{\mathbb{Z}}$

$\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers : symbol → $\boxed{\mathbb{Q}}$

- any number that can be written as a FRACTION with the denominator not equal to zero.
- any terminating or repeating (in pattern) decimal may be expressed as a fraction.

examples of Rational numbers:

3 , -3 , 0 , $\frac{2}{3}$, $-\frac{4}{5}$,

2.3 , $-5.\overline{2}$, $\sqrt{4}$, $-\sqrt{81}$, $6.13\overline{45}$

Explanation for each:

$3 = \frac{3}{1}$, $-3 = \frac{-3}{1}$, $0 = \frac{0}{1}$,

$2.3 = 2\frac{3}{10} = \frac{23}{10}$, $-5.\overline{2} = -5\frac{2}{9} = \frac{-47}{9}$,

$\sqrt{4} = 2 = \frac{2}{1}$, $-\sqrt{81} = -9 = \frac{-9}{1}$,

$6.13\overline{45} = 6 + \frac{13}{100} + \frac{45}{9900} = 6\frac{37}{275}$

Irrational Numbers: symbol $\rightarrow$ $\overline{\mathbb{Q}}$

- any number that cannot be written as a fraction.

  - thus, any non-terminating and non-repeating decimal is irrational.

eg: $1.62974\ldots$, π

Also, the square root of any non-perfect square is irrational.

eg: $\boxed{\sqrt{2}, \sqrt{3}, \sqrt{5}, \text{etc...}}$

<u>Real Numbers:</u> symbol → <u>$\mathbb{R}$</u>

- any number that is either <u>RATIONAL</u> or <u>IRRATIONAL</u>

<u>Exceptions</u>?

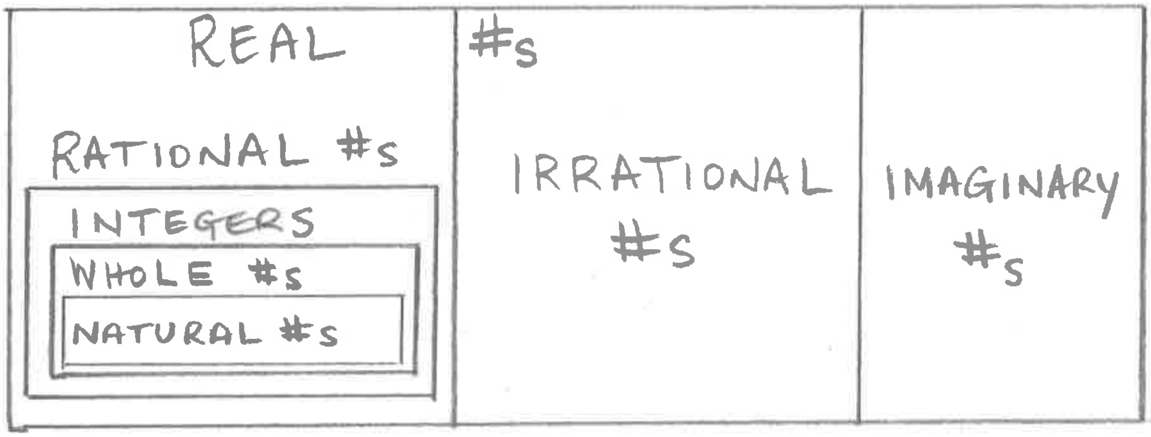
<u>i</u> - the <u>IMAGINARY</u> number

$$\sqrt{-1} = \boxed{i}$$

$$\sqrt{-9} = \boxed{\sqrt{-1} \cdot \sqrt{9} = 3i}$$

<u>∞</u> - <u>INFINITY</u> - has no definitive value.

$\boxed{\dfrac{\text{any number}}{0}}$ - <u>UNDEFINED</u> in $\mathbb{R}$.


REAL #s
RATIONAL #s
INTEGERS
WHOLE #s
NATURAL #s
IRRATIONAL #s
IMAGINARY #s

p. 6-7

1-3

Ch.1.2 - Square Roots

$$2 \times 2 = 2^2 \text{ ("2 squared")} = 4$$

What if you were asked to find a number that, when squared, equals 9?

- you would attempt to find $\sqrt{9}$ ("the square root of 9").

$$\boxed{\sqrt{9} = \sqrt{3^2} = \sqrt{3 \cdot 3} = 3}$$

Thus, square rooting 'undoes' squaring.

$$4 \boxed{\xrightarrow{\text{SQUARED}} 4^2 = 16 \xrightarrow[\text{ROOTED}]{\text{SQUARE}} \sqrt{16} = \sqrt{4^2} = 4}$$

The symbol $\boxed{\sqrt{}}$ is called a RADICAL sign and is used to indicate the square root of a number.

- the value 'inside' $\sqrt{}$ is called the RADICAND.

- a knowledge of perfect square natural numbers is helpful.

1, 4, 9, 16, 25, 36,
49, 64, 81, 100, 121, 144,
169, 196, 225.

Also: 400, 625

Furthermore, there exist perfect square rational numbers. The square root of such numbers result in rational numbers as well.

eg: $\sqrt{\frac{4}{9}} = \boxed{\frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}}$

eg1: A square has an area of 9 cm^2. What must its side length be?

$\text{Area} = lw = l \cdot l = l^2$

$9 = l^2$

$l = \sqrt{9} = \boxed{3 \text{ cm}}$

eg2: Determine the exact square root of each of the following, if possible:

a) $\sqrt{25} = \sqrt{5^2} = \sqrt{5 \cdot 5} = 5$

b) $-\sqrt{16} = -\sqrt{4^2} = -\sqrt{4 \cdot 4} = -4$

c) $\sqrt{-4} =$ no solution

* no two IDENTICAL real numbers can be multiplied to get -4.

d) $\sqrt{0} = 0$

e) $\sqrt{\frac{49}{81}} = \frac{\sqrt{49}}{\sqrt{81}} = \frac{7}{9}$

f) $\sqrt{\frac{2}{18}} = \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3}$

g) $\sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4} \leftarrow$ IRRATIONAL

h) $\sqrt{9+16} = \sqrt{25} = 5$

i) $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$

j) $\sqrt{0.64} = \sqrt{\frac{64}{100}} = \frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10} = \frac{4}{5}$

p. 10–12 # 1–8, 10, 12, 13

Ch. 1.3 – Square Roots of Non-Perfect Squares

Recall: - square roots of PERFECT SQUARES are rational numbers.

- square roots of NON-PERFECT SQUARES are irrational numbers

for instance, $\sqrt{5}$ is an irrational number because no two equal rational numbers can be multiplied to equal 5.

- thus, a calculator can be utilized to create a decimal approximation.

eg: $\sqrt{5} = \boxed{2.236067977 \doteq 2.24}$

nearest hundredth.

- this makes sense since 5 is between the perfect squares 4 and 9, but is closer to 4.

$\sqrt{4} = 2 \qquad \sqrt{5} \doteq 2.24 \qquad \sqrt{9} = 3$

Approximating Square Roots (without calc.)

- relies upon knowledge of perfect squares.

eg 1: Find $\sqrt{11}$ to the nearest tenth without a calculator.

$9 < 11 < 16$

$\sqrt{9} < \sqrt{11} < \sqrt{16}$

$3 < \sqrt{11} < 4$

* compare to closest perfect squares

→ 11 closer to 9 than 16.

$\sqrt{11} = 3.3$

Check: $(3.3)^2 = 10.89$

eg 2: Find $\sqrt{110}$ to nearest hundredth without a calculator.

$100 < 110 < 121$

$\sqrt{100} < \sqrt{110} < \sqrt{121}$

$10 < \sqrt{110} < 11$

$\sqrt{110} = 10.5 \longrightarrow (10.5)^2 = 110.25$

10.48? $(10.48)^2 = 109.83$

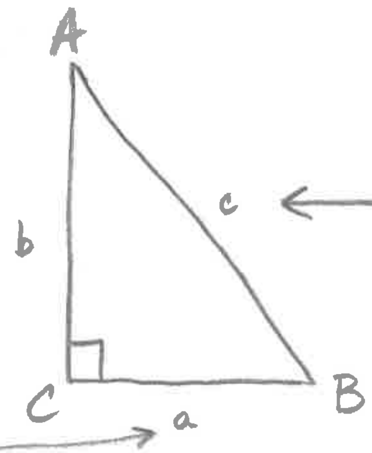
10.49? $(10.49)^2 = 110.04$ ✓

$\sqrt{110} = \boxed{10.49}$

The Pythagorean Theorem

- describes the relationship between the sides of any **RIGHT** triangle.

Given ΔABC :



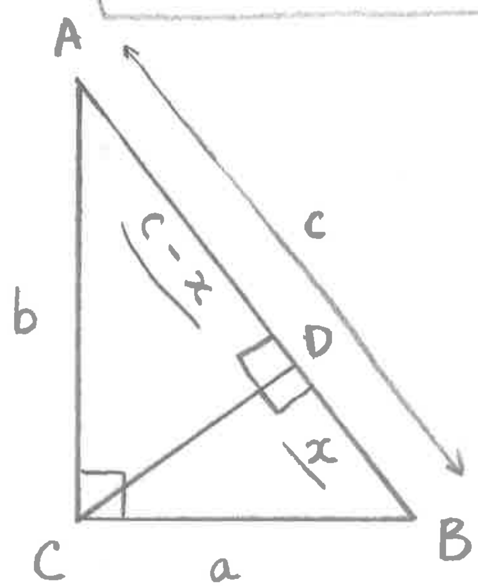
LEGS
- shorter than c
- interchangeable

HYPOTENUSE
- longest side
- opposite the 90° angle.

the Pythagorean Theorem states :

$$\boxed{a^2 + b^2 = c^2}$$

PROOF :



ΔABC , ΔACD, ΔCBD are SIMILAR Δs.

Thus, all corresponding side lengths are proportional.

$$\frac{c}{a} = \frac{a}{x} \rightarrow a^2 = cx$$

$$\frac{c}{b} = \frac{b}{c-x} \rightarrow b^2 = c(c-x)$$

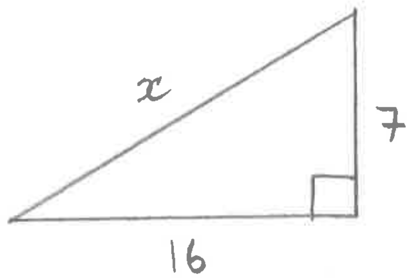
$$\rightarrow b^2 = c^2 - cx$$

$$a^2 + b^2 = cx + c^2 - cx$$

$$* \; a^2 + b^2 = c^2$$

eg 1: Solve for x to the nearest tenth, in each of the following:

a)



$a^2 + b^2 = c^2$

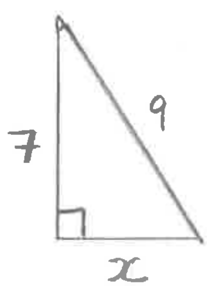
$7^2 + 16^2 = x^2$

$49 + 256 = x^2$

$305 = x^2$

$\boxed{x = 17.5}$

b)



$a^2 + b^2 = c^2$

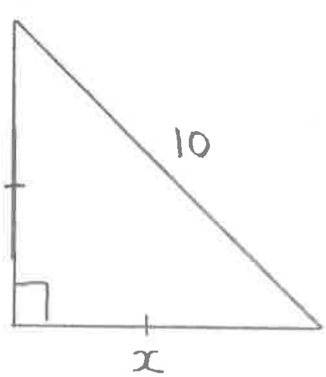
$x^2 + 7^2 = 9^2$

$x^2 + 49 = 81$

$x^2 = 32$

$\boxed{x = 5.7}$

c)



$a^2 + b^2 = c^2$

$x^2 + x^2 = 10^2$

$2x^2 = 100$

$x^2 = 50$

$\boxed{x = 7.1}$

p. 17-20 # 1-4, 6-11

* take careful note of <u>Pythagorean Triples</u> in hwk.

Ch. 1.4 – Defining a Power

Scenario: In a five-question T/F quiz, how many different answer sequences are possible?

$$\boxed{\underset{Q1}{2} \times \underset{Q2}{2} \times \underset{Q3}{2} \times \underset{Q4}{2} \times \underset{Q5}{2} = 32}$$

ie. multiplying a number by itself multiple times.

- powers / exponents can help!

$$\boxed{2 \times 2 \times 2 \times 2 \times 2 = 2^5 \quad (\text{"2 to the 5th power"})}$$

2^5 ← EXPONENT (POWER)

BASE

$$a \cdot a \cdot a \cdot \ldots (n \text{ times}) = \boxed{a^n}$$

Note: $a = \underline{a^1}$

The Product of Negative Numbers

- very important to know the <u>base</u>.

$(-x)^n \longrightarrow$ base is $\underline{-x}$

$-x^n \longrightarrow$ base is $\underline{x}$

BEDMAS says that 'E' (exponents) comes before 'M' (multiplication).

$(-2)^4 \rightarrow$ multiply 2 by -1 first, then raise to 4^{th} power.

$-2^4 \rightarrow$ raise 2 to 4^{th} power first, then multiply by -1.

- also, a negative number multiplied by itself an EVEN number of times results in a <u>POSITIVE</u> number.

  - whereas, a negative number multiplied by itself an ODD number of times results in a <u>NEGATIVE</u> number.

eg: $(-2)\cdot(-2)\cdot(-2)\cdot(-2) = \underline{(-2)^4 = 16}$

$(-2)\cdot(-2)\cdot(-2) = \underline{(-2)^3 = -8}$

eg1: Determine if the following, when simplified, would be $\oplus$ or $\ominus$:

a) $(-1)^2$ $\underline{\oplus}$

d) -2^4 $\underline{\ominus}$

b) $(-3)^7$ $\underline{\ominus}$

e) $-(-5)^2$ $\underline{\ominus}$

c) 4^9 $\underline{\oplus}$

f) $-(-1)^9$ $\underline{\oplus}$

SUMMARY: Given $x > 0$,

$$(-x)^{\text{even}} = \oplus$$

$$(-x)^{\text{odd}} = \ominus$$

$$-x^{\text{odd or even}} = \ominus$$

One and Zero as Exponents

$$2 \times 2 \times 2 \times 2 = 2^4 = 16$$

$$2 \times 2 \times 2 = 2^3 = 8$$

$$2 \times 2 = 2^2 = 4$$

$$2 = 2^1 = 2$$

$$1 = 2^0 = 1$$

$$\frac{1}{2} = 2^{-1} = \frac{1}{2}$$

so, $a^1 = a$, for any number a.

$a^0 = 1$, for any non-zero number a.

Note: 0^0 is <u>UNDEFINED</u>.

eg 2: Evaluate:

a) $5^0 = 1$

b) $3^1 = 3$

c) $\left(\frac{1}{4}\right)^0 = 1$

d) $(-5)^0 = 1$

e) $-5^0 = -1$

f) $0^0 =$ undefined

g) $(a+b)^0 = 1$

h) $a + b^0 = a + 1$

i) $a^0 + b^0 = 1 + 1 = 2$

j) $(a \cdot b)^0 = 1$

k) $a \cdot b^0 = a \cdot 1 = a$

* Note the following:

i) $\left(\frac{x}{y}\right)^n = \boxed{\frac{x^n}{y^n}}$

ii) $-\frac{x}{y} = \boxed{\frac{-x}{y}} = \boxed{\frac{x}{-y}}$

p. 23-26 #1-12

Ch. 1.5 – Order of Operations

Formal Rules:

1. Perform all calculations within brackets FIRST! If more than one set of brackets are 'layered' within one another, perform calculations from the innermost brackets to the outermost.

2. Evaluate all exponential expressions

3. Perform all multiplication and division IN ORDER from L to R.

4. Perform all addition and subtraction IN ORDER from L to R.

B RACKETS

E XPONENTS

D IVISION

M ULTIPLICATION

A DDITION

S UBTRACTION

eg 1: Simplify each of the following:

a) $5 + 4 \times 3 = \boxed{5 + 12 = \boxed{17}}$

b) $6 - (2+3)^2 = 6 - (5)^2$

$= 6 - 25 = \boxed{-19}$

c) $(3 - 2 \times 4)^2 - \left(3 + \frac{6^2}{2}\right)$

$= (3-8)^2 - \left(3 + \frac{36}{2}\right)$

$= (-5)^2 - (3 + 18)$

$= 25 - 21 = \boxed{4}$

d) $\dfrac{42 - 18}{2} = \dfrac{24}{2} = \boxed{12}$

OR $= \dfrac{42}{2} - \dfrac{18}{2} = 21 - 9$

$= \boxed{12}$

p. 28 - 30 # 1-9

Ch. 1.6 - Exponent Laws

Multiplying with Exponents

eg: $2^3 \cdot 2^4 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2)$

$= 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$

$= 2^7$

Check: $2^3 \cdot 2^4 = 2^7$

$8 \cdot 16 = 128$ ✓

Note: $3 + 4 = 7$

(the exponents of an identical base)

The Product Rule

If x is a REAL number and, m and n are INTEGERS, then:

$$x^m \cdot x^n = \underline{x^{m+n}}$$

Note: $x \neq \underline{0}$

Why? If $m \leq 0$ or $n \leq 0$, then x^m or x^n is UNDEFINED.

eg1: Simplify (but do not evaluate):

a) $3^5 \cdot 3^4 = 3^{5+4} = \boxed{3^9}$

b) $(2^2)(2^4)(2^3) = 2^{2+4+3} = \boxed{2^9}$

c) $2^5 \cdot 2^3 \cdot 3^2 = (2^{5+3})(3^2) = \boxed{2^8 \cdot 3^2}$

Common Errors while using Product Rule:

i) $2^2 \cdot 2^3 \neq 4^{2+3} = 4^5$

ii) $2^2 \cdot 2^3 \neq 2^{2 \cdot 3} = 2^6$

iii) $2^2 \cdot 2^3 \neq 4^{2 \cdot 3} = 4^6$

Dividing with Exponents

eg: $2^5 \div 2^2 = \dfrac{2^5}{2^2} = \dfrac{(\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2)}{(\cancel{2} \cdot \cancel{2})}$

$= \dfrac{2 \cdot 2 \cdot 2}{1} = 2^3$

Check: $2^5 \div 2^2 = 2^3$

$32 \div 4 = 8$ ✓

Note: $5 - 2 = 3$

The Quotient Rule

If x is a REAL number, and m and n are INTEGERS, then:

$$x^m \div x^n = \boxed{\frac{x^m}{x^n} = x^{m-n}}$$

Note: $x \neq \underline{0}$

eg2: Simplify (but do NOT evaluate):

a) $5^8 \div 5^4 = \frac{5^8}{5^4} = 5^{8-4} = \boxed{5^4}$

b) $\frac{2^7 \cdot 3^2}{2^4} = \frac{2^7}{2^4} \cdot 3^2 = 2^{7-4} \cdot 3^2$

$= \boxed{2^3 \cdot 3^2}$

c) $\frac{3^4 \cdot 3^5}{3^6} = \frac{3^{4+5}}{3^6} = \frac{3^9}{3^6} = 3^{9-6}$

$= \boxed{3^3}$

OR $= 3^{4+5-6}$

$= \boxed{3^3}$

Common Errors while using Quotient Rule:

i) $\dfrac{3^8}{3^2} \neq 1^{8-2} = 1^6$

ii) $\dfrac{3^8}{3^2} \neq 3^{8\div 2} = 3^4$

iii) $\dfrac{3^8}{3^2} \neq 1^{8\div 2} = 1^4$

Summary:

a) When multiplying, if the bases are the same, keep the base and <u>ADD</u> the exponents.

b) When dividing, if the bases are the same, keep the base and <u>SUBTRACT</u> the exponents.

Zero Exponent Proof → Why does $x^0 = 1$?

$$\boxed{\begin{aligned} \frac{x^3}{x^3} &= x^{3-3} = x^0 \\ \frac{x^3}{x^3} &= 1 \end{aligned}}$$

eg3: Simplify (do <u>not</u> evaluate):

a) $(-2)^5 \cdot 2^3$ $= -2^5 \cdot 2^3$

$= -1\left(2^{5+3}\right)$

$= \boxed{-2^8}$

b) $\dfrac{(-3)^8}{3^4} = \dfrac{3^8}{3^4} = 3^{8-4} = \boxed{3^4}$

c) $2^7 - 2^2 \cdot 2^3 = 2^7 - \left(2^{2+3}\right)$

$= \boxed{2^7 - 2^5}$

d) $3 \cdot 3^4 + \dfrac{3^5}{3^3} = 3^{1+4} + 3^{5-3}$

$= \boxed{3^5 + 3^2}$

p. 34-37 #1-9

Ch. 1.7 – Power Rules

eg: $(5^3)^2$

$= (5^3)(5^3) = 5^{3+3} = 5^6$

* NOTICE : $3 \cdot 2 = 6$

The Power Rule

For any REAL number x, and any INTEGERS m and n:

$$(x^m)^n = x^{m \cdot n} = x^{mn} \quad (x \neq 0)$$

eg1: Simplify (but do NOT evaluate):

a) $(2^4)^2 = 2^{4 \cdot 2} = \boxed{2^8}$

b) $(3^3)^5 = 3^{3 \cdot 5} = \boxed{3^{15}}$

c) $(x^2)^3 = x^{2 \cdot 3} = \boxed{x^6}$

Raising a Product to a Power

eg: $(3 \cdot 4)^2 = (3 \cdot 4) \cdot (3 \cdot 4)$

$= 3 \cdot 3 \cdot 4 \cdot 4$

$= 3^2 \cdot 4^2$

Check:

$(3 \cdot 4)^2 = 3^2 \cdot 4^2$

$(12)^2 = 9 \cdot 16$

$144 = 144$ ✓

Rule:

For any REAL number x and y, and any INTEGER n:

$$(xy)^n = \boxed{x^n y^n \quad \begin{matrix}(x \neq 0)\\(y \neq 0)\end{matrix}}$$

eg 2: Simplify, then evaluate (where applicable):

a) $(2 \cdot 3)^4 = 2^4 \cdot 3^4$

$= 16 \cdot 81 = \boxed{1296}$

b) $(a^2 b^3)^2 = (a^2)^2 \cdot (b^3)^2$

$= a^{2 \cdot 2} \cdot b^{3 \cdot 2}$

$\boxed{= a^4 b^6}$

Raising a Quotient to a Power

eg: $\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$

$= \left(\frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3}\right)$

$= \boxed{\frac{2^3}{3^3}}$

Check:

$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$

$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$

$\frac{8}{27} = \frac{8}{27}$ ✓

Rule:

For any REAL number x and y, and any INTEGER n:

$$\left(\frac{x}{y}\right)^n = \boxed{\frac{x^n}{y^n} \quad \begin{pmatrix} x \neq 0 \\ y \neq 0 \end{pmatrix}}$$

eg3: Simplify, and evaluate (when applicable):

a) $\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \boxed{\frac{9}{25}}$

b) $\left(\frac{a^3}{b^2}\right)^4 = \frac{(a^3)^4}{(b^2)^4} = \frac{a^{3\cdot 4}}{b^{2\cdot 4}} = \boxed{\frac{a^{12}}{b^8}}$

Common Errors:

i) $(x+y)^n \neq x^n + y^n$

eg: $(2+3)^2 \neq 2^2 + 3^2$

$5^2 \neq 4 + 9$

$25 \neq 13$

ii) $x^m \cdot x^n \neq x^{mn}$

but

$(x^m)^n = x^{mn}$

ie. students often confuse the Product Rule and the Power Rule.

p. 40-43 #1-9, 10-11 for 'fun' + Ch. Review