Ch. 6.5 Word Problem Solutions

(1) Let x = 1st odd #
then $x+2$ = next odd #

$$x^2 + (x+2)^2 = 130$$
$$2x^2 + 4x + 4 = 130$$
$$2x^2 + 4x - 126 = 0$$
$$x^2 + 2x - 63 = 0$$
$$(x+9)(x-7) = 0$$
$$x = -9, 7$$
$$x+2 = -7, 9$$

Two sets of solutions:
-9 and -7 | 7 and 9

(2) Let x = the number
then $\frac{1}{x}$ = the reciprocal

$$x + 3\left(\frac{1}{x}\right) = 4$$
$$x + \frac{3}{x} = 4 \quad \text{(multiply every term by } x\text{)}$$
$$x^2 + 3 = 4x$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$

$x = 3$ and 1

(3) $4 \times 8 = 32\ ft^2$ (area)

Let x = amt. added to l & w

$(4+x)(8+x) = 2 \cdot 32$

$x^2 + 12x + 32 = 64$

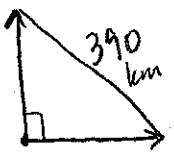
$x^2 + 12x - 32 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-12 \pm \sqrt{(12)^2-4(1)(-32)}}{2(1)} = \frac{-12 \pm \sqrt{272}}{2}$

$= \frac{-12 \pm 16.5}{2}$

$= 2.25\ ft$ and ~~$-14.25\ ft$~~

(4)



Let x = speed of slower plane

then $x+210$ = speed of faster plane

time = 1 hour

$d = st$

d (slower plane) $= x \cdot 1 = x$

d (faster plane) $= (x+210) \cdot 1 = x+210$

Using Pythagorean Theorem:

$x^2 + (x+210)^2 = 390^2$

$2x^2 + 420x + 44100 = 152100$

$2x^2 + 420x - 108000 = 0$

$x^2 + 210x - 54000 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-210 \pm \sqrt{(210)^2-4(1)(-54000)}}{2(1)}$

$= \frac{-210 \pm 510}{2} = 150, -360$

speed of slower plane $= 150$ km/h

(5) Let x = speed of current

so... speed up-river = $15 - x$

speed down-river = $15 + x$

$d = st \Rightarrow t = \frac{d}{s}$

time up-river: $\frac{36}{15-x}$ − time down-river: $\frac{36}{15+x} = 1$

$36(15+x) - 36(15-x) = 1(15-x)(15+x)$

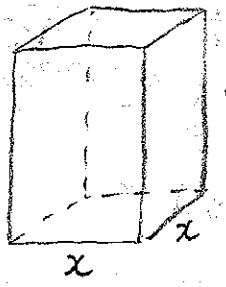
$36x + 540 + 36x - 540 = -x^2 + 225$

$x^2 + 72x - 225 = 0$

$(x+75)(x-3) = 0$

$x = -75$, $\boxed{3}$ $\boxed{\text{current speed} = 3 \text{ km/h}}$

(6)



Let x = length & width of base

$SA = 2x^2 + 4(9x)$

$350 = 2x^2 + 36x$

$0 = 2x^2 + 36x - 350$

$0 = x^2 + 18x - 175$

$0 = (x+25)(x-7)$

$x = -25$, $\boxed{7}$ $\boxed{\text{length} = 7 \text{ cm}}$

(7.) Find John's rate: Weeded garden = (speed)(time)

$1 = s(4)$

$s = \frac{1}{4}$ garden/hr.

Find Mike's rate: $1 = s(6)$

$s = \frac{1}{6}$ garden/hr.

Together, their rates are ADDITIVE: $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

So, together, in one hour, they can weed $5/12$ of the garden.

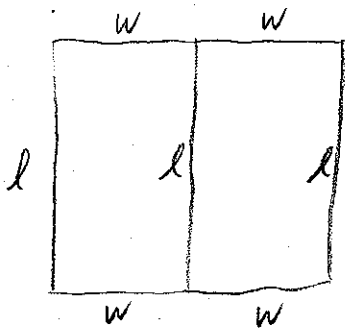
Find t: $1 = \frac{5}{12}t$

$1 = \frac{5t}{12}$

$12 = 5t$

$t = \frac{12}{5} = 2.4$ hrs = 2 hrs 24 mins

(8)



Let l = length and let w = width.

① $3l + 4w = 200$

② $l \cdot 2w = 1400$

MUST solve by SUBSTITUTION.

$3l = 200 - 4w$

$l = \frac{200 - 4w}{3}$ SUBSTITUTE into ②

$\left(\frac{200 - 4w}{3}\right) \cdot 2w = 1400$

$\frac{400w - 8w^2}{3} = 1400$

$0 = 8w^2 - 400w + 4200$

$0 = w^2 - 50w + 525$

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \frac{50 \pm 20}{2} = 35, 15$

over page →

$2w = 70$ and 30

$$l = \frac{200 - 4w}{3} = \frac{200 - 4(35)}{3} = 20$$

one answer:

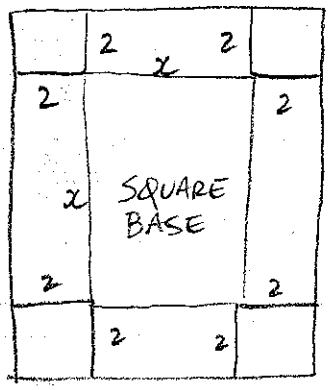
$\boxed{70\text{m} \times 20\text{m}}$

$$l = \frac{200 - 4w}{3} = \frac{200 - 4(15)}{3} = \frac{140}{3}$$

one answer:

$\boxed{30\text{m} \times \frac{140}{3}\text{m}}$

(9)



Let x = length of sides of newly formed base.

$x \cdot x \cdot 2 = 200$

$2x^2 = 200$

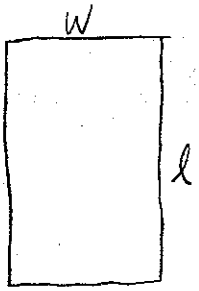
$x^2 = 100$

$x = \pm 10$

$10 + 4 = 14$

$\boxed{14 \times 14}$

(10)



Same area $= 72\text{m}^2$

① $A = lw$

② $A = (w+2)(l-3)$

$72 = lw$

$l = \frac{72}{w}$ (SUBSTITUTE into ②)

$72 = (w+2)\left(\frac{72}{w} - 3\right)$

$72 = 72 - 3w + \frac{144}{w} - 6$

$3w - \frac{144}{w} + 6 = 0$

multiply both sides by w

$3w^2 + 6w - 144 = 0$

$w^2 + 2w - 48 = 0$

$(w+8)(w-6) = 0$

$w = -8, \boxed{6}$

$l = \frac{72}{w} = \frac{72}{6} = \boxed{12}$

(11) Let j = James' original speed

$d = st \Rightarrow t = \frac{d}{s}$

longer time − shorter time = 1

$$\left(\frac{300}{j}\right) - \left(\frac{300}{j+10}\right) = 1$$

$$300(j+10) - 300(j) = 1(j)(j+10)$$

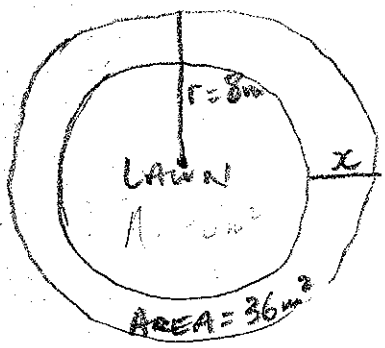
$$300j + 3000 - 300j = j^2 + 10j$$

$$0 = j^2 + 10j - 3000$$

$$0 = (j+60)(j-50)$$

$j = -\cancel{60}, \boxed{50}$ Orig. speed = $\boxed{50 \text{ km/h}}$

(12)



* diagram in book is wrong!

Let x = width of flower bed

Area (flower bed) = Area (big circle) − Area (lawn)

$$36 = \pi r^2 - \pi r^2$$

$$36 = \pi(8)^2 - \pi(8-x)^2$$

$$36 = 64\pi - \pi x^2 + 16\pi x - 64\pi$$

$$\pi x^2 - 16\pi x + 36 = 0$$

$a = \pi$
$b = -16\pi$
$c = 36$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{16\pi \pm \sqrt{(-16\pi)^2 - 4(\pi)(36)}}{2\pi}$$

$$x = \frac{16\pi \pm 45.54}{2\pi} = \boxed{0.75 \text{ m}} \text{ or } \cancel{15.25 \text{ m}}$$

(13) Carpenter alone rate : Completed job = (speed)(time)

$$1 = s(6)$$

$$s = \frac{1}{6}$$

Carpenter and helper rate: $1 = \left(\frac{1}{6} + h\right)4$
(Let h = helper's rate)

$$\frac{1}{4} = \frac{1}{6} + h$$

$$\frac{3}{12} - \frac{2}{12} = h$$

$$\frac{1}{12} = h$$

$$1 = \frac{1}{12}t$$

$t = 12$ hours for helper alone

(14) Let x = Scott's rate in still water

$$d = st \Rightarrow t = \frac{d}{s}$$

$$10 = \underbrace{\frac{9}{x+4}}_{\text{downstream time}} + \underbrace{\frac{9}{x-4}}_{\text{upstream time}}$$

$$10(x+4)(x-4) = 9(x-4) + 9(x+4)$$

$$10x^2 - 160 = 18x$$

$$10x^2 - 18x - 160 = 0$$

$$5x^2 - 9x - 80 = 0$$

$$(5x^2 - 25x) + (16x - 80) = 0$$

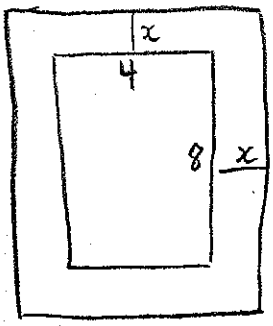
$$5x(x-5) + 16(x-5) = 0$$

$$(x-5)(5x+16) = 0$$

$$x = 5, \ -\frac{16}{5}$$

Scott's rate is 5 km/h

(15)



Let x = width of border

Area (border) = Area (big) − Area (small)

$$28 = [(4+2x)(8+2x)] - (8)(4)$$

$$28 = 4x^2 + 24x + 32 - 32$$

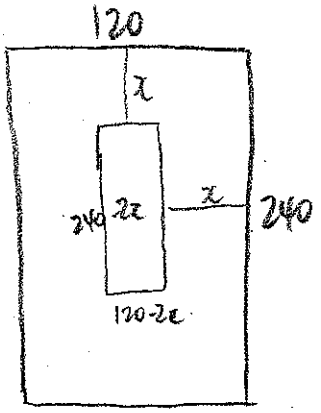
$$0 = 4x^2 + 24x - 28$$

$$0 = x^2 + 6x - 7$$

$$0 = (x+7)(x-1)$$

$x = \cancel{-7}$, $\boxed{1}$

(16)



Let x = width of strip mowed

$$\frac{1}{3} = \frac{(240-2x)(120-2x)}{(240)(120)}$$

$$9600 = 4x^2 - 720x + 28800$$

$$0 = 4x^2 - 720x + 19200$$

$$0 = x^2 - 180x + 4800$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{180 \pm \sqrt{(-180)^2 - 4(1)(4800)}}{2(1)}$$

$$x = \frac{180 \pm 114.9}{2} = \boxed{32.6}, \cancel{147.5}$$

(17) let c = speed of current

$$t = \frac{d}{s}$$

Longer time − shorter time = 1

$$\frac{12}{5-c} - \frac{12}{5+c} = 1$$

$$12(5+c) - 12(5-c) = 1(5-c)(5+c)$$

$$60 + 12c - 60 + 12c = -c^2 + 25$$

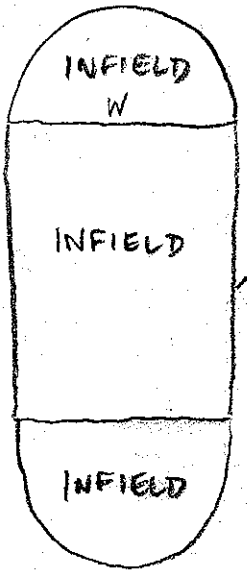
$$c^2 + 24c - 25 = 0$$

$$(c-1)(c+25) = 0$$

$$c = 1 \quad -25$$

Speed of current = 1 km/h.

(18)



$A(\text{infield}) = lw$

$9430 = lw + \pi\left(\frac{w}{2}\right)^2$

$9430 = lw + \frac{\pi w^2}{4}$

Perimeter $= l + l + \frac{\pi w}{2} + \frac{\pi w}{2}$

$400 = 2l + \pi w$

$2l = 400 - \pi w$

$l = 200 - \frac{\pi}{2}w$

SUBSTITUTE INTO OTHER EQUATION

$9430 = \left(200 - \frac{\pi}{2}w\right)(w) + \frac{\pi w^2}{4}$

$9430 = 200w - \frac{\pi}{2}w^2 + \frac{\pi}{4}w^2$

$9430 = 200w - \frac{\pi}{4}w^2$

$\frac{\pi}{4}w^2 - 200w + 9430 = 0$

$\pi w^2 - 800w + 37720 = 0$

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2\pi}$

$w = \frac{800 \pm \sqrt{(-800)^2 - 4(\pi)(37720)}}{2(\pi)}$

$w = \frac{800 \pm 407.4}{2\pi}$

$w = 62.5$, ~~192.2~~

$l = 200 - \frac{\pi}{2}(62.5)$

$l = 101.8$