Solutions to Qs 7-14 p. 244

7) $y = x^2 + kx + 4$

$$y - 4 = x^2 + kx \qquad \boxed{\tfrac{k}{2}} \quad \tfrac{k^2}{4}$$

$$y - 4 + \frac{k^2}{4} = x^2 + kx + \frac{k^2}{4}$$

$$y + \frac{k^2}{4} - 4 = \left(x + \frac{k}{2}\right)^2$$

$$y = \left(x + \frac{k}{2}\right)^2 + 4 - \frac{k^2}{4}$$

$$\boxed{V = \left(-\frac{k}{2},\ 4 - \frac{k^2}{4}\right)}$$

8) $y = 2x^2 + kx + k^2$

$$y - k^2 = 2\left(x^2 + \frac{k}{2}x\right) \qquad \boxed{\tfrac{k}{4}} \quad \tfrac{k^2}{16}$$

$$y - k^2 + \frac{2k^2}{16} = 2\left(x^2 + \frac{k}{2}x + \frac{k^2}{16}\right)$$

$$y - \frac{14k^2}{16} = 2\left(x + \frac{k}{4}\right)^2$$

$$y = 2\left(x + \frac{k}{4}\right)^2 + \frac{7k^2}{8}$$

$$\boxed{V = \left(-\frac{k}{4},\ \frac{7k^2}{8}\right)}$$

9) $y = 2x^2 + ax + b^2$

$$y - b^2 = 2\left(x^2 + \frac{a}{2}x\right) \qquad \boxed{\tfrac{a}{4}} \quad \tfrac{a^2}{16}$$

$$y - b^2 + \frac{a^2}{8} = 2\left(x^2 + \frac{a}{2}x + \frac{a^2}{16}\right)$$

$$y = 2\left(x + \frac{a}{4}\right)^2 + b^2 - \frac{a^2}{8}$$

$$\boxed{V = \left(-\frac{a}{4},\ b^2 - \frac{a^2}{8}\right)}$$

10) $y = px^2 - 3x + p$

$$y - p = p\left(x^2 - \frac{3}{p}x\right) \qquad \boxed{\tfrac{-3}{2p}} \quad \tfrac{9}{4p^2}$$

$$y - p + \frac{9}{4p} = p\left(x^2 - \frac{3}{p}x + \frac{9}{4p^2}\right)$$

$$y + \frac{9 - 4p^2}{4p} = p\left(x - \frac{3}{2p}\right)^2$$

$$y = p\left(x - \frac{3}{2p}\right)^2 + \frac{4p^2 - 9}{4p}$$

$$\boxed{V = \left(\frac{3}{2p},\ \frac{4p^2 - 9}{4p}\right)}$$

11) $y = kx(8 - x)$

$$y = -kx^2 + 8kx \qquad \boxed{-4} \quad 16$$

$$y = -k(x^2 - 8x)$$

$$y - 16k = -k(x^2 - 8x + 16)$$

$$y = -k(x - 4)^2 + 16k$$

$$\boxed{V = (4,\ 16k)}$$

12) $y = ax^2 + 4x - 4$ Vertex x-value $= \underline{\underline{6}}$

$$y + 4 = a\left(x^2 + \frac{4}{a}x\right) \quad \boxed{\frac{2}{a}} \quad \frac{4}{a^2}$$

$$y + 4 + \frac{4}{a} = a\left(x + \frac{4}{a}x + \frac{4}{a^2}\right)$$

$$y = a\left(x + \frac{2}{a}\right)^2 - \frac{4 + 4a}{a}$$

$$y = a\left(x - \left(-\frac{2}{a}\right)\right)^2 - \frac{4 + 4a}{a}$$

$$-\frac{2}{a} = 6$$

$$\boxed{a = -\frac{1}{3}}$$

13) $y = 2x^2 + bx - 3$ y-value of vertex $= -5$

$$y + 3 = 2\left(x^2 + \frac{b}{2}x\right) \quad \boxed{\frac{b}{4}} \quad \frac{b^2}{16}$$

$$y + 3 + \frac{b^2}{8} = 2\left(x^2 + \frac{b}{2}x + \frac{b^2}{16}\right)$$

$$y = 2\left(x + \frac{b}{4}\right)^2 - \left(3 + \frac{b^2}{8}\right)$$

$$-\left(3 + \frac{b^2}{8}\right) = -5$$

$$3 + \frac{b^2}{8} = 5$$

$$\frac{b^2}{8} = 2$$

$$b^2 = 16$$

$$\boxed{b = \pm 4}$$

14) $y = 0.1x^2 + 7x + c$ y-value of vertex $= -120.5$

$y - c = 0.1(x^2 + 70x)$ $\boxed{35}$ 1225

$y - c + 122.5 = 0.1(x^2 + 70x + 1225)$

$y = 0.1(x+35)^2 + c - 122.5$

$c - 122.5 = -120.5$

$\boxed{c = 2.0}$

Ch. 5.6 - Applications of Quad. Fcns
Solutions (p. 256-258 # 1-18)

1) $R = -\frac{1}{2}p^2 + 2000p$ Find p by completing the square. (p is the x-value of the vertex)

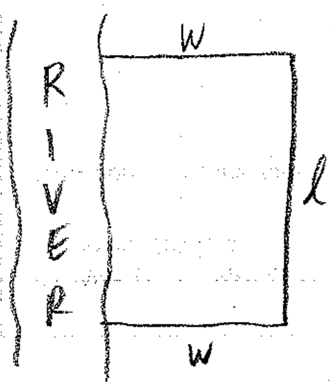
$R = -\frac{1}{2}(p^2 - 4000p)$ $\boxed{-2000}$ 4000 000

$R - 2000000 = -\frac{1}{2}(p^2 - 4000p + 4000000)$ (max. revenue is the y-value of the vertex).

$R = -\frac{1}{2}(p-2000)^2 + 2000000$

$$\boxed{\begin{array}{l} p = \text{unit price} = \$2000 \\ \text{max revenue} = 2000000 \end{array}}$$

2)



$P = 6000 = 2w + l$

$l = 6000 - 2w$

Area $= lw$

Area $= (6000 - 2w)(w)$

$A = -2w^2 + 6000w$

$A = -2(w^2 - 3000w)$ $\boxed{-1500}$ 2250 000

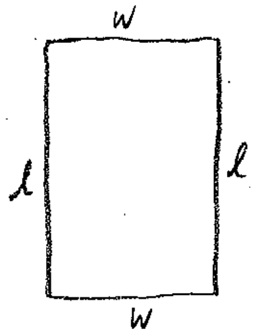
$A - 4500000 = -2(w^2 - 3000w + 2250000)$

$A = -2(w-1500)^2 + 4500000$

largest area $= \boxed{4\,500\,000 \text{ m}^2}$

dimensions : $\boxed{\begin{array}{l} w = 1500 \\ l = 3000 \end{array}}$

3)



$$P = 36 = 2l + 2w$$

$$2l = 36 - 2w$$

$$l = 18 - w$$

$$A = w(18 - w)$$

$$A = -w^2 + 18w$$

$$A = -1(w^2 - 18w) \quad \boxed{-9} \; 81$$

$$A - 81 = -1(w^2 - 18w + 81)$$

$$A = -(w-9)^2 + 81$$

Max area = $81\ cm^2$ $\quad w = 9$; $l = 9$ (SQUARE)

4) Let x = one number
then y = the other

$$x - y = 8$$

$$x = 8 + y$$

Product $= xy$

$$P = (8+y)(y)$$

$$P = y^2 + 8y \quad \boxed{4} \; 16$$

$$P + 16 = y^2 + 8y + 16$$

$$P = (y+4)^2 - 16$$

$y = -4$; $x = 4$

PRODUCT $= -16$

5) $h = -0.005x^2 + x + 100$

$h - 100 = -0.005(x^2 - 200x)$ $\boxed{-100}$ 10000

$h - 100 - 50 = -0.005(x^2 - 200x + 10000)$

$h - 150 = -0.005(x - 100)^2$

$h = -0.005(x - 100)^2 + 150$

a) weirdly worded... but $\boxed{x = 100\text{ m}}$

b) Max height = $\boxed{150\text{ m}}$

c) Set $h = 0$ and find x

$0 = -0.005(x - 100)^2 + 150$

$-150 = -0.005(x - 100)^2$

$30000 = (x - 100)^2$

$x - 100 = \pm 173.2$

$\boxed{x = 273.2\text{ m}}$ and $-\cancel{73.2}$

6) Let x = # of \$1 increases. Revenue = # of people × admission fee

Revenue $= (80 - 5x)(10 + 1x)$

$R = -5x^2 + 30x + 800$

$R - 800 = -5(x^2 - 6x)$ $\boxed{-3}$ 9

$R - 800 - 45 = -5(x - 3)^2$

$R = -5(x - 3)^2 + 845$

$x = 3$, 3 \$1 increases need to be made

so $10 + 3 = \boxed{\$13}$ $\boxed{\text{max rev.} = \$845}$

7) Income = Stereos sold × Price

$$I = x(500 - x)$$

$$I = -x^2 + 500x \qquad \boxed{-250} \quad 62500$$

$$I = -1(x^2 - 500x)$$

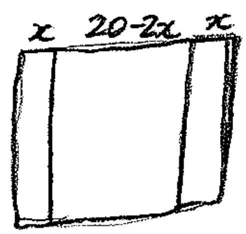
$$I - 62500 = -1(x^2 - 500x + 62500)$$

$$I = -1(x - 250)^2 + 62500$$

$$\boxed{\begin{array}{l} \text{MAX. INCOME} = \$62500 \\ \text{Stereos sold} = 250 \end{array}}$$

8) $V = l \cdot w \cdot h$ $\qquad W = 20 \text{ cm}$

let x = height of trough



$$V = l \cdot (20 - 2x)x$$

$$V = -2x^2 l + 20xl$$

$$V = -2l(x^2 - 10x) \qquad \boxed{-5} \quad 25$$

$$V + 50l = -2l(x^2 - 10x + 25)$$

$$V = -2l(x - 5)^2 - 50l$$

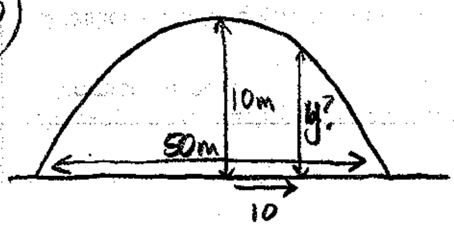
$$\boxed{x = 5 \text{cm}}$$

9) Let x = one integer
Let y = the other

$$x + y = 10$$
$$y = 10 - x$$

$$S = x^2 + y^2$$
$$S = x^2 + (10-x)^2$$
$$S = 2x^2 - 20x + 100$$
$$S - 100 = 2(x^2 - 10x) \quad \boxed{-5} \; 25$$
$$S - 100 + 50 = 2(x^2 - 10x + 25)$$
$$S - 50 = 2(x-5)^2$$
$$S = 2(x-5)^2 + 50$$

$$\boxed{\begin{array}{l} x = 5 \text{ so } y = 5 \\ \text{Sum of squares } (S) = 50 \end{array}}$$

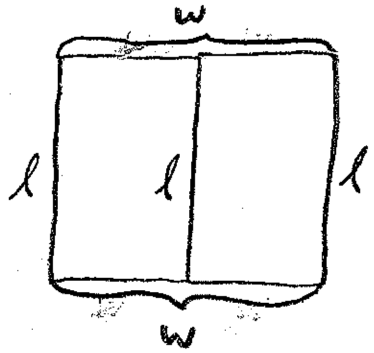
10) 

Let vertex = $(0, 10)$
x-intercepts = $(-25, 0), (25, 0)$

$$y = a(x-h)^2 + k$$
$$0 = a(25-0)^2 + 10$$
$$0 = 625a + 10$$
$$-10 = 625a$$
$$a = -0.016$$

$$y = -0.016(x)^2 + 10$$
Let $x = 10$, find y
$$y = -0.016(10)^2 + 10$$
$$\boxed{y = 8.4\text{m}}$$

11)



$$2w + 3l = 1200$$
$$2w = 1200 - 3l$$
$$w = 600 - \tfrac{3}{2}l$$

$$A = lw$$
$$A = l\left(600 - \tfrac{3}{2}l\right)$$
$$A = -\tfrac{3}{2}l^2 + 600l$$
$$A = -\tfrac{3}{2}\left(l^2 - 400l\right) \quad \boxed{-200} \quad 40000$$
$$A - 60000 = -\tfrac{3}{2}\left(l^2 - 400l + 40000\right)$$
$$A = -\tfrac{3}{2}(l-200)^2 + 60000$$
$$\boxed{l = 200\,m} \quad w = 600 - \tfrac{3}{2}(200) = \boxed{300\,m}$$
$$\text{Max area} = \boxed{60000\,m^2}$$

12) Revenue = (# of members)(membership fee)

Let x = # of member increases over 60

$$R = (60 + x)(200 - 2x)$$
$$R = -2x^2 + 80x + 12000 \quad \boxed{-20} \quad 400$$
$$R - 12000 = -2(x^2 - 40x)$$
$$R - 12000 - 800 = -2(x^2 - 40x + 400)$$
$$R - 12800 = -2(x-20)^2$$
$$R = -2(x-20)^2 + 12800$$

max. revenue

20 increases so ... $60 + 20 =$ 80 members

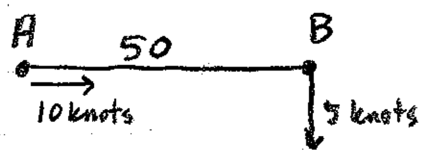
13) Let x = # of dollars increase in price

$$R = (10+x)(300-10x)$$
$$R = -10x^2 + 200x + 3000$$
$$R - 3000 = -10(x^2 - 20x) \quad \boxed{-10} \quad 100$$
$$R - 3000 = -10(x^2 - 20x + 100)$$
$$R - 3000 - 1000 = -10(x-10)^2$$
$$R = -10(x-10)^2 + 4000$$

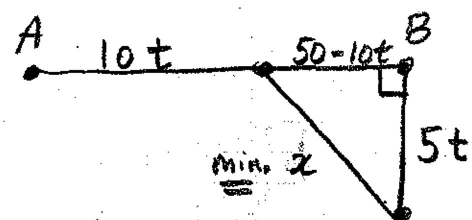
$x = 10$ increases ... so, $\$10 + \$10 = \boxed{\$20}$

Max. revenue $= \boxed{\$4000}$

14)



$d = st$



Let x = minimum distance

$$x^2 = (50-10t)^2 + (5t)^2$$
$$x^2 = 100t^2 - 1000t + 2500 + 25t^2$$
$$x^2 - 2500 = 125t^2 - 1000t \quad \boxed{-4} \quad 16$$
$$x^2 - 2500 = 125(t^2 - 8t)$$
$$x^2 - 2500 + 2000 = 125(t^2 - 8t + 16)$$
$$x^2 = 125(t-4)^2 + 500$$

After $\boxed{4 \text{ hrs,}}$ the ships will be $\sqrt{x^2} = \sqrt{500}$ $\boxed{= 22.4 \text{ mi apart}}$

15) $P = x + y + 60 + x + y$

$300 = 2x + 2y + 60$

$240 = 2x + 2y$

$120 = x + y$

$y = 120 - x$

$A = (60+x)y$

$A = (60+x)(120-x)$

$A = -x^2 + 60x + 7200$

$A - 7200 = -1(x^2 - 60x)$ $\boxed{-30}$ 900

$A - 7200 - 900 = -1(x^2 - 60x + 900)$

$A = -1(x-30)^2 + 8100$

$\boxed{x = 30\text{ft} \quad y = 90\text{ft}}$ $\boxed{\text{Max area} = 8100\text{ ft}^2}$

16) Let vertex $= (0, 10)$
another point $= (100, 60)$

$y = a(x-h)^2 + k$

$60 = a(100-0)^2 + 10$

$60 = 10000a + 10$

$50 = 10000a$

$a = 0.005$

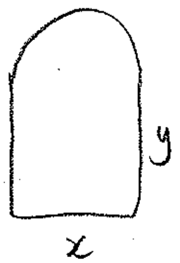
$\therefore$ x-coordinate of longest support cable is 75.

$y = 0.005(x)^2 + 10$

$y = 0.005(75)^2 + 10$

$\boxed{y = 38.1 \text{ ft}}$

17.



Let x = width (diameter)

y = length

$$P = 2y + x + \frac{\pi x}{2}$$

$$24 = 2y + x + \frac{\pi x}{2}$$

$$2y = 24 - x - \frac{\pi x}{2}$$

$$y = 12 - \frac{x}{2} - \frac{\pi x}{4}$$

$$\text{Area} = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$A = x\left(12 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8}$$

$$A = 12x - \frac{1}{2}x^2 - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$

$$A = -\frac{1}{2}x^2 - \frac{2\pi x^2}{8} + \frac{\pi x^2}{8} + 12x$$

$$A = -\frac{1}{2}x^2 - \frac{\pi x^2}{8} + 12x$$

$$A = -\frac{4x^2 - \pi x^2}{8} + 12x$$

$$A = -\frac{7.14x^2}{8} + 12x$$

$$A = -0.893x^2 + 12x \qquad \boxed{-6.72} \quad 45.16$$

$$A = -0.893(x^2 - 13.44x)$$

$$A - 40.33 = -0.893(x^2 - 13.44x + 45.16)$$

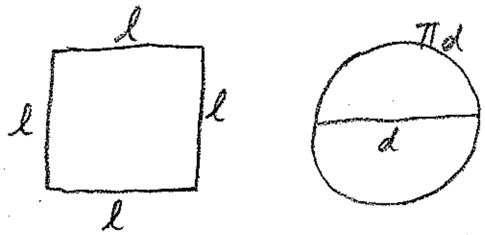
$$A = -0.893(x - 6.72)^2 + 40.33$$

$$\boxed{x = 6.72 \text{ ft}}$$

$$y = 12 - 3.36 - 5.28$$

$$\boxed{y = 3.36 \text{ ft}}$$

18)



$$36 = 4l + \pi d$$

$$36 - 4l = \pi d$$

$$d = \frac{36 - 4l}{\pi} = 11.46 - 1.27l$$

$$A^{sum} = l^2 + \pi\left(\frac{d}{2}\right)^2$$

$$A = l^2 + \pi\left(\frac{11.46 - 1.27l}{2}\right)^2$$

$$A = l^2 + \pi(5.73 - 0.635l)^2$$

$$A = l^2 + \pi(0.403l^2 - 7.28l + 32.83)$$

$$A = l^2 + 1.27l^2 - 22.87l + 103.14$$

$$A - 103.14 = 2.27l^2 - 22.87l \qquad \boxed{5.035} \quad 25.35$$

$$A - 103.14 = 2.27(l^2 - 10.07l)$$

$$A - 103.14 + 57.54 = 2.27(l^2 - 10.07l + 25.35)$$

$$A = 2.27(l - 5.035)^2 + 45.6$$

$$l = 5.035$$
$$d = 5.05$$

$$\boxed{\begin{array}{l} \text{Square} = 20.14 \text{ cm} \\ \text{circle} = 15.87 \text{ cm} \end{array}}$$