Ch. 5.1 - Defining Polynomials

A <u>polynomial</u> is a monomial or the sum and/or difference of monomials.

A <u>monomial</u> is a <u>number</u>, a <u>variable</u>, or the product of <u>numbers and variables</u>.

(mono- = <u>ONE</u> ; poly- = <u>MANY</u>)

Important definitions:

<u>VARIABLE</u> - a letter (or symbol) whose value is an <u>unknown real number</u>.

* most commonly used: x, y, z.

others: θ, n, a, b, c

$\boxed{\text{TERM}}$ - a term is a number, or the product of a number and variable(s) raised to a power.

eg: $5,\ 2x,\ -3x^2,\ -2xy,\ 0.$

COEFFICIENT - the coefficient of a term is the numerical factor.

eg:

Term	Coefficient
$5x$	5
$\frac{x^2}{3}$	$\frac{1}{3}$
$-1.3xy$	-1.3
$-y$	-1
3	3

also referred to as a CONSTANT (ie. just a number, no variable)

$$\underbrace{2x^3}_{\text{TERM}}$$

coefficient

exponent

Variable

note: $4x^0 = \boxed{4}$

MONOMIAL - an expression of the form ax^n, where a is a real number coefficient, x is a variable, and n is a non-negative integer.

eg: $\frac{3}{4}y^2, -4, 2x, x^0$ non-eg: $\frac{1}{x}, \sqrt{x}, x^{\frac{2}{3}}, x^{-2}$

- so, a monomial is a <u>term</u> and vice versa!

POLYNOMIAL - a polynomial is a monomial, or a combination of sums and/or differences of monomials.

eg: $3x$, $2x - 1$, $4x^2 + x + 2$, $4x^2 + y^2$

Degree and Leading Term of a Polynomial

The degree of a $\boxed{\text{term}}$ is the sum of the exponents of its variables.

The leading term of a polynomial is the term possessing the highest degree. Thus, the degree of a polynomial is the same as the degree of its leading term.

eg1: For each of the following polynomials, list each term and each term's degree, then list the leading term and the degree of the polynomial:

a) $5x^2 - 3x + 4$

Three terms:	$5x^2$	$-3x$	4
degree:	2	1	0

over →

Leading term: $5x^2$

Degree of Polynomial: 2

b) $-5 + xy$

Two terms: -5 and xy

degree: 0 and 2

Leading term: xy

Degree of Polynomial: 2

c) $4x^2y - 5x^3y^4 + 3xy - 6x$

Four terms: $4x^2y$, $-5x^3y^4$, $3xy$, $-6x$

degree: 3, 7, 2, 1

Leading term: $-5x^3y^4$

Degree of Polynomial: 7

Note: A polynomial is usually written in descending order of powers.

eg: $2x + 4 - 3x^2$ is written as:

$$-3x^2 + 2x + 4.$$

Note: A polynomial with more than one variable is written in alphabetical order.

eg. $xy + y^2 - x^2$ is written as:

$$\boxed{-x^2 + xy + y^2}$$

More terminology:

Monomial - a polynomial of one term.

Binomial - a polynomial of two terms.

eg: $\boxed{x+1 \quad , \quad 3x-2}$

Trinomial - a polynomial of three terms.

eg: $\boxed{3x^2 + x - 2, \quad 5y^2 - 2ay - a^2}$

(bi- = TWO , tri- = THREE)

Like Terms

Terms with the same variable raised to exactly the same powers are called LIKE TERMS.

eg:

LIKE TERMS	UNLIKE TERMS
$5x, 3x$	$5x^2, 3x$
$2x^2, -x^2$	$3x^2, 3y^2$
$2xy, -4xy$	$2xy, 2xz$
$\underbrace{xy^2, y^2x}_{\text{ie. } xy = yx}$	xy^2, x^2y

- we are able to ADD/SUBTRACT <u>like</u> terms, but we cannot add/subtract <u>unlike</u> terms.

$4x + 3x$ is <u>NOT</u> a binomial!

Why not?

$$\boxed{4x + 3x = 7x \quad (\text{MONOMIAL!})}$$

<u>eg2:</u> Simplify each of the following:

a) $4x^2 + 3x^2 - 2x^2$

$$\boxed{= 5x^2}$$

b) $4x + 3x^2 - 2x^2$

$= 4x + x^2$

$$\boxed{= x^2 + 4x}$$

c) $2xy^2 + 3x^2y - 4x^2y + xy^2$

$= 2xy^2 + xy^2 + 3x^2y - 4x^2y$

$= 3xy^2 - x^2y$

$$\boxed{= -x^2y + 3xy^2}$$

p. 161 – 166
#1–5, 8, 10

Ch. 5.2 - Adding and Subtracting Polynomials

Adding Polynomials Horizontally

- try to group like terms together before adding

eg1: Add: $(-2x^2-4)+(3x^2-2x+2)$

$= -2x^2 - 4 + 3x^2 - 2x + 2$

$= -2x^2 + 3x^2 \quad -2x \quad -4+2$ grouping like terms

$\boxed{= x^2 - 2x - 2}$

Adding Polynomials Vertically

- place like terms in the same columns (in descending order of powers) → if there exists a term(s) with no match, leave a blank space (term with coefficient of 0).

same eg1: Add $(-2x^2-4)+(3x^2-2x+2)$

$$\begin{array}{rrrr} & -2x^2 & +0x & -4 \\ + & 3x^2 & -2x & +2 \\ \hline & \boxed{x^2} & \boxed{-2x} & \boxed{-2} \end{array}$$

eg2: Add vertically:

$$(6x^2 - 7x + 1) + (4x^2 + 6x)$$

$$\begin{array}{r} 6x^2 - 7x + 1 \\ + \quad 4x^2 + 6x + 0 \\ \hline \boxed{10x^2 - x + 1} \end{array}$$

Subtracting Polynomials Horizontally

- when subtracting an entire polynomial, change the sign of each term to its opposite (this is the equivalent of multiplying the polynomial by $\underline{-1}$).

eg: $-(-2x^2 + 4x + 1) = \underline{2x^2 - 4x - 1}$

eg3: Subtract: $(2x^2 - 4x - 3) - (-x^2 + 2x - 1)$

$$= 2x^2 - 4x - 3 + x^2 - 2x + 1$$

$$= 2x^2 + x^2 - 4x - 2x - 3 + 1$$

$$\boxed{= 3x^2 - 6x - 2}$$

Subtracting Polynomials Vertically

- similar in set-up to adding vertically, but multiply negative sign outside the brackets into the polynomial, then add the result.

Same eg3: Subtract $(2x^2-4x-3)-(-x^2+2x-1)$

$$\begin{array}{r} 2x^2-4x-3 \\ -(-x^2+2x-1) \end{array}$$

$$\begin{array}{rr} & 2x^2-4x-3 \\ + & x^2-2x+1 \\ \hline & \boxed{3x^2-6x-2} \end{array}$$

eg4: Subtract: $-7x^2-6x+2-(4x-6x^2-5)$

$$\begin{array}{r} -7x^2-6x+2 \\ -(-6x^2+4x-5) \end{array}$$

$$\begin{array}{rr} & -7x^2-6x+2 \\ + & 6x^2-4x+5 \\ \hline & \boxed{-x^2-10x+7} \end{array}$$

p.169-176 1-7, 8a-e (ignore algebra tiles)

Ch. 5.3 - Multiplying Polynomials

Recall Ch. 2.3: we multiplied 'constant monomials' by other 'constant monomials' $\Rightarrow 2^3 \times 2^4$

$$= 2^{3+4}$$
$$= 2^7$$

So... <u>MULTIPLYING MONOMIALS</u>:

To multiply two monomials, multiply the <u>numerical</u> (<u>coefficient</u>) factors, then multiply the <u>variable</u> factors.

eg: a) Multiply $(2x^2)(-3x)$

$$= (2)(-3)(x^2)(x)$$
$$= -6(x^{2+1}) = \boxed{-6x^3}$$

b) Multiply $(7x^2)(5x^3y)$

$$= (7)(5)(x^2)(x^3)(y)$$
$$= 35(x^{2+3})y$$
$$\boxed{= 35x^5y}$$

- to find the product of a monomial and a polynomial with more than one term, we use the <u>DISTRIBUTIVE PROPERTY</u>:

$$a(b+c) = \underline{ab + ac}$$

Check: $2(3+4) = (2)(3) + (2)(4)$

$= 6 + 8$

$= \underline{\underline{14}}$

<u>OR</u>

$2(3+4) = 2(7) = \underline{\underline{14}}$

So... <u>Multiplying Monomials by Polynomials</u>:

- use distributive property, then simplify.

eg 2: Multiply:

a) $-4(3-x)$

$= (-4)(3) + (-4)(-x)$

$= -12 + 4x$

$\boxed{= 4x - 12}$

b) $-4x(x^2 - 3x + 1)$

$= (-4x)(x^2) + (-4x)(-3x) + (-4x)(1)$

$\boxed{= -4x^3 + 12x^2 - 4x}$

$\boxed{\text{p. 181 – 186 \# 1–3, 8}}$

Ch. 5.4 - Dividing Polynomials

Dividing a Polynomial by a Constant

- when dividing a polynomial by a constant, be sure to divide <u>each</u> <u>term</u> by the constant!

ie: $\dfrac{a+b+c+\dots z}{\#} = \boxed{\dfrac{a}{\#}+\dfrac{b}{\#}+\dfrac{c}{\#}+\dots+\dfrac{z}{\#}}$

(# = CONSTANT) (where $\# \neq 0$)

eg1: Divide $9x^2-3x+6$ by 3.

ie. $(9x^2-3x+6) \div 3$

OR

$$\frac{9x^2-3x+6}{3} = \frac{9x^2}{3} - \frac{3x}{3} + \frac{6}{3}$$

$$\boxed{= 3x^2 - x + 2}$$

eg2: Divide: $\dfrac{20x^2+6x-4}{4}$

$$= \frac{20x^2}{4} + \frac{6x}{4} - \frac{4}{4}$$

$$\boxed{= 5x^2 + \frac{3}{2}x - 1}$$

Dividing a Polynomial by a Monomial

- when dividing a polynomial by a monomial, be sure to divide <u>each</u> <u>term</u> by the monomial!

ie: $$\frac{a+b+c+\dots+z}{☺} = \boxed{\frac{a}{☺}+\frac{b}{☺}+\frac{c}{☺}+\dots+\frac{z}{☺} \quad (☺ \neq 0)}$$

(☺ = monomial)

<u>eg3:</u> Divide: $\frac{-9x^2+6xy}{3x}$

$$= \frac{-9x^2}{3x} + \frac{6xy}{3x}$$

$$= -3x^{2-1} + 2x^{1-1}y$$

$$\boxed{= -3x + 2y}$$

* show "cross-out" method (canceling)

<u>eg4.</u> Divide: $\frac{14x^4y+18xy^2-6x^2y^3}{6xy}$

$$= \frac{14x^4y}{6xy} + \frac{18xy^2}{6xy} - \frac{6x^2y^3}{6xy}$$

$$\boxed{= \frac{7}{3}x^3 + 3y - xy^2}$$

p.189–193
#1–3, 6, 10,

Also: CHAPTER REVIEW!

*Ch. 5.5 – Multiplying Binomials by Binomials

- employs the concept of **F.O.I.L.**

First
Outside
Inside
Last

eg: $(x+1)(x+2)$

$= x(x+2) + 1(x+2)$ *using the distributive principle.

Using FOIL:

$(x+1)(x+2) = x^2 + 2x + 1x + 2$

$\boxed{= x^2 + 3x + 2}$

eg: Multiply $(x-3)(x+4)$

$= x^2 + 4x - 3x - 12$

$\boxed{= x^2 + x - 12}$

eg: Multiply $(x+5)(x-2)$

$= x^2 - 2x + 5x - 10$

$\boxed{= x^2 + 3x - 10}$

eg: Multiply $(x-4)(x-6)$

$= x^2 - 6x - 4x + 24$

$\boxed{= x^2 - 10x + 24}$

eg: Multiply $(3x-2)(2x+1)$

$= 6x^2 + 3x - 4x - 2$

$\boxed{= 6x^2 - x - 2}$

eg: Multiply: $(x+2)(x-2)$

$= x^2 - 2x + 2x - 4$

$\boxed{= x^2 - 4}$

Worksheet 1-32

*Ch.5.6 – Multiplying Binomials by Trinomials

Recall Multiplying Binomials:

$(x+1)(x+3)$ ... we use F.O.I.L.

FOIL allows us to 'skip' writing out the step showcasing the DISTRIBUTIVE PROPERTY:

$$(x+1)(x+3) = \underbrace{(x+1)(x) + (x+1)(3)}_{\text{expanded using Distributive Property.}}$$

$$= x^2 + x + 3x + 3$$

$$= \boxed{x^2 + 4x + 3}$$

Using the Distributive Property, we can multiply polynomials with more terms easily:

$$(x+1)(x^2+2x+1)$$

$$= (x+1)(x^2) + (x+1)(2x) + (x+1)(1)$$

$$= x^3 + x^2 + 2x^2 + 2x + x + 1$$

$$\boxed{= x^3 + 3x^2 + 3x + 1}$$

- however, like FOIL, we can recognize a pattern and streamline our work:

$$(x+1)(x^2+2x+1) = x^3+2x^2+x+x^2+2x+1$$

* in ANY order!

$$= \boxed{x^3+3x^2+3x+1}$$

BUT, if you would like to have 'like terms' grouped together in your expansion, use this order:

$$(x+1)(x^2+2x+1) = x^3+\underbrace{x^2+2x^2}+\underbrace{2x+x}+1$$

$$= \boxed{x^3+3x^2+3x+1}$$

eg1: Expand each of the following:

a) $(x-2)(x^2-3x-5)$

$$= x^3-2x^2-3x^2+6x-5x+10$$

$$= \boxed{x^3-5x^2+x+10}$$

b) $(x-5)(2x^2-x+2)$

$= 2x^3-10x^2-x^2+5x+2x-10$

$= \boxed{2x^3-11x^2+7x-10}$

c) $(4x+7)(3x^2+4x-2)$

$= 12x^3+21x^2+16x^2+28x-8x-14$

$= \boxed{12x^3+37x^2+20x-14}$

eg2: Expand: $(x+3)(2x-1)(x-10)$

$= (2x^2-x+6x-3)(x-10)$

$= (2x^2+5x-3)(x-10)$

$= 2x^3-20x^2+5x^2-50x-3x+30$

$= \boxed{2x^3-15x^2-53x+30}$

Worksheet

1-30