Solutions to Word Problems Ch. 2.6 p. 91-93

1a) $x + \frac{1}{x} = \frac{13}{6}$

$6x^2 + 6 = 13x$

$6x^2 - 13x + 6 = 0$

$6x^2 - 9x - 4x + 6 = 0$

$3x(2x-3) - 2(2x-3) = 0$

$(2x-3)(3x-2) = 0$

$\boxed{x = \frac{3}{2}, \frac{2}{3}}$

b) $x + \frac{1}{x} = \frac{65}{8}$

$8x^2 + 8 = 65x$

$8x^2 - 65x + 8 = 0$

$8x^2 - 64x - 1x + 8 = 0$

$8x(x-8) - 1(x-8) = 0$

$(8x-1)(x-8) = 0$

$x = \cancel{\frac{1}{8}}, \boxed{8}$

not an INTEGER!

c) $\frac{1}{x} + \frac{1}{x+1} = \frac{7}{12}$

$12(x+1) + 12x = 7(x)(x+1)$

$24x + 12 = 7x^2 + 7x$

$\frac{x}{-84}$ $\frac{+}{-17}$ $0 = 7x^2 - 17x - 12$

$0 = 7x^2 - 21x + 4x - 12$

$0 = 7x(x-3) + 4(x-3)$

$0 = (x-3)(7x+4)$

$x = 3, \cancel{-\frac{4}{7}}$

3 and 4

d) $\frac{1}{x} + \frac{1}{x+2} = \frac{8}{15}$

$15(x+2) + 15x = 8x(x+2)$

$30x + 30 = 8x^2 + 16x$

$0 = 8x^2 - 14x - 30$

$0 = 4x^2 - 7x - 15$

$0 = 4x^2 - 12x + 5x - 15$

$0 = 4x(x-3) + 5(x-3)$

$0 = (x-3)(4x+5)$

$x = 3, \cancel{-\frac{5}{4}}$

3 and 5

e) $\frac{1}{x} + \frac{1}{x+2} = \frac{7}{24}$

$24(x+2) + 24x = 7x(x+2)$

$48x + 48 = 7x^2 + 14x$

$0 = 7x^2 - 34x - 48$

$0 = 7x^2 - 42x + 8x - 48$

$0 = 7x(x-6) + 8(x-6)$

$0 = (x-6)(7x+8)$

$x = 6, -\frac{8}{7}$

6 and 8

f) $\frac{x}{5} - \frac{3}{2} = \frac{x}{2}$

$2x - 15 = 5x$

$-15 = 3x$

$x = -5$

Typo in text key!

g) $x + \frac{6}{x} = -5$

$x^2 + 6 = -5x$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3, -2$ another TYPO in text.

h) $\frac{12}{x} + \frac{7x-5}{x+1} = 8$

$12(x+1) + x(7x-5) = 8x(x+1)$

$12x + 12 + 7x^2 - 5x = 8x^2 + 8x$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x = -4, 3$

2a) JOB = RATE • TIME

Sue's rate = $\frac{1 \text{ job}}{4 \text{ hrs.}}$

Bert's rate = $\frac{1}{5}$

Let t = time together to complete.

Rate together = $\frac{1}{t}$

So... $\frac{1}{4} + \frac{1}{5} = \frac{1}{t}$ LCD = $20t$

$5t + 4t = 20$

$9t = 20$

$t = \frac{20}{9}$ = 2 hrs. 13 mins 20 secs.

b) Worker 1's rate = $\frac{1}{20}$ job/h ; worker 2's rate = $\frac{1}{15}$ job/h

Let x = time for third worker.

then $\frac{1}{x}$ = worker 3's rate

rate = $\frac{\text{JOB}}{h}$

$\frac{1}{20} + \frac{1}{15} + \frac{1}{x} = \frac{1}{6}$

Rate all together = $\frac{1}{6}$

LCD = $60x$

$3x + 4x + 60 = 10x$

$60 = 3x$

$x = 20$ hours for third worker

2c) Let a = Jane's time to clean
then $2a$ = Anna's time to clean

$$\text{Rate} = \frac{\text{Job}}{\text{time}}$$

15 mins. together...

$$\frac{1}{a} + \frac{1}{2a} = \frac{1}{15} \qquad \text{LCD} = 30a$$

$$30 + 15 = 2a$$

$$45 = 2a$$

$$a = \frac{45}{2}$$

$$2a = \frac{2(45)}{2} = \frac{90}{2} = \boxed{45 \text{ mins. for Anna}}$$

d) Let t = Ken's time alone
Let $t-3$ = Hans' time alone

2 hours together...

$$\text{Rate} = \frac{\text{Job}}{\text{time}}$$

$$\frac{1}{t} + \frac{1}{t-3} = \frac{1}{2} \qquad \text{LCD} = 2t(t-3)$$

$$2(t-3) + 2t = t(t-3)$$

$$2t - 6 + 2t = t^2 - 3t$$

$$0 = t^2 - 7t + 6$$

$$(t-6)(t-1) = 0$$

$t = 6, \cancel{1}$

$\boxed{\begin{array}{l}\text{Ken} = 6 \text{ hrs.} \\ \text{Hans} = 3 \text{ hrs.}\end{array}}$

2e) let x = time to fill tub

Rate cold = $\frac{1}{6}$

Rate hot = $\frac{1}{8}$

Rate drain = $\frac{1}{10}$

$\frac{1}{x}$ = rate to fill tub

$$\frac{1}{6} + \frac{1}{8} - \frac{1}{10} = \frac{1}{x} \qquad LCD = 120x$$

$$20x + 15x - 12x = 120$$

$$23x = 120$$

$$x = \frac{120}{23} \text{ mins} \quad 5 \text{ hrs. } 13 \text{ mins}$$

f) let x = bricklayer's time $\frac{1}{6}$ = bricklayer rate

then $x+5$ = apprentice's time $\frac{1}{16}$ = apprentice rate

$$\frac{1}{6}x + \frac{1}{16}(x+5) = 1 \text{ job}$$

$LCD = 48$

$$8x + 3x + 15 = 48$$

$$11x = 33$$

$$x = 3$$

$$x+5 = 8$$

Bricklayer = 3 hrs.
Apprentice = 8 hrs

3.a) Let s = speed in still water $\quad t = \frac{d}{s}$

$$\frac{40}{s+6} = \frac{30}{s-6}$$

$$40(s-6) = 30(s+6)$$
$$40s - 240 = 30s + 180$$
$$10s = 420$$
$$\boxed{s = 42 \text{ km/h}}$$

b) Let c = speed of current $\quad t = \frac{d}{s}$

$$5 = \frac{24}{10+c} + \frac{24}{10-c}$$

$$5(10+c)(10-c) = 24(10-c) + 24(10+c)$$
$$-5c^2 + 500 = 480$$
$$0 = 5c^2 - 20$$
$$20 = 5c^2$$
$$c^2 = 4 \qquad \boxed{c = \pm 2 \text{ mph}}$$

c) Let x = avg. speed for round trip. | distances are the same.

$t = \frac{d}{s}$ TOTAL TIME = time to work + time to home

$$\frac{2d}{x} = \frac{d}{50} + \frac{d}{30}$$

$$3000d = 30xd + 50xd$$
$$3000d = 80xd$$
$$3000 = 80x \qquad \boxed{x = 37.5 \text{ mph}}$$

3d) Let x = avg. speed for entire trip. $\quad t = \frac{d}{s}$

TOTAL TIME = time there + time back

$$\frac{100}{x} = \frac{50}{40} + \frac{50}{60}$$

$$\frac{100}{x} = \frac{5}{4} + \frac{5}{6}$$

$$1200 = 15x + 10x$$

$$1200 = 25x$$

$$\boxed{x = 48 \text{ km/h.}}$$

e) Let x = # of km walked

TOTAL TIME = time @ faster rate + time @ slower rate $\quad t = \frac{d}{s}$

$$2 = \frac{8-x}{7} + \frac{x}{3}$$

$$42 = 3(8-x) + 7(x)$$

$$42 = 24 - 3x + 7x$$

$$18 = 4x$$

$$\boxed{x = 4.5 \text{ km walked}}$$

f) Let x = driver A's speed

Then $x+10$ = driver B's speed

10 mins = $\frac{1}{6}$ h.

$t = \frac{d}{s}$

Slower (A) time − faster (B) time = $\frac{1}{6}$

$$\frac{80}{x} - \frac{100}{x+10} = \frac{1}{6}$$

$$(6)(x+10)(80) - (6)(x)(100) = x(x+10)$$

$$480x + 4800 - 600x = x^2 + 10x$$

$$0 = x^2 + 130x - 4800$$

$$0 = (x-30)(x+160)$$

$$x = 30, \cancel{-160}$$

Driver A = 30 km/h.

Driver B = 40 km/h.

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7) Rate of cold = $\frac{1}{t}$ tub/min

Rate of hot = $\frac{1}{t+2}$ tub/min

Let x = amount filled in 4 mins.

$$\frac{1}{t} + \frac{1}{t+2} = \frac{x}{4}$$

$$4(t+2) + 4t = x(t)(t+2)$$

$$4t + 8 + 4t = x(t^2 + 2t)$$

$$\boxed{\frac{8t+8}{t^2+2t}} = x$$

8) Let y = # of rooms painted in 8 hrs.

$$\frac{1}{x} + \frac{1}{x+1} = \frac{y}{8}$$

$$8x + 8 + 8x = y(x)(x+1)$$

$$16x + 8 = y(x^2 + x)$$

$$y = \frac{16x+8}{x^2+x}$$

9) Let T = total time
Let x = speed

$t = \frac{d}{s}$

$$T = \frac{100}{x} + \frac{200}{x+10}$$

$$T = \frac{100x + 1000 + 200x}{x(x+10)}$$

$$\boxed{T = \frac{300x + 1000}{x(x+10)} \text{ hours}}$$