Ch. 3.1 - Rational Numbers

- recall: A Rational number is any number that can be written as a fraction (with the denominator not equal to 0).

  Furthermore, a rational number can also be expressed as a terminating or a repeating decimal.

More formally now:

A real number x is rational if $x = \boxed{\frac{a}{b}}$, with a and b both being integers, and $b \neq 0$.

* when we hear "fraction", we think:

$\boxed{\frac{1}{2}, \frac{2}{3}, \frac{-5}{6}, \text{etc...}}$

but, all integers are rational because they can all be written as a fraction with a denominator of 1.

eg: $5 = \boxed{\frac{5}{1}}$, $-2 = \boxed{\frac{-2}{1}}$

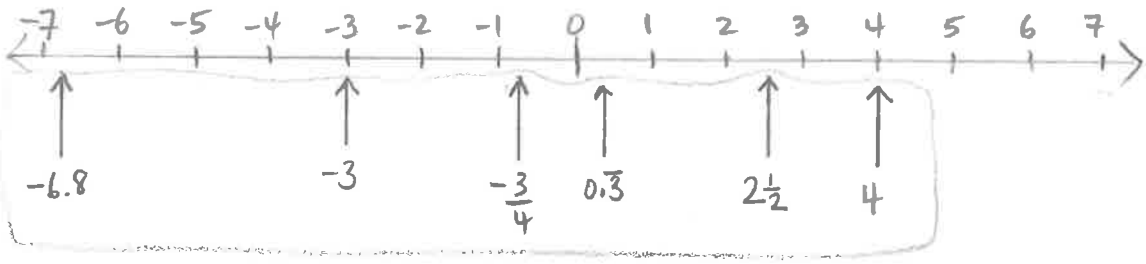
Fraction-to-Decimal examples:

$\frac{1}{4} = 0.25$ (a terminating decimal)

$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$ (a repeating decimal)

$\frac{7}{11} = 0.6363\ldots = 0.\overline{63}$ (a repeating decimal)

eg1: Given a number line, locate the following numbers: -3, 4, $-\frac{3}{4}$, $0.\overline{3}$, $2\frac{1}{2}$, -6.8



- larger numbers exist to the RIGHT of smaller numbers on the number line.

- to make a comparative statement about two numbers, we use INEQUALITY symbols:

$\boxed{<}$, when read from L to R, means "LESS THAN."

$\boxed{>}$, when read from L to R, means "GREATER THAN."

examples of usage:

$-3 < 2$ reads "negative 3 is less than 2" when read Ⓛ to Ⓡ. ← DEFAULT
(It also means "2 is greater than -3" when read Ⓡ to Ⓛ).

$-4 > -9$ reads "-4 is greater than -9"

Note: If a real number x is positive, then $\underline{x > 0}$.

If a real number x is negative, then $\underline{x < 0}$.

eg2: Use $<$ or $>$ to make the following true:

a) $3 \underline{<} 7$

c) $5 \underline{>} -8$

b) $-6 \underline{<} 3$

d) $-3 \underline{>} -5$

eg3: Use < or > to make the following true:

a) $\frac{2}{7}$ __<__ $\frac{3}{8}$ * see below

b) -4.5 __<__ $3\frac{1}{3}$ * easy

c) $\frac{22}{7}$ __>__ $-1.\overline{76}$ * easy

d) $-\frac{2}{7}$ __>__ -0.375 * see below

a) Two methods to approach:

① Give each fraction a COMMON DENOMINATOR, then compare the numerators.

L.C.D. = 56

$$\frac{2}{7} \cdot \frac{8}{8} = \frac{16}{56} \qquad \frac{3}{8} \cdot \frac{7}{7} = \frac{21}{56}$$

$16 < 21$, so $\frac{2}{7} < \frac{3}{8}$

② Change each fraction to a decimal, then compare

$$\frac{2}{7} = 0.\overline{285714} \qquad \frac{3}{8} = 0.375$$

d) Use method ②: $-\frac{2}{7} = -0.\overline{285714} > -0.375$

p. 90–93 * 1–11

Ch. 3.2 - Adding and Subtracting Rationals

Assume that a and b are rational numbers. We will consider the following expression, $a + b$, to draw some conclusions.

i) If b is positive, we move to the <u>RIGHT</u> on a number line. (this occurs regardless of <u>a's value</u>).

ii) If b is negative, we move to the <u>LEFT</u> on a number line (also, regardless of <u>a's value</u>).

iii) If b is <u>zero</u>, we stay at a. (we do NOT move).

<u>eg1:</u> Using a number line, find each of the following:

a) $-2 + 5$

Start at -2 and move 5 right:

$-2 + 5 = \boxed{3}$

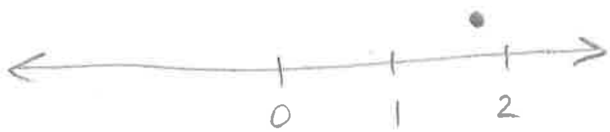
b) $4 + (-6)$

Start at 4 and move 6 LEFT:

$4 + (-6) = \boxed{-2}$

c) $1.75 + 0$

Start at 1.75 and do NOT move:



$1.75 + 0 = \boxed{1.75}$

Rules for Adding Rational Numbers:

i) Two POSITIVE numbers: add the numbers, the answer is positive.

ii) Two NEGATIVE numbers: add the numbers, the answer is negative.

iii) A POSITIVE and a NEGATIVE number:

a) If the larger number is positive, then the result of adding is positive.

b) If the larger number is negative, then the result is negative.

Rule for Subtracting Rational Numbers

For any rational numbers a and b,

$$a - b = \underline{a + (-b)}$$

so, to subtract, add the opposite of the number being subtracted.

What is opposite? $\underline{-3 \text{ is the opposite of } 3}$

eg2: Find:

a) $4 - 7 = 4 + (-7) = \boxed{-3}$

b) $-5 - 3 = -5 + (-3) = \boxed{-8}$

c) $-6 - (-8) = -6 + 8 = \boxed{2}$

d) $-4 - (-3) = -4 + 3 = \boxed{-1}$

therefore, the rule for subtractn is the same as the rule for addition.

Adding or Subtracting Decimals

- follow the same rules as we do for integers.

eg3: Will the following result in a positive or negative solution?

a) $3.42 + 2.71$

POSITIVE $\Rightarrow$ 6.13

b) $3.42 + (-2.71)$

POSITIVE $\Rightarrow$ 0.71

Also, what is an equivalent expression to this?

$3.42 - 2.71$

What is another?

$-2.71 + 3.42$

c) $-3.42 + 2.71$

NEGATIVE $\Rightarrow$ -0.71

Give two equivalent expressions:

$2.71 - 3.42$

$2.71 + (-3.42)$

d) $-3.42 - 2.71$

NEGATIVE $\Rightarrow$ -6.13

e) $3.42 - (-2.71)$

* same as a!

POSITIVE $\Rightarrow$ $3.42 + 2.71$

$= 6.13$

p. 97–100 # 1–7, 9–13

Ch. 3.3 - Adding and Subtracting Fractions

Converting MIXED FRACTIONS to IMPROPER FRACTIONS:

A mixed fraction example: $2\frac{1}{3}$

numerator

denominator

whole number

An improper fraction's numerator is <u>larger</u> than its denominator.

Steps to convert:

1. Multiply the whole number by the denominator;
2. Add the product to the numerator;
3. The sum is the 'new' numerator. Keep the same denominator.

eg1: Convert each mixed fraction to an improper fraction:

a) $4\frac{3}{5}$

$$= \frac{(4 \times 5 + 3)}{5}$$

$$= \boxed{\frac{23}{5}}$$

b) $-9\frac{2}{7}$

keep the neg. separate!

$$= -\frac{(9 \times 7 + 2)}{7}$$

$$= \boxed{\frac{-65}{7}}$$

error:

$$\frac{-9 \times 7 + 2}{7} = \frac{-61}{7}$$

X wrong!

Converting Improper Fractions to Mixed Fractions:

1. Divide the denominator into the numerator to produce the whole number;

2. Multiply the deduced whole number by the denominator and subtract this product from the numerator.

3. Write the remainder in the numerator and keep the same denominator.

eg2: Convert each improper fraction into a mixed fraction:

a) $\frac{23}{5}$

$23 \div 5 = 4$

$4 \times 5 = 20$

$23 - 20 = 3$

$\boxed{4\frac{3}{5}}$

b) $\frac{37}{6}$

$37 \div 6 = 6$

$6 \times 6 = 36$

$37 - 36 = 1$

$\boxed{6\frac{1}{6}}$

c) $-\frac{29}{3}$

Adding Fractions with Like Denominators

Steps:
1. Add the numerators
2. Keep denominator the same
3. Simplify, if possible.

eg3: Add the following:

*HINT: when adding mixed fractions, first convert to improper fractions.

a) $\frac{5}{3} + \frac{2}{3}$

$= \frac{5+2}{3} = \boxed{\frac{7}{3}}$

b) $1\frac{2}{9} + 2\frac{4}{9}$

$= \frac{11}{9} + \frac{22}{9} = \frac{33}{9} = \frac{11 \cdot 3}{3 \cdot 3} = \boxed{\frac{11}{3}}$

c) $\frac{5}{9} + \left(-\frac{7}{9}\right)$

Note: $\frac{-7}{9} = -\frac{7}{9} = \frac{7}{-9} \neq \frac{-7}{-9}$

$= \frac{5+(-7)}{9} = \frac{5-7}{9} = \boxed{\frac{-2}{9}}$

Subtracting Fractions with Like Denominators

Steps:
1. Subtract the numerators
2. Keep denominator the same
3. Simplify, if possible

eg4: Subtract the following:

a) $\frac{4}{9} - \frac{1}{9}$

$= \frac{4-1}{9} = \frac{3 \div 3}{9 \div 3} = \boxed{\frac{1}{3}}$

b) $\frac{5}{7} - \left(-\frac{4}{7}\right)$

$= \frac{5-(-4)}{7} = \frac{5+4}{7}$

$\boxed{= \frac{9}{7} \text{ or } 1\frac{2}{7}}$

c) $3\frac{1}{5} - 5\frac{3}{5}$

$= \frac{3 \cdot 5 + 1}{5} - \frac{5 \cdot 5 + 3}{5}$

$= \frac{16}{5} - \frac{28}{5}$

$\boxed{= \frac{-12}{5} \text{ or } -2\frac{2}{5}}$

Adding and Subtracting Fractions with Unlike Denominators

- requires re-writing the fractions with a COMMON denominator (preferably, the L.C.D. – Lowest Common Denominator).
  - → requires finding the L.C.M. – Least Common Multiple of all denominators involved.

Finding the LCM of a Set of Numbers

LCM - the smallest number that is a **multiple** of each number.

Prime Number - a number is considered to be PRIME if it is divisible only by **1 and itself.**

eg: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc.

Note: 1 is not prime.

Method 1 - works for two or more numbers...

1. Write each number as a product of prime factors.
2. Select the greatest number of times each prime factor occurs in any one number.
3. Multiply the values.

eg5: Determine the LCM of 24 and 36.

Using a Factor Tree

36

4×9

$2 \times 2 \quad 3 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

24

6×4

$3 \times 2 \quad 2 \times 2$

$24 = 2 \times 2 \times 2 \times 3$

$36 = 3^2 \times 2^2$

$24 = 3 \times 2^3$

$\text{LCM} = 3^2 \times 2^3$

$= 9 \times 8 = \boxed{72}$

eg6: Determine the LCM of 12, 25, and 35.

12 → 4 × 3 → 2 × 2 × 3

25 → 5 × 5

35 → 5 × 7

$2 \times 2 \times 3$ 5×5 5×7

$$\left.\begin{array}{l} 2^2 \times 3 \\ 5^2 \\ 5 \times 7 \end{array}\right\} \quad 2^2 \times 3 \times 5^2 \times 7$$

$$\boxed{= 2100}$$

eg7: Find:

a) $\frac{7}{12} + \frac{3}{5}$

$$= \frac{7 \times 5}{12 \times 5} + \frac{3 \times 12}{5 \times 12}$$

$$= \frac{35}{60} + \frac{36}{60}$$

$$= \boxed{\frac{71}{60}}$$

b) $\frac{7}{8} - \frac{1}{3}$

$$= \frac{7 \times 3}{8 \times 3} - \frac{1 \times 8}{3 \times 8}$$

$$= \frac{21}{24} - \frac{8}{24}$$

$$= \boxed{\frac{13}{24}}$$

c) $3\frac{2}{3} + 4\frac{5}{6}$

$= \frac{11}{3} + \frac{29}{6}$

$= \frac{11 \times 2}{3 \times 2} + \frac{29}{6}$

$= \frac{22}{6} + \frac{29}{6}$

$= \frac{51}{6}$

$= \frac{51 \div 3}{6 \div 3}$

$= \boxed{\frac{17}{2}}$

d) $\frac{19}{7} - \frac{13}{12}$

$= \frac{19}{7} \times \frac{12}{12} - \frac{13}{12} \times \frac{7}{7}$

$= \frac{228}{84} - \frac{91}{84}$

$= \boxed{\frac{137}{84}}$

p. 105 – 109
\# 1–18

Ch. 3.4 - Multiplying and Dividing Rational Numbers

* see p. 110 for a visual tutorial.

Multiplying Fractions

- simply multiply the numerators, multiply the denominators, and simplify (if possible).

↳ put fraction into LOWEST TERMS

eg1: Find:

a) $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \boxed{\frac{10}{21}}$

* multiplying a number by something less than one will make the number smaller.

b) $3 \times \frac{2}{7} = \frac{3}{1} \times \frac{2}{7} = \boxed{\frac{6}{7}}$

Dividing Fractions

- multiply by the reciprocal

↳ a "flipped-over" fraction

eg2: Find:

a) $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2}$

$\boxed{= \frac{15}{8}}$

* $\frac{5}{2}$ is the reciprocal of $\frac{2}{5}$ and vice versa.

b) $\frac{4}{7} \div 3 = \frac{4}{7} \div \frac{3}{1} = \frac{4}{7} \times \frac{1}{3}$

$\boxed{= \frac{4}{21}}$

eg3: Find:

a) $\frac{3}{8} \times \frac{4}{15} = \frac{3 \times 4}{8 \times 15} = \frac{12}{120} = \boxed{\frac{1}{10}}$

OR

$= \frac{3 \times 2 \times 2}{2 \times 2 \times 2 \times 5 \times 3} = \frac{\cancel{3} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 2 \times 5 \times \cancel{3}}$

$= \boxed{\frac{1}{10}}$

b) $\frac{12}{25} \times \frac{8}{15} = \frac{12 \times 8}{25 \times 15} = \frac{96}{375} = \boxed{\frac{32}{125}}$

OR

$= \frac{2 \times 2 \times \cancel{3} \times 2 \times 2 \times 2}{5 \times 5 \times \cancel{3} \times 5} = \boxed{\frac{32}{125}}$

c) $5\frac{2}{3} \times 1\frac{2}{5}$ MUST convert to improper!

$= \frac{17}{3} \times \frac{7}{5} = \boxed{\frac{119}{15}} = \boxed{7\frac{14}{15}}$

d) $5\frac{2}{3} \div 1\frac{2}{5}$

$= \frac{17}{3} \div \frac{7}{5} = \frac{17}{3} \times \frac{5}{7} = \frac{85}{21} = \boxed{4\frac{1}{21}}$

p. 113-116 # 1-8, 10 + Ch. Review