Ch. 6.1 – Linear Equations

- within the context of mathematics (including algebra), an 'equal' sign (=) is considered a 'balance' sign, in that it verifies that the expressions on either side of the equal sign are identical in value (ie. represent the same number).

Are the following equations true, false, or neither?

i) $4 + 3 = 7$ TRUE

ii) $7 - 3 = 5$ FALSE

iii) $3 \times 4 = 12$ TRUE

iv) $x + 2 = 7$ NEITHER, since x is unknown.

- replacing the variable, x, in iv with a constant that makes the equation true would SOLVE the equation.

What # + 2 = 7 ?

$\boxed{5}$ $\therefore$ $x = 5$

- we need to develop skills that allow us to isolate a variable on one side of an equation, in order to solve for it.

The Addition Principle

For any real numbers a, b, and c:

$a = b$ implies that

$a + c = b + c$

eg 1: Solve each of the following using the Addition Principle:

a) $x + 2 = 5$ * to "solve" means to solve for x

What can be added to both sides to isolate x?

-2

$x + 2 + (-2) = 5 + (-2)$

$\boxed{x = 3}$

over →

* What could be subtracted from either side to isolate x?

2

$x + 2 - 2 = 5 - 2$

$\boxed{x = 3}$

So...

$a = b$ implies that

$a - c = b - c$

Check: 'Plug' 3 in for x in the original question:

$(3) + 2 = 5$

$5 = 5$ TRUE!

b) $x - 5 = -3$

$x - 5 + 5 = -3 + 5$

$\boxed{x = 2}$ Check: $(2) - 5 = -3$

$-3 = -3$ ✓

c) $4 = 5 + x$

$4 + (-5) = 5 + x + (-5)$

$4 - 5 = 5 + x - 5$

$-1 = x$ Check: $4 = 5 + (-1)$

SAME AS:

$\boxed{x = -1}$

$4 = 5 - 1$

$4 = 4$ ✓

d) $3 - x = 5$

$3 - x + (-3) = 5 + (-3)$

$3 - x - 3 = 5 - 3$

$-x = 2$ hmm...

Multiply (or divide) both sides by -1.

Golden Rule: What you do to one side of an equation, you do to the other.

$$\frac{-x}{-1} = \frac{2}{-1}$$

$$\boxed{x = -2}$$

OR

$3 - x = 5$

$3 - x + x = 5 + x$

$3 = 5 + x$

$3 + (-5) = 5 + x + (-5)$

$3 - 5 = 5 + x - 5$

$-2 = x$

$\boxed{x = -2}$

e) $2x - 3 = x + 5$

$2x - 3 + 3 = x + 5 + 3$

$2x = x + 8$

$2x - x = x + 8 - x$

$\boxed{x = 8}$

OR

$2x - 3 = x + 5$

$2x - 3 - 5 = x + 5 - 5$

$2x - 8 = x$

$2x - 8 - 2x = x - 2x$

$-8 = -x$

$\frac{-8}{-1} = \frac{-x}{-1}$

$8 = x$

$\boxed{x = 8}$

The Multiplication Principle

For any real numbers a, b, and c:

$a = b$ implies that $a \times c = b \times c$

Likewise:

$a = b$ implies that $a \div c = b \div c$ $(c \neq 0)$

Connection?

Requires knowledge of RECIPROCALS:

Two fractions are reciprocals if their product is 1.

ie. The reciprocal of $\frac{a}{b}$ is $\boxed{\frac{b}{a}}$

where $a, b \neq 0$.

eg2: Find the reciprocal of each of the following:

a) $\frac{2}{3} \rightarrow \boxed{\frac{3}{2}}$ b) $\frac{-3}{4} \rightarrow \frac{4}{-3} = \boxed{\frac{-4}{3}}$

c) $4 = \frac{4}{1} \rightarrow \boxed{\frac{1}{4}}$ d) $\frac{-1}{2} \rightarrow \frac{2}{-1} = \boxed{-2}$

So: dividing a value by a number is the same as multiplying that value by the number's reciprocal.

eg: $3 \div \frac{3}{4} = 4$

$3 \times \frac{4}{3} = \frac{3}{1} \times \frac{4}{3} = \frac{12}{3} = 4$

eg3: Solve each of the following by the Multiplication (or Division) Principle:

a) $3x = 15$ (ie. 3 times what equals 15?)

i) Using Mult. Principle: (3 times what is 1?) (to isolate x)

$3x \times \frac{1}{3} = 15 \times \frac{1}{3}$

$\frac{3x}{3} = \frac{15}{3}$

$\boxed{x = 5}$

Check: $3(5) = 15$

$15 = 15$ ✓

ii) Using Div. Principle:

$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$

$\boxed{x = 5}$

ie. $\times \frac{1}{3}$ same as $\div 3$

b) $\frac{-2}{3}x = 6$ same as: $\left(-\frac{2}{3}x = 6\right)$ $\left(\frac{-2x}{3} = 6\right)$

i) $-\frac{2}{3}x = 6$

$-\frac{2}{3}x \times \frac{-3}{2} = 6 \times \frac{-3}{2}$

$\frac{6x}{6} = \frac{-18}{2}$

$\boxed{x = -9}$

Check: $\left(-\frac{2}{3}\right)(-9) = 6$

$\frac{18}{3} = 6$

$6 = 6$ ✓

ii) $-\frac{2}{3}x = 6$

$\frac{-\frac{2}{3}x}{\left(-\frac{2}{3}\right)} = \frac{6}{\left(\frac{-2}{3}\right)}$

$-\frac{2}{3}x \cdot \frac{-3}{2} = 6 \cdot \frac{-3}{2}$

$\boxed{x = -9}$

c) $\frac{2}{7x} = 4$

$\frac{2}{7x} \times \frac{7x}{1} = 4 \times \frac{7x}{1}$

$\frac{14x}{7x} = \frac{28x}{1}$

$2 = 28x$

$2 \times \frac{1}{28} = (28x)\left(\frac{1}{28}\right)$

$\frac{2}{28} = \frac{28x}{28}$

$\boxed{\frac{1}{14} = x}$

Check: $\frac{2}{7\left(\frac{1}{14}\right)} = 4$

$\frac{2}{\left(\frac{7}{14}\right)} = 4$

$\frac{2}{\left(\frac{1}{2}\right)} = 4$

$2 \cdot 2 = 4$ ✓

Putting It All Together

eg 4: Solve the following:

a) $5x - 4 = 2x - 1$

$5x - 4 - 2x = 2x - 1 - 2x$

$3x - 4 = -1$

$3x - 4 + 4 = -1 + 4$

$3x = 3$

$\frac{3x}{3} = \frac{3}{3}$

$\boxed{x = 1}$

Check: $5(1) - 4 = 2(1) - 1$

$5 - 4 = 2 - 1$

$1 = 1$ ✓

b) $5x - 3x + 7 = 8x + 5$

$2x + 7 = 8x + 5$

$2x + 7 - 2x = 8x + 5 - 2x$

$7 = 6x + 5$

$7 - 5 = 6x + 5 - 5$

$2 = 6x$

$\frac{6x}{6} = \frac{2}{6}$

$\boxed{x = \frac{1}{3}}$

Check:

$5(\frac{1}{3}) - 3(\frac{1}{3}) + 7$

$= 8(\frac{1}{3}) + 5$

$\frac{5}{3} - 1 + 7 = \frac{8}{3} + 5$

$\frac{5}{3} + 6 = \frac{8}{3} + 5$

$\frac{5}{3} + \frac{18}{3} = \frac{8}{3} + \frac{15}{3}$

$\frac{23}{3} = \frac{23}{3}$ ✓

p. 203 - 207 # 1-5, 7, 8

Ch. 6.1 Continued

See p. 207 #8.

a) $a = bc$

$$\frac{a}{c} = \frac{bc}{c}$$

$$\frac{a}{c} = b$$

$$\boxed{b = \frac{a}{c}}$$

b) $a = b + c$

$$a - c = b + c - c$$

$$a - c = b$$

$$\boxed{b = a - c}$$

Try on your own

Applications of Linear Equations

ie. "Word Problems"

Involves translating verbal statements into algebraic equations $\Rightarrow$ TOUGHEST part!

* must learn to recognize key phrases.

Addition

The sum of a number and three:

$$x + 3$$

Five more than a number:

$$x + 5$$

Ten added to a number:

$$x + 10$$

A number increased by four:

$$x + 4$$

Subtraction

Four less than a number:

$$x - 4$$

Eight minus a number:

$$8 - x$$

A number decreased by seven:

$$x - 7$$

The difference between six and a number:

$6 - x$

Multiplication

Seven times a number: $7 \cdot x = 7x$

Twelve percent of a number: $0.12x$

Twice a number: $2x$

The product of a number and 9: $9x$

Division

Half a number: $\frac{1}{2}x = \frac{x}{2}$

The quotient of a number and two:

$$\frac{x}{2}$$

The quotient of two and a number:

$$\frac{2}{x}$$

Do # 9 p. 207

ANSWERS to #9:

a) $x + 3 = 12$

b) $2x - 5 = 15$

c) $5x = 2x + 8$

d) $\frac{x}{3} + 2x = 10$

e) $\frac{x}{5} = 7$

f) $x + 3x = 12$

eg 1: The sum of 13 and three consecutive even numbers is 43. Find the integers.

Let x = first integer

Then next two are: $x+2$ and $x+4$

$$x + (x+2) + (x+4) + 13 = 43$$

$$3x + 19 = 43$$

$$3x + 19 - 19 = 43 - 19$$

$$\frac{3x}{3} = \frac{24}{3}$$

Check: $8 + 10 + 12 + 13 = 43$

$43 = 43$ ✓

$x = 8$

$x+2 = 10$

$x+4 = 12$

8, 10, and 12

or

Let x = middle

$x-2$

$x+2$

$$x + (x-2) + (x+2) + 13 = 43$$

$$3x = 30 \qquad x = 10$$

or

Let x = last

$x-2$

$x-4$

$$x + (x-2) + (x-4) + 13 = 43$$

$$3x - 6 + 13 = 43$$

$$3x = 36$$

$$x = 12$$

eg2: A husband is two years older than his wife. Their son is half the age of his mother. If the sum of all three of their ages is 97, how old is the son?

Let x = age of son

then $2x$ = age of mother

and $2x+2$ = age of father

$$x + 2x + (2x+2) = 97$$

$$5x + 2 = 97$$

$$5x = 95$$

$$x = 19$$ Son is 19

Check:

$19 + 38 + 40 = 97$

$97 = 97$ ✓

or: Let x = age of mother

$\frac{x}{2}$ = son

$x+2$ = father

$$x + \frac{x}{2} + (x+2) = 97$$

$$2x + 2 + \frac{x}{2} = 97$$

$$\frac{4x+4+x}{2} = 97$$

$$5x + 4 = 194$$

$$5x = 190$$

$$x = 38$$

or: x = father $x-2$ = mother

$\frac{x-2}{2}$ = son

$$x + (x-2) + \left(\frac{x-2}{2}\right) = 97$$

$$2x - 2 + \left(\frac{x-2}{2}\right) = 97$$

$$\frac{4x-4+x-2}{2} = 97$$

$$5x - 6 = 194$$

$$5x = 200$$

$$x = 40$$

eg3: A board 70 cm in length is cut into two pieces. One piece is 8 cm shorter than three times the length of the other piece. Find the length of the two pieces.

Let x = length of other piece

then $3x - 8$ = length of one piece.

} use the language

$$x + (3x - 8) = 70$$

$$4x - 8 = 70$$

$$4x = 78$$

$$x = \frac{78}{4} = \frac{39}{2} = \boxed{19\tfrac{1}{2} \text{ cm}}$$

$$3x - 8 = 3(19\tfrac{1}{2}) - 8$$

$$= 58\tfrac{1}{2} - 8 = \boxed{50\tfrac{1}{2} \text{ cm}}$$

OR

Let x = length of one piece

then $\frac{x+8}{3}$ = length of other piece

p. 208 # 10-17

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HANDOUT

Ch. 6.2 - Removing Fractions, Decimals, and Parentheses

Helpful Hints:

1. Eliminate any fraction or decimal by multiplying each term on both sides by the lowest common denominator;
2. Remove brackets by multiplication (Distributive Property);
3. Combine like terms;
4. Use the Addition Principle to get all terms with a variable to one side and all constants to the other side;
5. Combine terms;
6. Use the Multiplication / Division Principle to make the variable's coefficient 1.
7. Check answer!

eg 1: Solve $\frac{2}{3}x - \frac{1}{6} = \frac{3}{4}x$

$\frac{2x}{3} - \frac{1}{6} = \frac{3x}{4}$ LCD = 12

$\frac{12(2x)}{3} - \frac{12(1)}{6} = \frac{12(3x)}{4}$

$4(2x) - 2(1) = 3(3x)$

$8x - 2 = 9x$ (−8x)

$-2 = x$

$\boxed{x = -2}$

Check:

$\left(\frac{2}{3}\right)(-2) - \frac{1}{6} = \left(\frac{3}{4}\right)(-2)$

$\frac{-4}{3} - \frac{1}{6} = \frac{-6}{4}$

$\frac{-16}{12} - \frac{2}{12} = \frac{-18}{12}$

$\frac{-18}{12} = \frac{-18}{12}$ ✓

Another strategy:

$\frac{2}{3}x - \frac{1}{6} = \frac{3}{4}x$ LCD = 12

$3x\,?\,= 12$ → $\boxed{4}$

$4(2x) - 2(1) = 3(3x)$

eliminates 'middle' step.

eg2: Solve: $\frac{2x+5}{3} = \frac{1}{2} + \frac{x}{6}$

LCD = 6

$$\frac{6(2x+5)}{3} = \frac{6(1)}{2} + \frac{6(x)}{6}$$

$$2(2x+5) = 3 + x$$

$$4x + 10 = 3 + x \quad (-x, -10)$$

$$3x = -7$$

$$\boxed{x = \frac{-7}{3}}$$

Check:

$$\frac{2\left(\frac{-7}{3}\right)+5}{3} = \frac{1}{2} + \frac{\left(\frac{-7}{3}\right)}{6}$$

$$\frac{-\frac{14}{3}+\frac{15}{3}}{3} = \frac{1}{2} + \left(\frac{-7}{3}\right)\left(\frac{1}{6}\right)$$

$$\frac{\left(\frac{1}{3}\right)}{3} = \frac{9}{18} - \frac{7}{18}$$

$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{18}$$

$$\frac{1}{9} = \frac{1}{9} \quad \checkmark$$

eg3: Solve $0.002x + 0.05 = 0.03x - 0.006$

* convert decimals to fractions (PreCalc. vs. Science)
  ↓ fractions ↓ decimals

$$\frac{2}{1000}x + \frac{5}{100} = \frac{3}{100}x - \frac{6}{1000}$$

LCD = 1000

$$\frac{1000(2x)}{1000} + \frac{1000(5)}{100} = \frac{1000(3x)}{100} - \frac{1000(6)}{1000}$$

$$2x + 10(5) = 10(3x) - 6$$

$$2x + 50 \quad (-2x+6) = 30x - 6 \quad -2x + 6$$

OVER →

$56 = 28x$

$\frac{28x}{28} = \frac{56}{28}$

$\boxed{x = 2}$

Check:

$0.002(2) + 0.05 = 0.03(2) - 0.006$

$0.004 + 0.05 = 0.06 - 0.006$

$0.054 = 0.054$ ✓

#4: Solve: $0.09x + 0.13(x + 10) = 20$

$\frac{9}{100}x + \frac{13}{100}(x + 10) = 20$

$\frac{9x}{100} + \frac{13x}{100} + \frac{130}{100} = 20$

$\frac{22x}{100} + \frac{130}{100} = 20$

$\frac{100(22x)}{100} + \frac{100(130)}{100} = 100(20)$

$22x + 130 = 2000$ (−130 −130)

$\frac{22x}{22} = \frac{1870}{22}$

$\boxed{x = 85}$

Check:

$0.09(85) + 0.13(85 + 10) = 20$

$7.65 + 0.13(95) = 20$

$7.65 + 12.35 = 20$

$20 = 20$ ✓

p. 212 - 215 # 1-4 (3 a-d use fractions)
(4 a-d use fractions)

Word Problems

~~Ex~~ 1: A parking meter contains $27.05 in quarters and dimes. There are 146 coins in total. How many of each coin?

Let d = # of dimes (worth \$0.10)

then $146 - d$ = # of quarters (worth \$0.25)

$$(0.10)(d) + (0.25)(146 - d) = 27.05$$

$$\frac{10}{100}d + \frac{25}{100}(146 - d) = 27\frac{5}{100}$$

$$\frac{10}{100}d + \frac{3650}{100} - \frac{25d}{100} = \frac{2705}{100}$$

$$\frac{-15d}{100} + \frac{3650}{100} = \frac{2705}{100}$$

$$-15d + 3650 = 2705$$

$$\frac{-15d}{-15} = \frac{-945}{-15}$$

$$\boxed{d = 63 \text{ dimes}}$$

$$146 - d = 146 - 63 = \boxed{83 \text{ quarters}}$$

Check:

$63(0.10) = \$6.30$

$83(0.25) = \$20.75$

$\$27.05$ ✓

eg 2: Valerie bought five packages of golf balls on sale for $29.50. Each package was discounted $2.75. What is the regular price of one package of golf balls?

Let g = reg. price

$$5(g - 2.75) = 29.50$$

$$5\left(g - 2\tfrac{75}{100}\right) = 29\tfrac{50}{100}$$

$$5\left(g - 2\tfrac{3}{4}\right) = 29\tfrac{1}{2}$$

$$5\left(g - \tfrac{11}{4}\right) = \tfrac{59}{2}$$

$$5g - \tfrac{55}{4} = \tfrac{118}{4}$$

$$5g = \tfrac{173}{4}$$

$$g = \tfrac{173}{20} = 8\tfrac{13}{20} = \boxed{\$8.65}$$

Check:

$5(8.65 - 2.75) = 29.50$

$5(5.90) = 29.50$

$29.50 = 29.50$ ✓

OR...

$$4(5g) - \frac{4(55)}{4} = \frac{4(118)}{4}$$

$$20g - 55 = 118$$

$$20g = 173$$

$$\boxed{g = \tfrac{173}{20}}$$

eg3: The second angle of a triangle is three times the measure of the first angle. The third angle is 15° more than the 1st angle. Find the three angles.

HINT: angles in a Δ add to $180°$

Let x = first angle

$3x$ = 2nd

$x + 15$ = third

$$x + 3x + (x + 15) = 180$$

$$5x + 15 = 180$$

$$5x = 165$$

$$x = 33°$$

$$3x = 99°$$

$$x + 15 = 48°$$

$\Big\}$ $180°$

p. 216 #5-12

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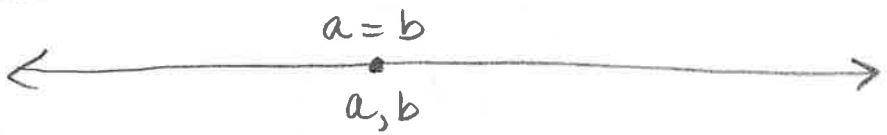
Worksheet #15-34
(p. 116-117)
(Answer Key attached)

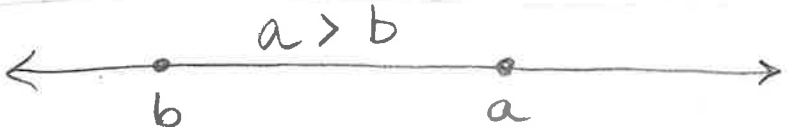
Ch. 6.3 - Linear Inequalities

Let a and b be real numbers. If we were to compare a and b in terms of their values, there exist only three relative possibilities:

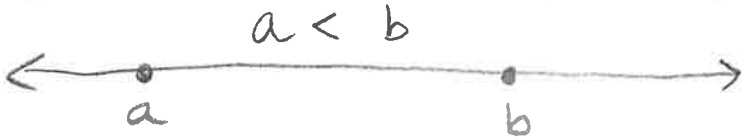
i) a is <u>equal to</u> b;



ii) a is <u>greater than</u> b;



iii) a is <u>less than</u> b;



<u>Notes:</u> In (ii), b is <u>less than</u> a.

In (iii), b is <u>greater than</u> a.

Inequality Symbols

SYMBOL	MEANING	EXAMPLE(S)
$\neq$	"not equal to"	$3 \neq 5$
$<$	"less than" (read fr. (L) to (R))	$-3 < 1$
$>$	"greater than"	$-2 > -4$
$\leqslant$	"less than or equal to"	$5 \leqslant 5$, $0 \leqslant 3$
$\geqslant$	"greater than or equal to"	$2 \geqslant -1$, $3 \geqslant 3$

Note: $=$ "equal to" $3 = 3$

* known as the EQUALITY symbol.

Graphs of Inequalities

If we are asked for a solution to the inequality $x > 2$, what is an answer?

2.1, 3, 100, π, $\sqrt{41}$, etc...

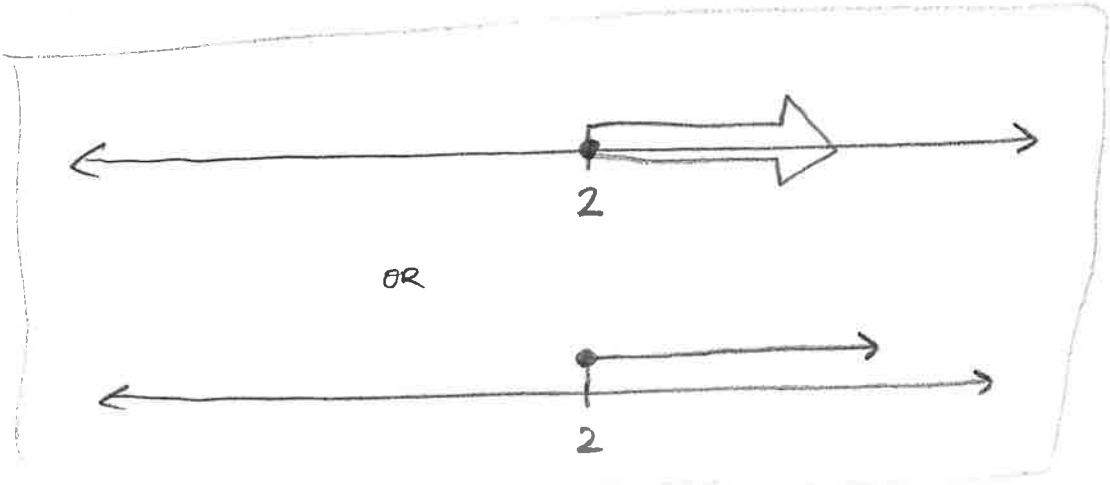
There are an infinite number of solutions!

Thus, the only way to properly represent the solution set is with a number line:

$x > 2$ OR

○ → "open" point → exclusive (non-inclusive)
(ie. point serves as a boundary but not part of solution set).

eg1: Graph solution of $x \geq 2$.



• → "closed" point → inclusive
(ie. point serves as boundary and a part of the solution set).

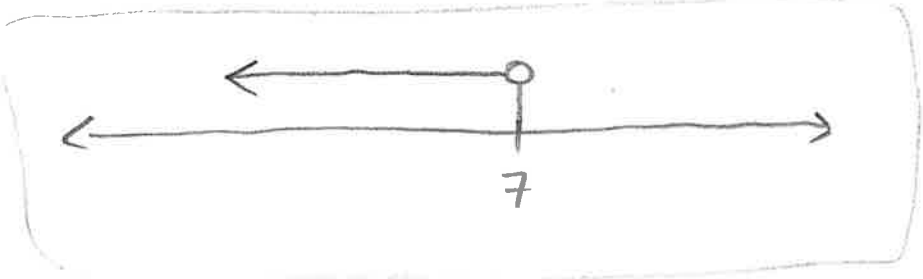
In general:

$<$, $>$ → exclusive inequalities → ○

$\leq$, $\geq$ → inclusive inequalities → •

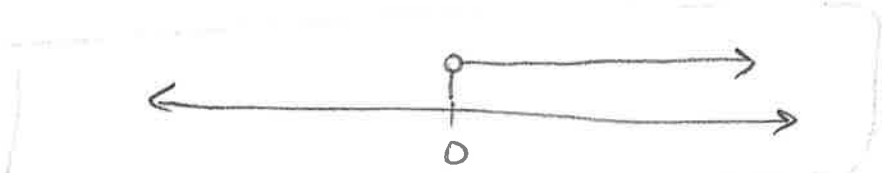
eg2: Graph the solutions to each of the following:

a) $x < 7$



b) $x \geq -4$

c) $x > 0$



d) $x \leq 97$

p. 219-221
1, 3-5.

Translating Phrases to Inequalities

Key Phrases to watch out for:

- at most
- at least
- must not exceed
- must exceed
- less than
- more than

eg3: Convert each of the following sentences into a mathematical statement:

a) At most, three students will not pass the test.

Let s = # of students that will not pass: $s \leq 3$

b) Sue is at least five years old

Let S = Sue's age: $S \geq 5$

c) A driver must not exceed 30 km/h in a school zone.

Let d = driver's speed in sch. zone $d \leq 30$

d) To make a profit, you must exceed sales of $\$800$.

Let x = sales to make profit: $x > 800$

e) Kate weighs less than 25 lbs.

Let k = Kate's weight: $k < 25$

f) Alex weighs more than 150 lbs.

Let a = Alex's weight: $a > 150$

Note: $3 < x$ reads 3 is less than x

OR

x is greater than 3

Hint: If the variable is written on the RIGHT side of an inequality, re-write it so that it is on the LEFT.

eg 4: Re-write the following so that the variable is on the LEFT:

a) $-5 > y$

$y < -5$

b) $7 \geq x$

$x \leq 7$

c) $-42 < t$

$t > -42$

d) $15 \leq p$

$p \geq 15$

eg 5: Write a mathematical inequality for each:

a) A number is more than 3

Let n = a number $\quad n > 3$

b) The cost is, at most, \$8.00.

Let C = the cost $\quad C \leq 8.00$

c) Vanessa must not work more than 4 hours.

Let V = hours Vanessa must work

$V \leq 4$

d) The value of the coffee grinder is more than $15.00.

Let c = value of grinder

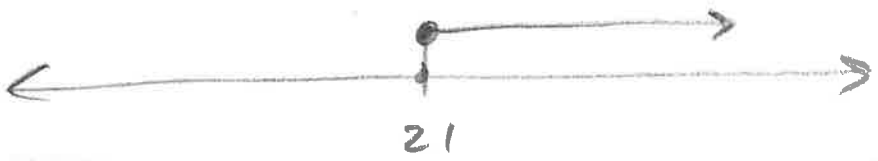
$c > 15.00$

eg6: Graph each statement on a number line:

a) Art is at least 21 years old

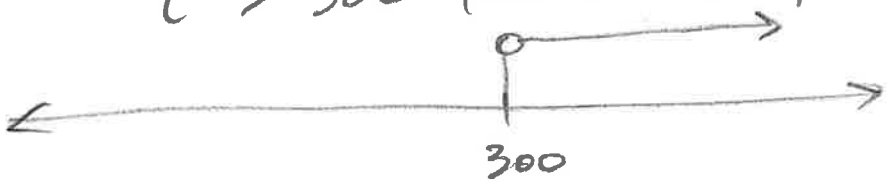
Let a = Art's age

$a \geq 21$

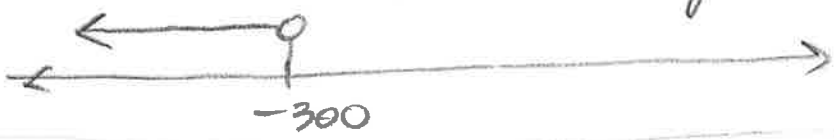


b) Tyler owes more than $300

$t > 300$ (where t = $ Tyler owes)



$t < -300$ (where t = Tyler's net worth)



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2, 6

Ch. 6.4 - Adding and Subtracting Linear Inequalities

The Addition Principle

- applies to inequalities just as it does to 'equalities' (equations).

For any real numbers a, b, and c:

$a < b$ is equivalent to:

$$a + c < b + c$$

(also: $a - c < b - c$)

$a > b$ is equivalent to:

$$a + c > b + c$$

(also: $a - c > b - c$)

* Similar statements hold true for $\leq$ and $\geq$.

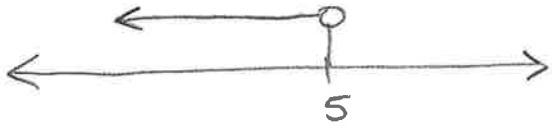
The key to determining the solution(s) to an inequality is to get the variable alone on one side (similar to solving equations).

eg 1: Solve and graph each of the following:

a) $x - 2 < 3$

$x - 2 + 2 < 3 + 2$

$\boxed{x < 5}$



Check: $4 - 2 < 3$ $\quad$ $6 - 2 < 3$

$2 < 3$ ✓ $\quad$ $4 < 3$ ×

b) $4x - 1 \geq 3x - 2$

$4x - 1 - 3x \geq 3x - 2 - 3x$

$x - 1 \geq -2$

$x - 1 + 1 \geq -2 + 1$

$\boxed{x \geq -1}$

Check: $4(0) - 1 \geq 3(0) - 2$

$-1 \geq -2$ ✓

$4(-2) - 1 \geq 3(-2) - 2$

$-9 \geq -8$ ×

Special Cases

Solve: $x < x + 1$

$$x - x < x + 1 - x$$

$$0 < 1$$

* the variables add/subtract to ZERO!
  so, evaluate the statement, $0 < 1$
  0 is ALWAYS less than 1, therefore
  any x value is a solution.

x can be any real number
all real numbers

$x \in \mathbb{R}$
"belongs to"

Solve: $x < x - 1$

$$x - x < x - 1 - x$$

$$0 < -1$$

0 is never less than -1, therefore
no x value is a solution.

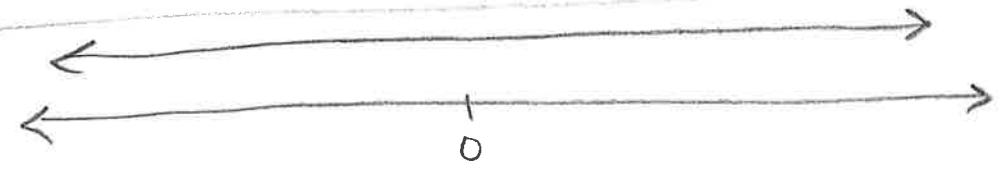
NO SOLUTION or $\emptyset$ (empty set)

$x \in \emptyset$

How to graph?

take $x < x + 1$

where $0 < 1$



$x \in \mathbb{R}$

take $x < x - 1$

where $0 < -1$

$x \in \emptyset$

CAUTION: → $2 < 3$ is identical to $3 > 2$,
but if $5 > 2$ then $-5 < -2$.

→ $2 < x$ is identical to $x > 2$,
but if $x < 3$ then $-x > -3$.

→ $y > x$ is identical to $x < y$,
but if $x < y$ then $-x > -y$.

* same for $\leq$ and $\geq$

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Ch. 6.5 - Multiplying Linear Inequalities

Dividing too!

The Multiplication Principle for Inequalities

For any real numbers a and b, and for any positive number c:

$a < b$ is equivalent to $ac < bc$

$a > b$ is equivalent to $ac > bc$

(also $a < b$ equivalent to $\frac{a}{c} < \frac{b}{c}$; $c \neq 0$

and $a > b$ equiv. to $\frac{a}{c} > \frac{b}{c}$ $c \neq 0$).

* same holds true for $\leq$ and $\geq$.

eg: $3 < 5 \longrightarrow$

$3(2) < 5(2)$

$6 < 10$ ✓

$12 > 8 \longrightarrow$

$\frac{12}{2} > \frac{8}{2}$

$6 > 4$ ✓

BUT...

For any real number a and b, and for any negative number c:

$a < b$ is equivalent to $ac > bc$

(also $a < b$ is equiv. to $\frac{a}{c} > \frac{b}{c}$)

$a > b$ is equivalent to $ac < bc$

(also $a > b$ is equiv. to $\frac{a}{c} < \frac{b}{c}$).

* same holds true for $\leq$ and $\geq$.

Thus, the multiplication rule for inequalities is not the same as the multiplication rule for equations!

eg: $3 < 7$ $\qquad$ $3 < 7$

$3(2) < 7(2)$ $\qquad$ $3(-2) < 7(-2)$

$6 < 14$ TRUE! $\qquad$ $-6 < -14$ FALSE!

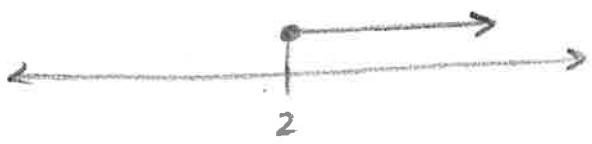
So, when multiplying or dividing an inequality by a negative number, the inequality symbol must be REVERSED!

ex 1: Solve and graph each:

a) $3x \geq 6$

$\frac{3x}{3} \geq \frac{6}{3}$

$\boxed{x \geq 2}$



b) $-3x \geq 9$

$\frac{-3x}{-3} \geq \frac{9}{-3}$ ✱ flip! Why? see below!

$\boxed{x \leq -3}$

OR

$-3x \geq 9$

$-3x + 3x \geq 9 + 3x$

$0 \geq 9 + 3x$

$0 - 9 \geq 9 + 3x - 9$

$-9 \geq 3x$

$\frac{-9}{3} \geq \frac{3x}{3}$

$-3 \geq x$

$\boxed{x \leq -3}$

Same!

c) $2 > \frac{2}{3}x$

$\frac{2x}{3} < 2$

$\frac{2x}{3} \cdot 3 < 2 \cdot 3$

$2x < 6$

$\frac{2x}{2} < \frac{6}{2}$

$\boxed{x < 3}$

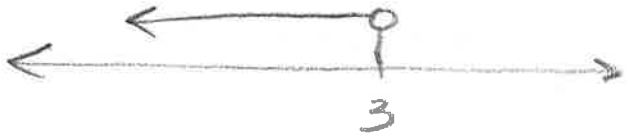
OR

$\frac{2}{3}x < 2$

$\left(\frac{2}{3}x\right)\left(\frac{3}{2}\right) < 2\left(\frac{3}{2}\right)$

$x < \frac{6}{2}$

$\boxed{x < 3}$



d) $-\frac{2}{3}x \leq 8$

$-\frac{2x}{3} \leq 8$

$-\frac{2x}{3} \cdot 3 \leq 8 \cdot 3$

$-2x \leq 24$

$\frac{-2x}{-2} \leq \frac{24}{-2}$ * flip!

$\boxed{x \geq -12}$

OR

$-\frac{2}{3}x \leq 8$

$\left(-\frac{2}{3}x\right)\left(-\frac{3}{2}\right) \geq 8\left(-\frac{3}{2}\right)$ * flip!

$x \geq -\frac{24}{2}$

$\boxed{x \geq -12}$

e) $3x - 3 \leq 4x - 2$

$3x - 3 - 4x \leq 4x - 2 - 4x$

$-x - 3 \leq -2$

$-x - 3 + 3 \leq -2 + 3$

$-x \leq 1$

$\frac{-x}{-1} \leq \frac{1}{-1}$ flip

$\boxed{x \geq -1}$

f) $-3(x+7) - 5x > 4x - 9$

$-3x - 21 - 5x > 4x - 9$

$-8x - 21 > 4x - 9$

$-8x - 21 + 8x > 4x - 9 + 8x$

$-21 > -9 + 12x$

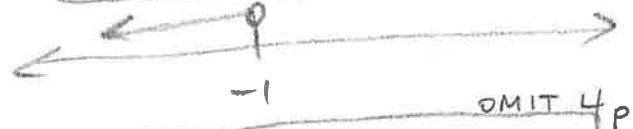
$-21 + 9 > -9 + 12x + 9$

$-12 > 12x$

$\frac{-12}{12} > \frac{12x}{12}$

$-1 > x$

$\boxed{x < -1}$



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