Chapter 9.2 - Similar Triangles

Two triangles are similar if:

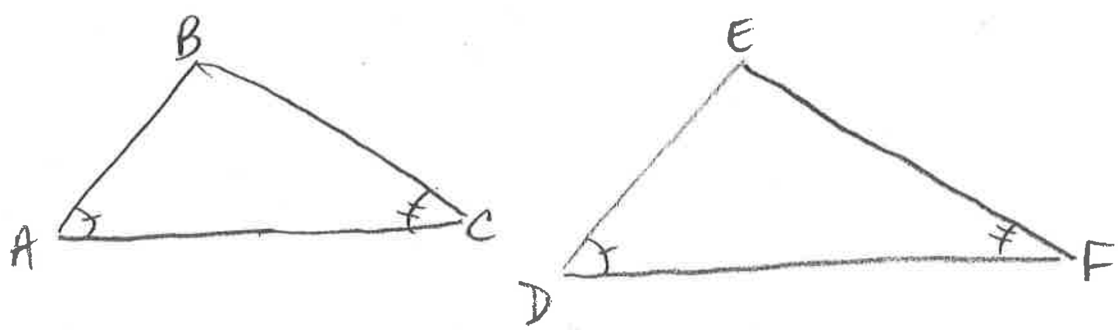
i) The corresponding angles are EQUAL,

OR

ii) The corresponding sides are PROPORTIONAL

Note: If TWO angles of one triangle are equal to the corresponding two angles of another triangle, then the triangles are SIMILAR ($\sim$).

eg:



Since $\angle A = \angle D$ and $\angle C = \angle F$, then

$\angle B$ MUST equal $\angle E$ since all angles in a Δ add to $180°$.

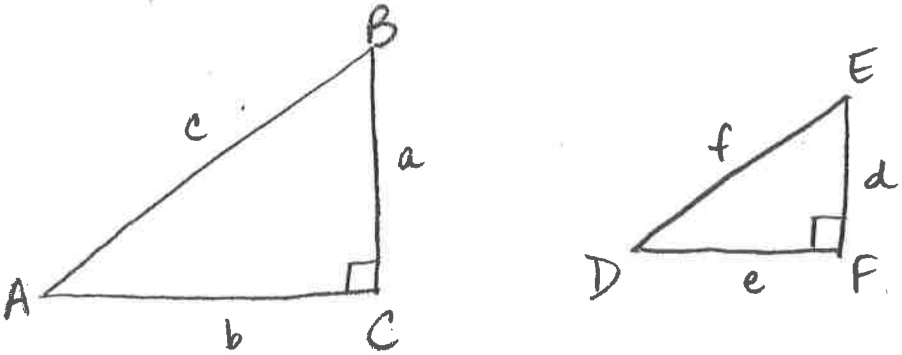
Thus $\Delta ABC \sim \Delta DEF$

Since $\triangle ABC \sim \triangle DEF$,

$$\boxed{\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}}$$

(ie. corresponding sides are <u>proportional</u>)

<u>eg1</u>: Name pairs of equal angles and equal ratios of sides in the two similar triangles below.



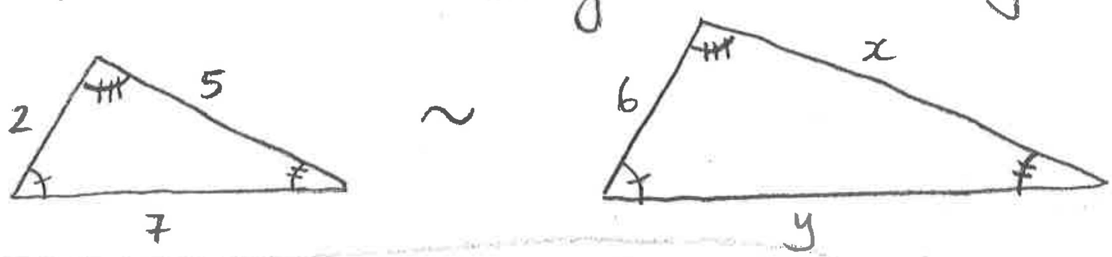
EQUAL ANGLES:

$$\boxed{\begin{aligned} \angle A &= \angle D \\ \angle B &= \angle E \\ \angle C &= \angle F \end{aligned}}$$

SIDE RATIOS:

$$\boxed{\frac{a}{d} = \frac{b}{e} = \frac{c}{f} \quad \text{and} \quad \frac{d}{a} = \frac{e}{b} = \frac{f}{c}}$$

<u>eg2</u>: Determine the length of x and y.



x: $\frac{2}{6} = \frac{5}{x}$

$2x = 30$

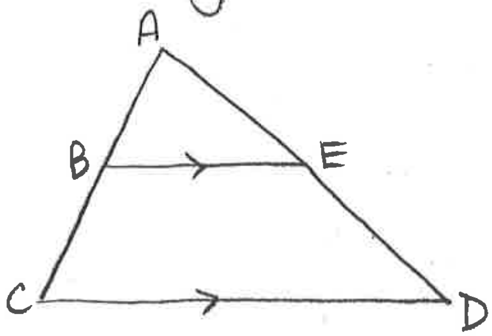
$\boxed{x = 15}$

y: $\frac{2}{6} = \frac{7}{y}$

$2y = 42$

$\boxed{y = 21}$

eg3: Name pairs of equal angles and equal ratios of sides in the two similar triangles below.



EQUAL ANGLES:

$\angle A = \angle A$

$\angle C = \angle ABE$

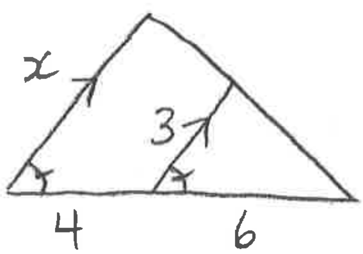
$\angle D = \angle AEB$

So... $\triangle ACD \sim \triangle ABE$

side ratios: $\dfrac{AB}{AC} = \dfrac{BE}{CD} = \dfrac{AE}{AD}$

eg4: For each of the following, calculate the value of x. Assume the two triangles are similar.

a)

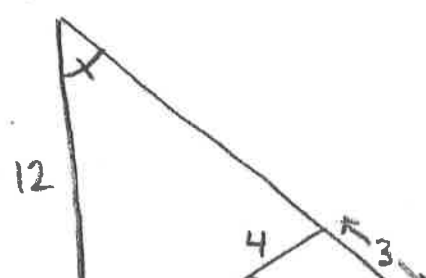


$$\frac{x}{3} = \frac{10}{6}$$

$$6x = 30$$

$$\boxed{x = 5}$$

b)



$$\frac{12}{4} = \frac{x+6}{3}$$

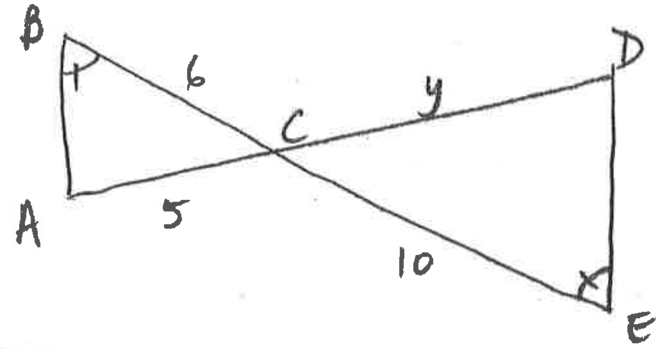
$$36 = 4(x+6)$$

$$36 = 4x + 24$$

$$12 = 4x$$

$$\boxed{x = 3}$$

eg 5: Calculate the value of y:



$\angle B = \angle E$

$\angle C = \angle C$

so, $\angle A = \angle D$

$\therefore \triangle ABC \sim \triangle DEC$

$$\frac{BC}{EC} = \frac{AC}{DC}$$

$$\frac{6}{10} = \frac{5}{y}$$

$$6y = 50$$

$$y = \frac{50}{6} = \boxed{\frac{25}{3}}$$

p. 328–332

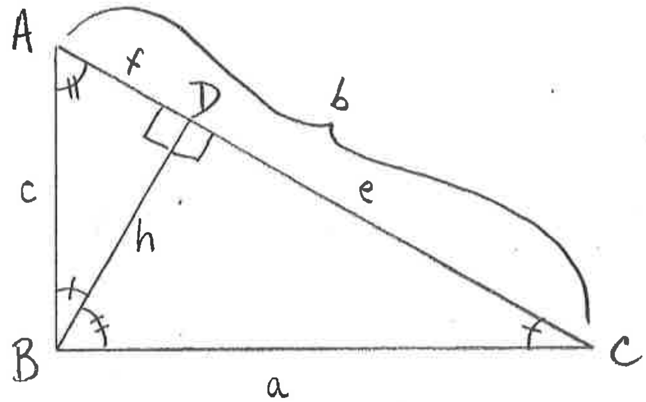
#1–3, 4 (omit c, f)

5, 6 (omit d)

7–12

Similarity Properties in Right Δs

The altitude (height) to the hypotenuse of a right triangle forms two Δs that are similar to each other, AND to the original triangle!



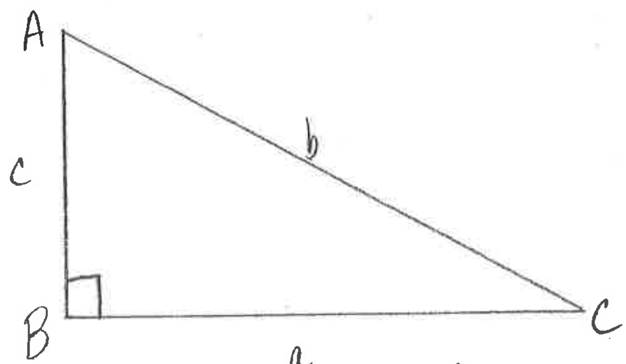
Note: $\measuredangle + \measuredangle = 90^\circ$

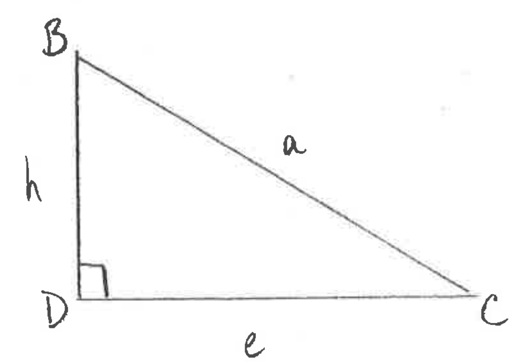
$e + f = b$

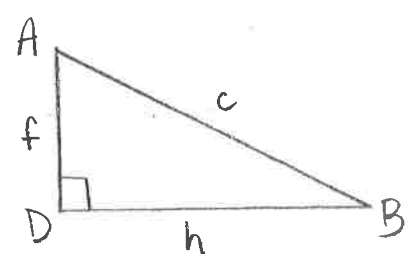
$\Delta ABC \sim \Delta BDC \sim \Delta ADB$

$$\frac{b}{c} = \frac{a}{h} = \frac{c}{f} \quad ; \quad \frac{b}{a} = \frac{a}{e} = \frac{c}{h} \quad ; \quad \frac{a}{c} = \frac{e}{h} = \frac{h}{f}$$

Help? Redraw the three Δs:

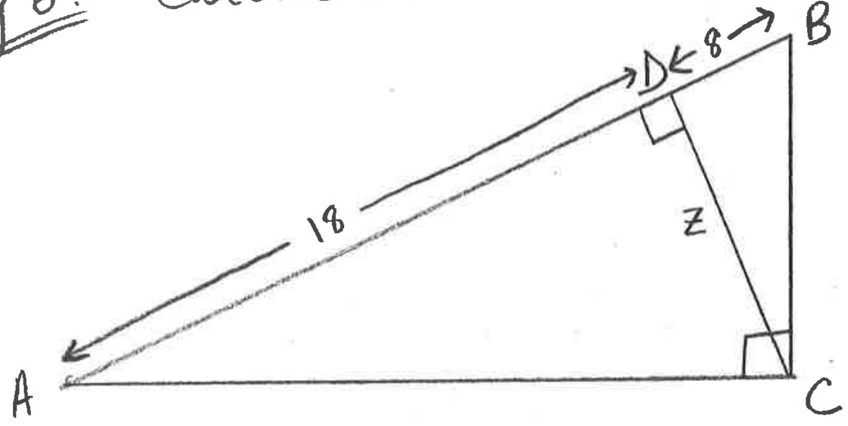






a-ha!

eg 6: Calculate the value of z.



z is an altitude (height) to a hypotenuse,

so $\triangle ABC \sim \triangle ACD \sim \triangle CBD$

$\therefore \quad \dfrac{z}{8} = \dfrac{18}{z}$

$z^2 = 144$

$\sqrt{z^2} = \sqrt{144}$

$\boxed{z = 12}$

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4f, 6d

Chapter 9.3 - Similar Polygons

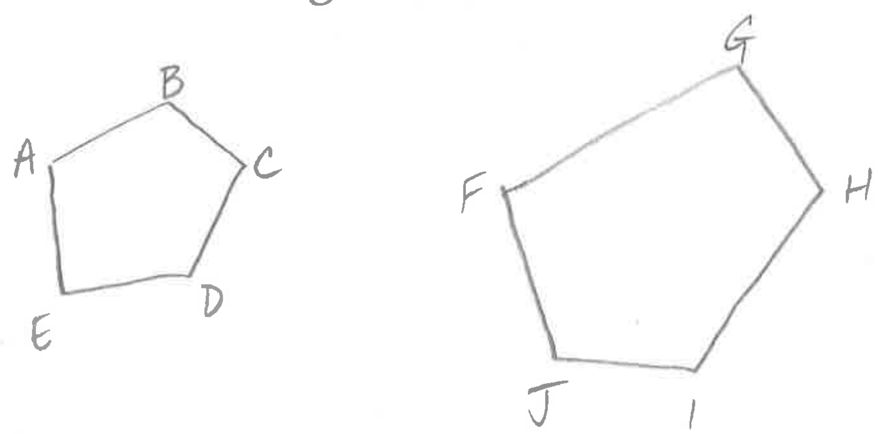
TWO polygons are SIMILAR if:

i) The corresponding angles are EQUAL;

AND

ii) The corresponding sides are PROPORTIONAL.

So, if Polygon ABCDE ~ Polygon FGHIJ



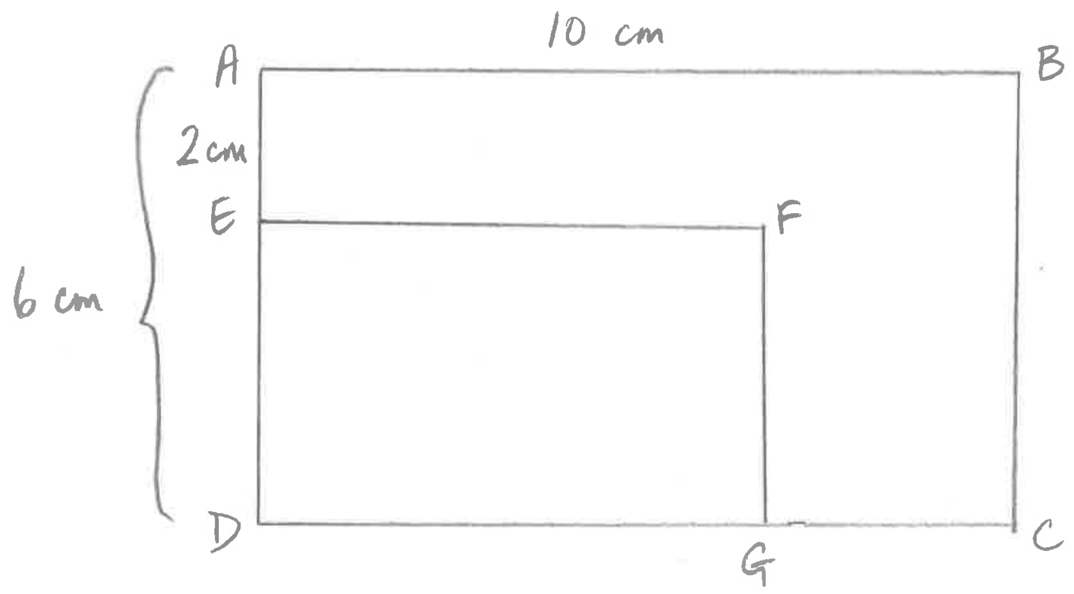
,

then $\boxed{\angle A = \angle F,\ \angle B = \angle G,\ \angle C = \angle H,\ \angle D = \angle I,\ \angle E = \angle J}$

AND

$$\frac{AB}{FG} = \frac{BC}{GH} = \frac{CD}{HI} = \frac{DE}{IJ} = \frac{EA}{JF}$$

eg 1: Given $\square ABCD \sim \square EFGD$, with $AB = 10$ cm, $AD = 6$ cm, and $AE = 2$ cm, determine EF.



$$\frac{AB}{EF} = \frac{AD}{ED}$$

$ED = 6 \text{ cm} - 2 \text{ cm}$

$ED = 4 \text{ cm}$

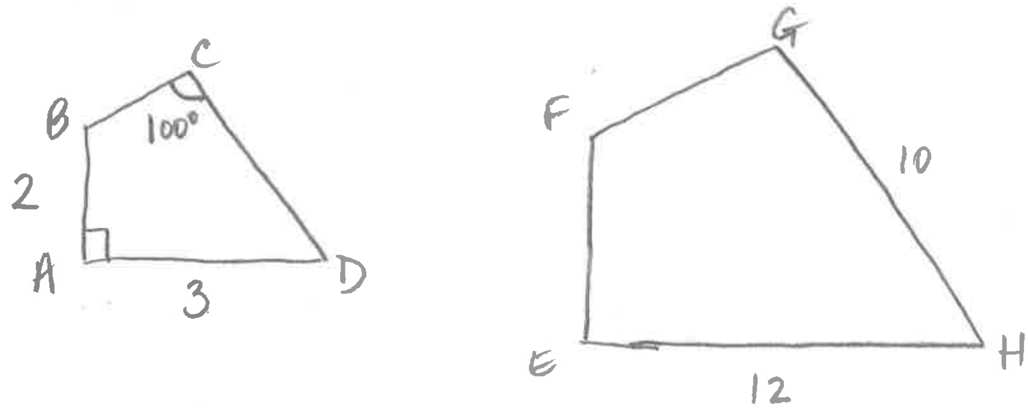
$$\frac{10 \text{ cm}}{EF} = \frac{6 \text{ cm}}{4 \text{ cm}}$$ let $EF = x$

$$\frac{10}{x} = \frac{6}{4}$$

$40 = 6x$

$$x = \frac{40}{6} = \boxed{\frac{20}{3} \text{ cm} = 6\tfrac{2}{3} \text{ cm}}$$

eg2: Given $\square ABCD \sim \square EFGH$:



Find the following:

a) $\angle E$

$\angle E$ corresponds to $\angle A$, $\therefore$ $\boxed{\angle E = 90°}$

b) $\angle G$

$\angle G$ corresponds to $\angle C$, $\therefore$ $\boxed{\angle G = 100°}$

c) EF

$$\frac{AB}{EF} = \frac{AD}{EH}$$

$$\frac{2}{EF} = \frac{3}{12}$$

$$24 = 3(EF)$$

$$\boxed{EF = 8}$$

d) CD

$$\frac{GH}{CD} = \frac{EH}{AD}$$

$$\frac{10}{CD} = \frac{12}{3}$$

$$30 = 12(CD)$$

$$CD = \frac{30}{12} = \frac{10}{4} = \boxed{\frac{5}{2}}$$

over →

e) The scale factor of image □ABCD to object □EFGH.

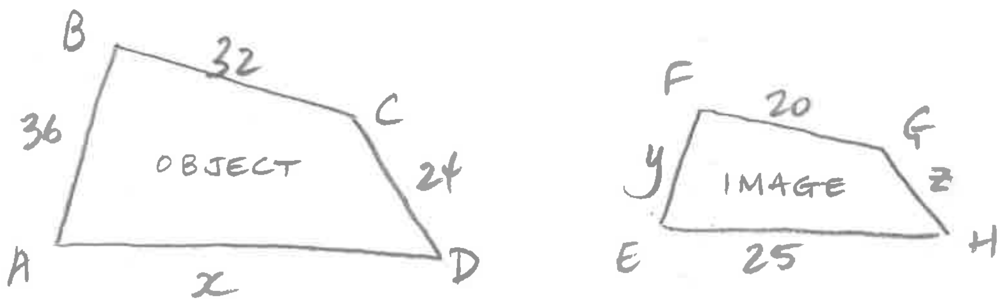
$$\text{Scale Factor} = \frac{\text{Image length}}{\text{Object length}}$$

$$= \frac{3}{12} = \boxed{\frac{1}{4}}$$

f) The scale factor of image □EFGH to object □ABCD.

4 (Reciprocal of (e)).

Q3: Given □ABCD ~ □EFGH,



Find: a) The scale factor

$$\text{Scale factor} = \frac{\text{IMAGE LENGTH}}{\text{OBJECT LENGTH}}$$

$$= \frac{20}{32} = \boxed{\frac{5}{8}}$$

Over →

b) x, y, and z

$$\frac{32}{20} = \frac{x}{25} \qquad \frac{32}{20} = \frac{36}{y}$$

$$20x = 800 \qquad 32y = 720$$

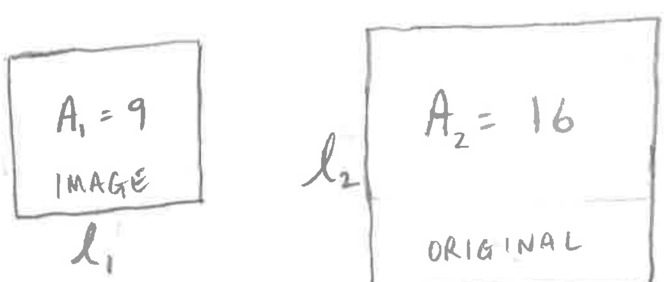
$$\boxed{x = 40} \qquad y = \frac{720}{32} = \boxed{\frac{45}{2}}$$

$$\frac{32}{20} = \frac{24}{z}$$

$$32z = 480$$

$$\boxed{z = 15}$$

eg 4: Two squares have the same shape, but different areas. If the areas have a scale factor of 9 to 16, what is the scale factor of the squares' sides?



$$\frac{l_1^2}{l_2^2} = \frac{A_1}{A_2} = \frac{9}{16} \qquad \sqrt{\left(\frac{l_1}{l_2}\right)^2} = \sqrt{\frac{9}{16}} \qquad \frac{l_1}{l_2} = \boxed{\frac{3}{4}}$$

$\boxed{\text{p.338–341 \# 1–11, 13}}$ $\qquad$ $\boxed{3 \text{ to } 4}$

Chapter 9.1 - Scale Factor

Some definitions:

POLYGON - the union of three or more line segments, such that each segment intersects exactly two others, one at each of its endpoints (vertices).

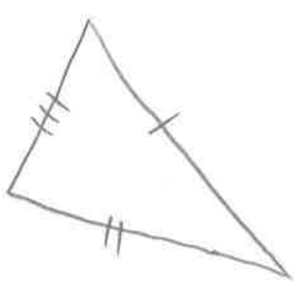
Types of Polygons

① TRIANGLE - a three-sided polygon where the sum of the interior angles is 180°

Three types: i) Scalene Δ - no sides equal.

ii) Isosceles Δ - at least two sides and two angles equal.

iii) Equilateral Δ - all three sides and angles equal.

i) 

SCALENE

ii) 

ISOSCELES

iii) 

EQUILATERAL

(2) QUADRILATERAL - a four-sided polygon where the sum of the interior angles is 360°.

Six Types: see p. 224 for descriptions

i) Trapezoid - one pair of parallel sides.

ii) Parallelogram - two pairs of parallel sides.

iii) Rhombus - four equal sides

iv) Rectangle - four equal angles

v) Square - four equal sides AND angles

vi) Kite - two distinct pairs of consecutive sides of the same length.

(3) PENTAGON - five sides - angles sum to 540°

(4) HEXAGON - six sides - angles sum to 720°

(5) HEPTAGON - seven sides - angles sum to 900°

(6) OCTAGON - eight sides - angles sum to 1080°

(7) NONAGON - nine sides - angles sum to 1260°

(8) <u>DECAGON</u> - 10 sides - angles sum to <u>1440°</u>

(9) <u>DODECAGON</u> - 12 sides - angles sum to <u>1800°</u>

<u>Similar Figures</u>

Two figures are SIMILAR if:

i) The <u>corresponding angles are equal</u>; <u>AND</u>

ii) The <u>corresponding side lengths are proportional.</u>

* we use the symbol <u>$\sim$</u> to represent 'similar.'

<u>Scale Factor</u> - applies to similar figures

- scale factor is applicable when dealing with maps, architects' plans, models of atoms, model trains, drawings of bacteria, etc.

eg: When comparing a model train to a real train, the model is SMALLER. This is called a <u>REDUCTION</u>.

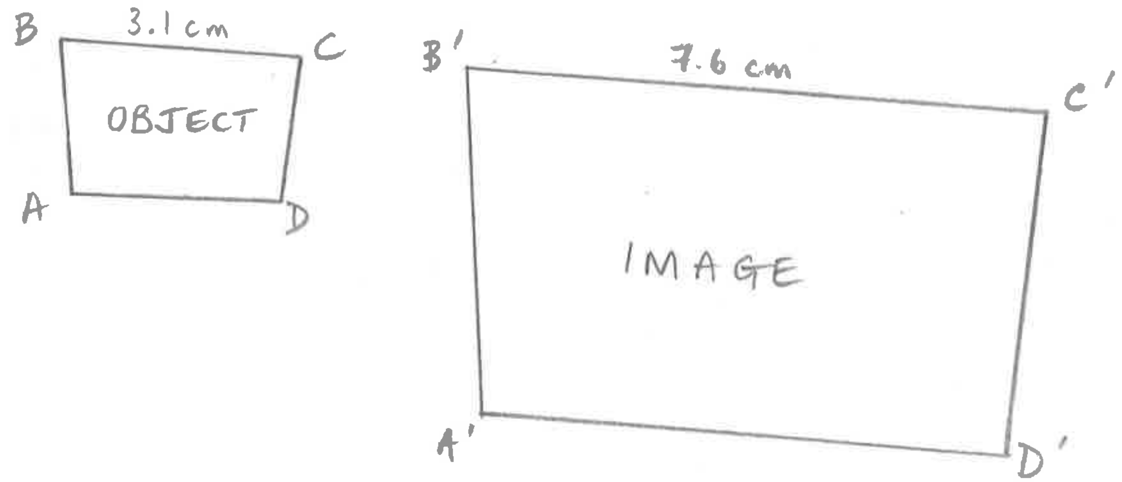
eg: When comparing the drawing of a bacterium to an actual bacterium, the drawing is LARGER. This is called an <u>ENLARGEMENT</u>.

$$\text{Scale Factor} = \boxed{\frac{\text{IMAGE (MODEL) LENGTH}}{\text{OBJECT (ACTUAL) LENGTH}}}$$

Scenarios:

1. If image smaller than object: <u>REDUCTION</u> (scale factor between <u>0 and 1</u>)
2. If image is larger than object: <u>ENLARGEMENT</u> (scale factor <u>greater than 1</u>)
3. If image and object are the same: <u>scale factor is 1</u>.

eg1: Consider the following diagrams:



a) How can scale factor be calculated?

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{D'A'}{DA} = \text{SCALE FACTOR}$$

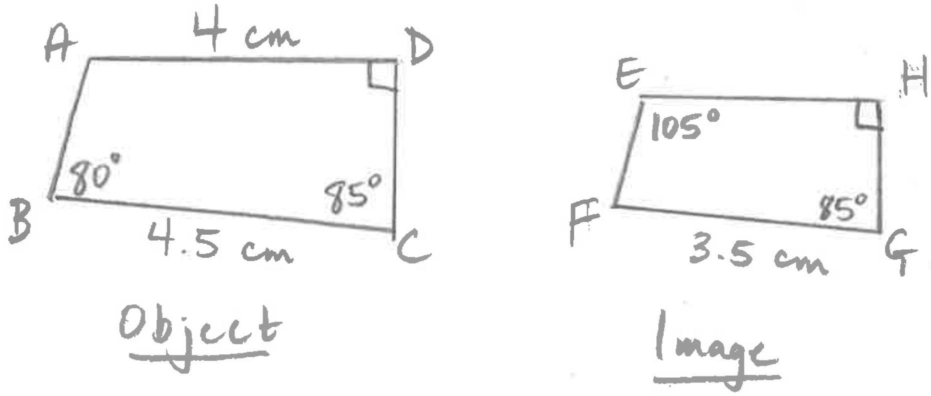
b) In this case, what scenario is represented?

An enlargement (Scale Factor > 1).

c) What is the scale factor? * nearest hundredth

$$\frac{7.6 \text{ cm}}{3.1 \text{ cm}} = \boxed{2.45}$$

eg2: Given:



a) Why is $\square ABCD \sim \square EFGH$?

$\angle A = 105°$ ($\angle$s add to 360°)

$\angle H = 80°$ (same)

$\therefore$ corresponding angles EQUAL!

b) Determine the scale factor. *nearest hundredth

$$\frac{HG}{BC} = \frac{3.5\text{ cm}}{4.5\text{ cm}} = \boxed{0.78}$$

c) Find length EH. *nearest hundredth

$$\frac{EF}{AD} = 0.78 \rightarrow EF = (AD)(0.78)$$

$$EF = 4(0.78)$$

$$\boxed{EF = 3.12\text{ cm}}$$

eg3: The distance from Vancouver to Kelowna is 385 km. A map details the distance to be 12.5 cm. What is the scale factor of the map?

$$\frac{385\text{ km}}{} \cdot \frac{1000\text{ m}}{1\text{ km}} \cdot \frac{100\text{ cm}}{1\text{ m}} = 38500000\text{ cm}$$

$$\frac{12.5\text{ cm}}{38500000\text{ cm}} = \boxed{0.000000324}$$

What would be a better way to represent it?

$$\frac{12.5\text{ cm}}{385\text{ km}} = \frac{1\text{ cm}}{x\text{ km}}$$

$$12.5x = 385$$

$$x = 30.8$$

$$\boxed{1\text{ cm} = 30.8\text{ km}}$$

p.320–323

#1–4, 7–13, 15 + 14, 16 check answers with me.

Scientific Notation

- a shorthand method of displaying very large or very small numbers.

eg: i) distance to the sun: 149 600 000 000 m

or

1.496×10^{11} m

ii) diameter of a hydrogen atom:

0.000000000106 m

or

1.06×10^{-10} m

- consists of a coefficient, a base of ten, and an exponent:

eg: 2.5×10^{4}

- the coefficient must be ≥ 1 and < 10, or else the number is not considered to be in proper scientific notation.

- if the exponent is POSITIVE, then the number is > 1
- if the exponent is NEGATIVE, then the number is < 1.

- if the exponent is 0, then the number is equal to the coefficient.
  (ie. $10^0 = 1$)

* a negative number's scientific notation allows the coefficient to be negative.
  ($-10 < \text{coefficient} \leq -1$)

BASE 10 REPRESENTATION	STANDARD
10^{12}	1 000 000 000 000 (trillion)
10^9	1 000 000 000 (billion)
10^6	1 000 000 (million)
10^5	100 000
10^4	10 000
10^3	1 000 (thousand)
10^2	100 (hundred)
10^1	10 (ten)
10^0	1
10^{-1}	0.1 (tenth)
10^{-2}	0.01 (hundredth)
10^{-3}	0.001 (thousandth)
10^{-4}	0.0001
10^{-5}	0.00001
10^{-6}	0.000001 (millionth)

Converting Standard Form to Sci. Notation

- each decimal 'jump' equals a factor 10 change.
  - if decimal moves LEFT, exponent gets LARGER
  - if decimal moves RIGHT, exponent gets SMALLER.

eg1: Convert each to scientific notation:

a) 3756

3756 3 jumps left to get 3.756 (between 1 and 10)

$$\boxed{= 3.756 \times 10^{3}}$$

b) 0.000493

0.000493 4 jumps right

$$\boxed{= 4.93 \times 10^{-4}}$$

c) 5.21

no jumps required

$$\boxed{= 5.21 \times 10^{0}}$$

Converting Sci. Notation to Standard Form

- if exponent is POSITIVE, move decimal <u>RIGHT</u>.
- if exponent is NEGATIVE, move decimal <u>LEFT</u>.

eg2: Convert each of the following to standard form:

a) 5.21×10^5 POSITIVE exponent

$5.21000. = \boxed{521000}$

b) 2.694×10^{-3} NEGATIVE exponent

$.002.694 = \boxed{0.002694}$

c) 8.01×10^1 POSITIVE exponent

$8.01 = \boxed{80.1}$

eg3: Re-write given values in proper Scientific Notation:

a) 34.79×10^3 — decimal must go LEFT, exponent ↑

$34.79 \Rightarrow \boxed{3.479 \times 10^4}$

b) 0.837×10^{-4} — decimal must go RIGHT, exponent ↓

$0.837 \rightarrow \boxed{8.37 \times 10^{-5}}$

Adding and Subtracting in Sci. Notation

- convert numbers to standard form, add or subtract, then convert back to sci. notation.

  (or, make sure exponents are equal, then +/− coefficients and adjust to sci. notation if necessary).

eg4: $(4.57 \times 10^3) + (3.4 \times 10^2)$

$4570 + 340$

$= 4910$

$= \boxed{4.91 \times 10^3}$

$(45.7 \times 10^2) + (3.4 \times 10^2)$

$45.7 + 3.4 = 49.1 \times 10^2$

$= \boxed{4.91 \times 10^3}$

Multiplying and Dividing

Multiplying

- multiply the coefficients
- ADD the exponents
- adjust if necessary

Dividing

- divide the coefficients
- SUBTRACT the exponents
- adjust if necessary..

eg 5:

$$\frac{(2.5 \times 10^3)(5.5 \times 10^4)}{(1.25 \times 10^5)}$$

$$= \frac{(2.5 \cdot 5.5) \times 10^{3+4}}{1.25 \times 10^5} = \frac{13.75 \times 10^7}{1.25 \times 10^5}$$

$$= \frac{13.75}{1.25} \times 10^{7-5} = 11 \times 10^2$$

$$= \boxed{1.1 \times 10^3}$$

Unit Conversions

- requires managing UNITS via UNIT ANALYSIS.
  ↳ eg: m, km, cm, g, L, mi., ft., etc...

- also requires knowledge and usage of CONVERSION FACTORS.
  eg: 1 km = 1000 m, 1 mi. = 5280 ft.

- refer to "Conversion Factors Info Sheet."

Examples within the SI (Metric) System:

eg1: How many cm in 5 km?

$$\frac{5\ \text{km}}{} \left|\ \frac{10^3\ \text{m}}{1\ \text{km}}\ \right|\ \frac{10^2\ \text{cm}}{1\ \text{m}} = \boxed{\begin{array}{l} 5 \times 10^5\ \text{cm} \\ = 500\,000\ \text{cm} \end{array}}$$

eg2: How many km in 4×10^5 mm?

$$\frac{4 \times 10^5\ \text{mm}}{} \left|\ \frac{1\ \text{m}}{10^3\ \text{mm}}\ \right|\ \frac{1\ \text{km}}{10^3\ \text{m}} = \boxed{\begin{array}{l} 4 \times 10^{-1}\ \text{km} \\ = 0.4\ \text{km} \end{array}}$$

eg3: How many μm in 7×10^{-3} Mm?

$$\frac{7 \times 10^{-3}\ \text{Mm}}{} \left|\ \frac{10^6\ \text{m}}{1\ \text{Mm}}\ \right|\ \frac{10^6\ \mu\text{m}}{1\ \text{m}} = \boxed{7 \times 10^9\ \mu\text{m}}$$

Conversion Factors Info Sheet

	Common Imperial	Imperial and SI	SI (Metric)
Length	1 mile = 1760 yards 1 mile = 5280 feet 1 yard = 3 feet 1 yard = 36 inches 1 foot = 12 inches	1 mile = 1.609 km 1 yard = 0.9144 m 1 foot = 30.48 cm 1 inch = 2.54 cm	1 km = 1000 m 1 m = 100 cm 1 cm = 10 mm 1 m = 1000 mm
Mass (Weight)	1 ton = 2000 pounds 1 pound = 16 oz.	2.2 pounds = 1 kg 1 pound = 454 g 1 ounce = 28.35 g	1 tonne = 1000 kg 1 kg = 1000 g 1 g = 1000 mg
Volume (Capacity)	All U.S. (not U.K.) 1 cup = 8 fluid oz. 1 pint = 2 cups 1 quart = 2 pints 1 gallon = 4 quarts	All U.S. (not U.K.) 1 gallon = 3.79 L 1 fluid oz = 29.6 mL 1 cup = 237 mL	1 L = 1000 mL 1 cL = 10 mL
Common Abbreviations	mile = mi. yard = yd. feet = ft. or ' inch = in. or " ton = tn. pound = lb. ounce = oz. cup = c. pint = pt. quart = qt. gallon = gal.		kilometer = km meter = m centimeter = cm millimeter = mm tonne (metric) = t gram = g kilogram = kg milligram = mg liter = L milliliter = mL centiliter = cL

Metric to Metric (demonstrated with length and its basic unit *m*, but can also refer to mass (*g*) or volume (*L*)):

1 Terameter (Tm) = 1000000000000m (1 trillion) = 1 x 10^{12}m

1 Gigameter (Gm) = 1000000000m (1 billion) = 1 x 10^{9}m

1 Megameter (Mm) = 1000000m (1 million) = 1 x 10^{6}m

1 kilometer (km) = 1000m = 1 x 10^{3}m

1 hectometer (hm) = 100m = 1 x 10^{2}m

1 decameter (dam) = 10m = 1 x 10^{1}m

1 meter (m) = 1m (base unit)

1 decimeter (dm) = 0.1m = 1 x 10^{-1}m

1 centimeter (cm) = 0.01m = 1 x 10^{-2}m

1 millimeter (mm) = 0.001m = 1 x 10^{-3}m

1 micrometer (μm) = 0.000001m (1 millionth) = 1 x 10^{-6}m

1 nanometer (nm) = 0.000000001m (1 billionth) = 1 x 10^{-9}m

1 picometer (pm) = 0.000000000001m (1 trillionth) = 1 x 10^{-12}m

Time:

1 min = 60 s

1 h = 60 min

1 d = 24 h

1 wk = 7 d

1 yr = 365 d

1 yr = 52 wk

1 yr = 12 mos

More examples: (round to nearest hundredth)

eg1: Convert 4.5 L to gallons:

$$\frac{4.5\text{ L}}{} \cdot \frac{1\text{ gal.}}{3.79\text{ L}} = \boxed{1.19\text{ gal.}}$$

eg2: A pipe has a 5 in. diameter. What is its diameter in mm?

$$\frac{5\text{ in.}}{} \cdot \frac{2.54\text{ cm}}{1\text{ in.}} \cdot \frac{10\text{ mm}}{1\text{ cm}} = \boxed{127\text{ mm}}$$

eg3: 1 liter of water weighs 1 kg. How many pounds does 4 L of water weigh?

If 1 L = 1 kg,
then 4L = 4 kg

$$\frac{4\text{ kg}}{} \cdot \frac{2.2\text{ lb}}{1\text{ kg}} = \boxed{8.80\text{ lb.}}$$

eg4: Convert 2 L to cups and fluid oz.
(nearest tenth oz.)

$$\frac{2\text{ L}}{} \left| \frac{10^3\text{ mL}}{1\text{ L}} \right| \frac{1\text{ c}}{237\text{ mL}} = 8.439 \text{ cups}$$

$$\frac{0.439\text{ c}}{} \left| \frac{8\text{ oz}}{1\text{ c}} \right. = 3.5 \text{ oz.}$$

$$\boxed{8 \text{ cups } 3.5 \text{ oz}}$$

eg5: Convert 6 ft. 7 in. to cm.
(nearest cm)

$$\frac{6\text{ ft}}{} \left| \frac{12\text{ in.}}{1\text{ ft}} \right. = 72 \text{ in} + 7 \text{ in} = 79 \text{ in.}$$

$$\frac{79\text{ in.}}{} \left| \frac{2.54\text{ cm}}{1\text{ in}} \right. = 200.66 \text{ cm}$$

$$= \boxed{201 \text{ cm}}$$

Double Unit Conversions:

eg1: A truck gets 30 mi./gal. How many km/L does it get?

$$\frac{30\text{ mi}}{1\text{ gal.}} \cdot \frac{1.609\text{ km}}{1\text{ mi.}} \cdot \frac{1\text{ gal.}}{3.79\text{ L}} = \boxed{12.74\text{ km/L}}$$

eg2: Change 88 ft./s to mi/h.

$$\frac{88\text{ ft.}}{1\text{ s}} \cdot \frac{1\text{ mi.}}{5280\text{ ft.}} \cdot \frac{60\text{ s}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ h}} = \boxed{60\text{ mi/h}}$$

eg3: Light travels at a rate of 3×10^8 m/s. It takes sunlight 1 hour and 18 minutes to reach Saturn. How many km separate the Sun and Saturn?

$$\frac{18\text{ min.}}{} \cdot \frac{1\text{ h}}{60\text{ min}} = 0.3\text{ h} \longrightarrow 1.3\text{ h}$$

$$\frac{1.3\text{ h}}{} \cdot \frac{60\text{ min.}}{1\text{ h}} \cdot \frac{60\text{ s}}{1\text{ min.}} \cdot \frac{3 \times 10^8\text{ m}}{1\text{ s}} \cdot \frac{1\text{ km}}{1000\text{ m}} = \boxed{1.4\text{ billion km}}$$