Ch. 1.2 - Radical Operations (part 1)

- changing integer exponents to rational exponents requires finding an <u>n^{th} root</u>.

n^{th} ROOT: If 'a' and x are real numbers and n is a positive integer, then x is an n^{th} root of 'a' if $\underline{x^n = a}$

INTRODUCTION of RADICAL SIGN:

if $x^n = a$, then $x = \boxed{a^{\frac{1}{n}}} = \sqrt[n]{a}$

index

radicand

RADICAL EXPRESSION

*Principal Square Root is ALWAYS <u>positive</u>!
(sq. rt. of non-variable positive real #)

n^{th} Root theorems: $\left(\text{using } x = \sqrt[n]{a}\right)$

i) If a is positive and n is even, then there exist <u>TWO real n^{th} roots</u>;

eg: a) $x^2 = 16$

$\sqrt{x^2} = \sqrt{16}$

$x = \sqrt{16}$ $\quad$ $x = -\sqrt{16}$

$\boxed{x = 4 \quad x = -4}$

$(-4)^2 = (4)^2 = 16$

b) $x^2 = 11$

$\sqrt{x^2} = \sqrt{11}$

$\boxed{x = \sqrt{11} \quad x = -\sqrt{11}}$

$(-\sqrt{11})^2 = (\sqrt{11})^2 = 11$

* decimals only when asked at this level.

c) $x^4 = 81$

$\sqrt[4]{x^4} = \sqrt[4]{81}$

$x = \sqrt[4]{81}$ $\quad$ $x = -\sqrt[4]{81}$

$\boxed{x = 3 \quad x = -3}$

$(-3)^4 = 3^4 = 81$

d) $x^4 = 5$

$\sqrt[4]{x^4} = \sqrt[4]{5}$

$\boxed{x = \sqrt[4]{5} \quad x = -\sqrt[4]{5}}$

b/c $(-\sqrt[4]{5})^4 = (\sqrt[4]{5})^4 = (5^{\frac{1}{4}})^4 = 5$

ii) If a is negative and n is even, then there are NO real number solutions; $x = \sqrt[n]{a}$

eg: a) $x^2 = -25$

$\sqrt{x^2} = \sqrt{-25}$

unsolvable

NO SOLUTION!

OR

$x = \emptyset$

b) $x^4 = -7$

$\sqrt[4]{x^4} = \sqrt[4]{-7}$

NO SOLUTION

OR

$x = \emptyset$

iii) If n is odd, then there is ONE real n^{th} root of a; $x = \sqrt[n]{a}$

eg: a) $x^3 = 8$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$\boxed{x = 2}$

b/c $2^3 = 8$

b) $x^3 = -8$

$\sqrt{x^3} = \sqrt{-8}$

$x = \sqrt[3]{-8}$

$\boxed{x = -2}$

b/c $(-2)^3 = -8$

c) $x^5 = -4$

$\sqrt[5]{x^5} = \sqrt[5]{4}$

$\boxed{x = \sqrt[5]{4}}$

b/c $\left(\sqrt[5]{-4}\right)^5 = \left(-4^{\frac{1}{5}}\right)^5 = -4$

iv) If a is zero, then there is ONE real n^{th} root of a, and it is 0; $x = \sqrt[n]{a}$

eg: $x^5 = 0$

$\sqrt[5]{x^5} = \sqrt[5]{0}$

$x = 0$

b/c $0^5 = 0$

Hwk: p. 9 # 1, 2, 3.

Ch. 1.2 cont'd (Part II)
Radical Properties

For any positive integer n:

i) $a^{\frac{1}{n}} = \sqrt[n]{a}$

ii) $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$

iii) $a^{-\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^{-m} = \left(\sqrt[n]{a}\right)^{-m} = \dfrac{1}{\left(\sqrt[n]{a}\right)^m}$

iv) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

v) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Square Roots of Positive Powers of x

CASE 1: When $x \geq 0$ in $\sqrt{x^n}$, with n a positive integer;

* if $x < 0$, $\sqrt{x}$ is UNDEFINED within $\mathbb{R}$.

$\boxed{\text{set of real \#s}}$

eg: $\sqrt{x^2} = \sqrt{(x \cdot x)} = x$

$\sqrt{x^3} = \sqrt{(x)(x \cdot x)} = x\sqrt{x}$

$\sqrt{x^4} = \sqrt{(x \cdot x)(x \cdot x)} = (x \cdot x) = x^2$

$\sqrt{x^5} = \sqrt{(x)(x \cdot x)(x \cdot x)} = \sqrt{x}\,(x \cdot x) = x^2\sqrt{x}$

etc...

CASE 2: When x is a real number in $\sqrt{x^n}$, with n a positive integer;

- we may require RESTRICTIONS to be placed on the expression.

- if the exponent of a radicand is even, then a NEGATIVE value will be changed into a positive value before the square root is taken;

eg: $\sqrt{(-3)^2} = \boxed{\sqrt{9} = 3}$

When variables exist in a radicand, it is not known if the variable represents a positive or a negative number. An ABSOLUTE VALUE is sometimes needed to ensure that the result is a positive number (required when x changes from an EVEN power to an ODD power).

eg: $\sqrt{x^2} = |x|$

$\sqrt{x^4} = x^2$ (no absolute value needed)

↳ even power to even power

$\sqrt{x^6} = |x^3|$

If the exponent in the radicand is ODD, then a negative value of x will make the value negative, which is undefined in $\mathbb{R}$. Therefore, $x \geq 0$ for all odd exponents;

eg: $\sqrt{x^3} = x\sqrt{x}\ ;\ x \geq 0$

$\sqrt{x^5} = x^2\sqrt{x}\ ;\ x \geq 0$

$\sqrt{x^7} = x^3\sqrt{x}\ ;\ x \geq 0$

More info:

FOR x, $a > 0$

① $\sqrt{-x}$ is <u>UNDEFINED</u> (unless $\underline{x \leq 0}$)

② $\sqrt{x} = -a$ has <u>no real solution</u>

$-\sqrt{x} = a$ has <u>no real solution</u>

③ $x^2 = -a$ has <u>no real solutions</u>

as well, through induction...

for n, a positive integer: $x^{2n} = -a$ has <u>no real solutions</u>

p. 10-12 # 4-17.

Ch. 1.3 - Simplifying Radicals

Remember: Radicals can be expressed with <u>fractional</u> exponents.

eg: $\sqrt{2} = \boxed{2^{\frac{1}{2}}}$ $\qquad \sqrt[n]{a} = \boxed{a^{\frac{1}{n}} \text{ where } n \text{ is a positive integer.}}$

$\sqrt[4]{x} = \boxed{x^{\frac{1}{4}}}$

Three Important Radical Relationships:

i) $\sqrt[n]{a^n} = \boxed{a}$; $\boxed{a \geq 0}$ $\qquad$ * this is also true for $\underline{a < 0}$ if n is an <u>ODD</u> integer.

Why? b/c $\underline{\sqrt[n]{a^n} = (a^n)^{\frac{1}{n}} = a}$

ii) $\sqrt[n]{ab} = \boxed{\sqrt[n]{a} \cdot \sqrt[n]{b}}$; $\boxed{a, b \geq 0}$ $\quad$ *

Why? b/c $\underline{\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = (a^{\frac{1}{n}})(b^{\frac{1}{n}}) = \sqrt[n]{a}\sqrt[n]{b}}$

iii) $\sqrt[n]{\frac{a}{b}} = \boxed{\frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$; $\boxed{a \geq 0,\ b > 0}$ $\quad$ *

Why? b/c $\boxed{\sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$

in notes

Simplifying Expressions comprised of Radicals $\quad$ if index is 2

- a radical is simplified when no <u>perfect square factor</u> (other than 1) remains in the radicand. $\quad \rightarrow$ 4, 9, 16 etc.

eg1: Simplify $\sqrt{20}$

$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = \boxed{2\sqrt{5}}$

OR, using prime factors:

$\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = \boxed{2\sqrt{5}}$

eg2: Simplify $\sqrt[3]{24}$ * look for perfect cube factors 8, 27, 64, etc...

$$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{8} \cdot \sqrt[3]{3} = \boxed{2\sqrt[3]{3}}$$

OR, using prime factors:

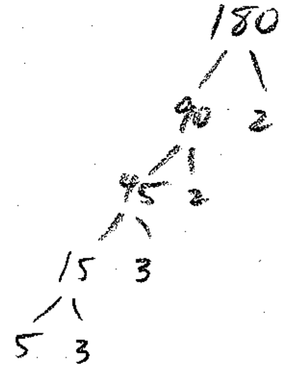
$$\sqrt[3]{24} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3} = \boxed{2\sqrt[3]{3}}$$

eg3: Simplify $\sqrt{180x^6y^3}$; $x, y \geq 0$

$$= \sqrt{(5 \cdot 3 \cdot 3 \cdot 2 \cdot 2)x^6y^3}$$

$$= 3 \cdot 2\sqrt{5} \quad x^3 y\sqrt{y}$$

$$= \boxed{6x^3y\sqrt{5y}}$$



eg4: Simplify $\sqrt[3]{\dfrac{x^{12}}{64}}$

$$= \frac{(x^{12})^{\frac{1}{3}}}{\sqrt[3]{64}} = \boxed{\frac{x^4}{4}}$$

Changing Mixed Radicals to Entire Radicals

* the reverse of simplifying!

eg5: Write $3\sqrt{5}$ as an entire radical

$$3\sqrt{5} = \sqrt{3 \cdot 3 \cdot 5} = \boxed{\sqrt{45}}$$

eg6: Write $-4x^2\sqrt{5x^3}$; $x \geq 0$ as an entire rad.

stays outside

$$= -\sqrt{(4x^2)(4x^2)(5x^3)}$$

$$\boxed{= -\sqrt{80x^7}}$$

eg7: Write $2xy\sqrt[3]{4x^2y^3}$; $x, y \geq 0$ as an entire rad.

$$= \sqrt[3]{(2xy)(2xy)(2xy)(4x^2y^3)}$$

$$\boxed{= \sqrt[3]{32x^5y^6}}$$

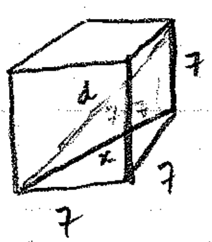
eg8: Write $\frac{3x^2y}{5}\sqrt[3]{2xy^2}$; $x, y \geq 0$ as an entire rad

$$= \sqrt[3]{\left(\frac{3x^2y}{5}\right)\left(\frac{3x^2y}{5}\right)\left(\frac{3x^2y}{5}\right)(2xy^2)}$$

$$= \sqrt[3]{\frac{54x^7y^5}{125}}$$

Hwk: Qs 1 – 16 p. 16 – 20 → Qs 9, 11, 13, 14 testable on unit test

eg9 ?? Find length of diagonal of a cube with side lengths of 7 units.



$7^2 + 7^2 = x^2$

$98 = x^2$

$x^2 + 7^2 = d^2$

$98 + 49 = d^2$

$147 = d^2$

$\boxed{d = 7\sqrt{3}}$

Ch. 1.4 - Adding and Subtracting Radical Expressions

- the concept "combining like terms", learned in the context of simplifying variable expressions, is also applicable when we add/subtract radicals.

  - 'like' radicals have the same radicand and index.

eg1: Simplify $\sqrt{27} + \sqrt{12} - \sqrt{8}$

* see if like radicals can be found:

$$= 3\sqrt{3} + 2\sqrt{3} - 2\sqrt{2}$$

$$\boxed{= 5\sqrt{3} - 2\sqrt{2}}$$

eg2: Simplify $\sqrt{27xy} + \sqrt{8xy}$

$$\boxed{= 3\sqrt{3xy} + 2\sqrt{2xy}}$$ * that's it!

eg3: Simplify $-3\sqrt{12} + 4\sqrt{75}$

$$= -3 \cdot 2\sqrt{3} + 4 \cdot 5\sqrt{3}$$

$$= -6\sqrt{3} + 20\sqrt{3}$$

$$\boxed{= 14\sqrt{3}}$$

eg4: Simplify $4\sqrt[3]{16} + 3\sqrt[3]{54}$

$$= 4\cdot 2\sqrt[3]{2} + 3\cdot 3\sqrt[3]{2}$$
$$= 8\sqrt[3]{2} + 9\sqrt[3]{2}$$
$$\boxed{= 17\sqrt[3]{2}}$$

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eg5: Simplify $3x\sqrt{63y} - 5\sqrt{28x^2y}$; $x, y \geq 0$

$$= (3x)(3)\sqrt{7y} - 5(2x)\sqrt{7y}$$

* don't worry about restrictions!

$$= 9x\sqrt{7y} - 10x\sqrt{7y}$$
$$\boxed{= -x\sqrt{7y}}$$

eg6: Simplify $\sqrt{20x^2y} - 2\sqrt{45y^3}$; $x, y \geq 0$

$$= 2x\sqrt{5y} - (2)(3)(y)\sqrt{5y}$$
$$= 2x\sqrt{5y} - 6y\sqrt{5y}$$
$$\boxed{= (2x - 6y)\sqrt{5y}}$$

eg7: Simplify $\sqrt{32x} + \sqrt{48y} - \sqrt{50x} + \sqrt{27y}$

$$= 4\sqrt{2x} + 4\sqrt{3y} - 5\sqrt{2x} + 3\sqrt{3y}$$
$$\boxed{= -\sqrt{2x} + 7\sqrt{3y}}$$

eg8: Simplify $\frac{5}{2}\sqrt[3]{16x^4y^5} - xy\sqrt[3]{54xy^2}$; $x, y \geq 0$

$$= \left(\frac{5}{2}\right)(2)(x)(y)\sqrt[3]{2xy^2} - (xy)(3)\sqrt[3]{2xy^2}$$
$$= 5xy\sqrt[3]{2xy^2} - 3xy\sqrt[3]{2xy^2}$$
$$\boxed{= 2xy\sqrt[3]{2xy^2}}$$

Hwk: p. 23–27 #1–9.

Ch. 1.5 - Multiplying and Dividing Radical Expressions

- when multiplying/dividing radical expressions, the coefficients and radicals are $\times/\div$ separately.

eg1: Multiply $2\sqrt{6} \cdot 5\sqrt{3}$

$$= (2\cdot 5)(\sqrt{6}\cdot\sqrt{3})$$
$$= 10\sqrt{6\cdot 3}$$
$$= 10\sqrt{18}$$
$$= 10(3)\sqrt{2}$$
$$\boxed{= 30\sqrt{2}}$$

eg2: Multiply $-3\sqrt{2x} \cdot 4\sqrt{3x}$; $x \geq 0$

$$= -12\sqrt{6x^2}$$
$$\boxed{= -12x\sqrt{6}}$$

eg3: Multiply $(2\sqrt{3} - 3\sqrt{2})(2\sqrt{3} + \sqrt{2})$

* many methods can be used here... see pp. 28-29

FOIL, distributive, vertical, rect.

using FOIL $= 4\sqrt{9} + 2\sqrt{6} - 6\sqrt{6} - 3\sqrt{4}$

$$= 12 - 4\sqrt{6} - 6$$
$$= \boxed{6 - 4\sqrt{6}}$$

eg4: Multiply $(\sqrt[3]{x} + \sqrt[3]{y})(\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2})$

using distr. meth. $= \sqrt[3]{x^3} - \cancel{\sqrt[3]{x^2y}} + \cancel{\sqrt[3]{xy^2}} + \cancel{\sqrt[3]{x^2y}} - \cancel{\sqrt[3]{xy^2}} + \sqrt[3]{y^3}$

$$\boxed{= x + y}$$

Multiplying and Dividing Terms with Different Indices:

Two APPROACHES:

① Convert all radical expressions to exponential notatn.

* offers more rad. practice!

② Use the radical property:

$$\sqrt[a]{x^b} = \sqrt[ac]{x^{bc}}$$

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eg5: Simplify $\sqrt[4]{x^2}$; $x \geq 0$

Method ①: $= (x^2)^{\frac{1}{4}} = x^{\frac{2}{4}}$

$= x^{\frac{1}{2}}$

$\boxed{= \sqrt{x}}$

Method ②: Find c such that 4 becomes 2: $c = \frac{1}{2}$

$= \sqrt[\frac{4}{2}]{x^{\frac{2}{2}}} = \sqrt{x^1} \boxed{= \sqrt{x}}$

eg6: Multiply $\sqrt{x^3} \cdot \sqrt[3]{x}$; $x \geq 0$

Method ①:

$= (x^3)^{\frac{1}{2}} \cdot x^{\frac{1}{3}}$

$= x^{\frac{3}{2}} \cdot x^{\frac{1}{3}}$

$= x^{\left(\frac{3}{2} + \frac{1}{3}\right)}$

$= x^{\frac{11}{6}}$

$= \sqrt[6]{x^{11}}$

$\boxed{= x\sqrt[6]{x^5}}$

Method ②: find c to make indices equal

$= \sqrt[2\cdot 3]{x^{(3\cdot 3)}} \cdot \sqrt[3\cdot 2]{x^{(1\cdot 2)}}$

$= \sqrt[6]{x^9} \cdot \sqrt[6]{x^2}$

$= \sqrt[6]{x^{11}}$

$\boxed{= x\sqrt[6]{x^5}}$

eg7: Divide $\dfrac{\sqrt{(2x-1)^4}}{\sqrt[4]{(2x-1)^3}}$, $x \geq 0$ * restriction: $x > \frac{1}{2}$ next section!

Method ①:

$= \dfrac{\left((2x-1)^4\right)^{\frac{1}{2}}}{\left((2x-1)^3\right)^{\frac{1}{4}}} = \dfrac{(2x-1)^2}{(2x-1)^{3/4}} = (2x-1)^{\left(2 - \frac{3}{4}\right)}$

$= (2x-1)^{\frac{5}{4}} = \sqrt[4]{(2x-1)^5}$

$\boxed{= (2x-1)\sqrt[4]{2x-1}}$

Hwk: p. 33–35 # 1ab – 5.

Ch. 1.5 cont'd

Rationalizing the Denominator

- the process of changing the denominator from an IRRATIONAL radical number to a rational number.
- never leave an answer with a radical in the denominator! * unless indicated.

eg 8: Simplify $\sqrt{\frac{2}{7}}$

$$= \frac{\sqrt{2}}{\sqrt{7}}$$

$$= \frac{\sqrt{2}}{\sqrt{7}} \cdot \left(\frac{\sqrt{7}}{\sqrt{7}}\right)$$

same as multiplying by 1 ⇒ does NOT change the value of the expression

$$\boxed{= \frac{\sqrt{14}}{7}}$$

eg 9: Simplify $\sqrt[3]{\frac{2}{y}}$; $y \neq 0$

$$= \frac{\sqrt[3]{2}}{\sqrt[3]{y}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}}$$

$$\boxed{= \frac{\sqrt[3]{2y^2}}{y}}$$

eg 10: Simplify $\frac{\sqrt[4]{5x}}{\sqrt[4]{10x^3}}$; $x > 0$

⇒ can be simplified before rationalizing denominator!

$$= \sqrt[4]{\frac{5x}{10x^3}} = \sqrt[4]{\frac{1}{2x^2}} = \frac{1}{\sqrt[4]{2x^2}} = \frac{1}{\sqrt[4]{2x^2}} \cdot \left(\frac{\sqrt[4]{8x^2}}{\sqrt[4]{8x^2}}\right)$$

$$= \frac{\sqrt[4]{8x^2}}{\sqrt[4]{16x^4}} = \boxed{\frac{\sqrt[4]{8x^2}}{2x}} \quad x > 0$$

$$\boxed{= \frac{\sqrt[4]{8}\,\sqrt{x}}{2x}}$$

both fine.

Using Conjugates to Rationalize a Denominator

The expressions $(a+b)$ and $(a-b)$ are called CONJUGATES.

- when conjugates are multiplied, a difference of two squares is produced, which represents a RATIONAL number.

eg 11: Simplify $\frac{3}{2-\sqrt{5}}$

$$= \frac{3}{2-\sqrt{5}} \cdot \frac{(2+\sqrt{5})}{(2+\sqrt{5})}$$

$$= \frac{6+3\sqrt{5}}{4-5}$$

$$= \frac{6+3\sqrt{5}}{-1}$$

$$\boxed{= -6-3\sqrt{5}} \overset{\text{or}}{=} \boxed{-3(2+\sqrt{5})}$$

eg 12: Simplify $\frac{\sqrt{x}-2}{\sqrt{x}+1}$; $x \geq 0$

$$= \frac{\sqrt{x}-2}{\sqrt{x}+1} \cdot \frac{(\sqrt{x}-1)}{(\sqrt{x}-1)} = \frac{x-2\sqrt{x}-\sqrt{x}+2}{x-1}$$

$$\boxed{= \frac{x-3\sqrt{x}+2}{x-1}}$$

R: $x \neq 1$

eg 13: Simplify: $\dfrac{\sqrt{a}+\sqrt{2b}}{\sqrt{a}-\sqrt{2b}}$; $a, b > 0$

$$= \frac{\sqrt{a}+\sqrt{2b}}{\sqrt{a}-\sqrt{2b}} \cdot \frac{(\sqrt{a}+\sqrt{2b})}{(\sqrt{a}+\sqrt{2b})}$$

$$= \frac{a+\sqrt{2ab}+\sqrt{2ab}+2b}{a-2b}$$

$$\boxed{= \frac{a+2\sqrt{2ab}+2b}{a-2b}}$$

p. 33 – 39 # 1c–e, 5–17

Ch. 1.6 - Radical Equations

- since it is not possible to find the square root of a <u>negative</u> number (within $\mathbb{R}$), expressions with a variable in the radicand will have <u>restrictions</u>.

eg1: Determine the restriction on x in

$$\sqrt{2x-3} = x-3$$

$2x-3$ cannot be negative, so...

$$2x-3 \geq 0$$

$$2x \geq 3$$

$$\boxed{x \geq \frac{3}{2}} \Leftarrow \text{restriction}$$

eg2: Determine the restriction(s) on x in

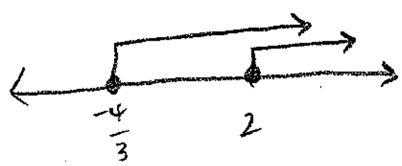
$$\sqrt{3x+4} - \sqrt{2x-4} = 2$$

$3x+4 \geq 0$ $2x-4 \geq 0$

$3x \geq -4$ $2x \geq 4$

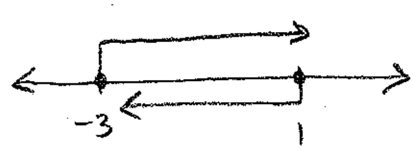
$x \geq \frac{-4}{3}$ $x \geq 2$ ✓

BOTH must be true



eg3: Determine the restriction(s) on x in

$$\sqrt{1-x} + \sqrt{x+3} = 4$$



$1-x \geq 0$ $x+3 \geq 0$

$-x \geq -1$ $x \geq -3$

$x \leq 1$ BOTH must be true, so...

$$\boxed{-3 \leq x \leq 1}$$

Solving Radical Equations

- goal is to rid the equation of radicals.
  * use the principle:

$$\text{If } a = b, \text{ then } a^2 = b^2$$

However, sometimes this "squaring of both sides" might introduce an EXTRANEOUS solution (a solution that does NOT satisfy the original equation), so you must ALWAYS check your answer(s)!

eg: If $x = 3$, then $x^2 = 3^2$ and $x^2 = 9$

$$x = \pm 3$$

$x = -3$ is EXTRANEOUS.

eg 4: Solve $\sqrt{x+1} = x - 1$ Restriction? $x \geq -1$

$$(\sqrt{x+1})^2 = (x-1)^2$$

$$x + 1 = x^2 - 2x + 1$$

$$0 = x^2 - 3x$$

$$0 = x(x-3)$$

$$\boxed{x = 0, 3}$$ BUT! check!

extr. true

$$\boxed{x = 3}$$

eg5: Solve $\sqrt{3-3x} = 3 + \sqrt{3x+2}$

Restr.?

$3-3x \geq 0$

$-3x \geq -3$

$x \leq 1$

$3x+2 \geq 0$

$x \geq \frac{-2}{3}$

$$\left(\sqrt{3-3x}\right)^2 = \left(3 + \sqrt{3x+2}\right)^2$$

$$3-3x = 9 + 6\sqrt{3x+2} + (3x+2)$$

$$3-3x = 11 + 3x + 6\sqrt{3x+2}$$

$$\frac{-8-6x}{2} = \frac{6\sqrt{3x+2}}{2}$$

$$(-4-3x)^2 = \left(3\sqrt{3x+2}\right)^2$$

$$16 + 24x + 9x^2 = 9(3x+2)$$

$$16 + 24x + 9x^2 = 27x + 18$$

x	+
-18	-3

$6, -3$

$$9x^2 - 3x - 2 = 0$$

$$9x^2 + 6x - 3x - 2 = 0$$

$$3x(3x+2) - 1(3x+2) = 0$$

$$(3x+2)(3x-1) = 0$$

$x = \cancel{-\frac{2}{3}}, \cancel{\frac{1}{3}}$ both extraneous!

$x = \phi$

eg6: Solve $\sqrt{2x+5} - \sqrt{x-1} = 2$

Rs?

$x \geq -\frac{5}{2}$

$x \geq 1$

$$\left(\sqrt{2x+5}\right)^2 = \left(2 + \sqrt{x-1}\right)^2$$

$$2x+5 = 4 + 4\sqrt{x-1} + (x-1)$$

$$2x+5 = 3 + x + 4\sqrt{x-1}$$

$$x+2 = 4\sqrt{x-1}$$

$$(x+2)^2 = \left(4\sqrt{x-1}\right)^2$$

$$x^2+4x+4 = 16(x-1)$$

$$x^2+4x+4 = 16x-16$$

$$x^2-12x+20 = 0$$

$$(x-10)(x-2) = 0$$

$\boxed{x = 10, 2}$ check!

p. 43 – 45 # 1–9

p. 283 # 6

Ch. Review p. 46 – 48 # 1–9