Ch. 2.1 – Properties of Rational Expressions

- a RATIONAL EXPRESSION is the quotient of two polynomials with the denominator not equal to zero.

  note: even a CONSTANT, by itself, is a polynomial.

- the DOMAIN of a rational expression is the set of $\mathbb{R}$ that can be used in place of the variable.
  - the domain is usually written in a form specifying what value(s) the variable cannot be.

- the denominator cannot equal zero or the expression is undefined

eg1: Determine the undefined values for x: then...
($\Rightarrow$ determine the restrictions on x
(ie: find the DOMAIN of x):

a) $\frac{2}{x}$; $\boxed{x = 0}$

$x \neq 0$

b) $\frac{x+2}{x-3}$; $x - 3 = 0$ $\boxed{x = 3}$

$x \neq 3$

c) $\frac{1}{x^2-9}$; $x^2 - 9 = 0$

$x^2 = 9$

$\boxed{x = \pm 3}$

$x \neq \pm 3$

d) $\frac{x-3}{x^2+9}$; $x^2 + 9 = 0$

$x^2 = -9$

$\boxed{x = \emptyset}$

no restrictions

$x \in \mathbb{R}$

To simplify a rational expression:

1. Completely FACTOR the numerator and denominator.
2. Divide both numerator and denominator by all common factors.

BUT: Restrictions are based on the original expression(s), not the simplified expression(s)

COMMON ERROR: only common FACTORS may be 'cancelled', not common TERMS!

eg: $\dfrac{\cancel{x}+1}{\cancel{x}}$; here, x is a common term

$\dfrac{\cancel{x^2}+x+\cancel{3}}{\cancel{x^2}+\cancel{3}}$; $\dfrac{\cancel{x}-2}{\cancel{x}+1}$; $\dfrac{\cancel{2}\cancel{x^2}^{x}-\cancel{9}^{3}}{\cancel{2}\cancel{x}-\cancel{3}}$

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eg2: Simplify $\dfrac{3x-3}{6x-6}$

$$= \frac{3(x-1)}{6(x-1)}$$

$$= \frac{3}{6} = \boxed{\frac{1}{2}}$$

$6x-6=0$

$6x=6$

$\boxed{x \neq 1}$ → RESTRICTION

but

*based upon ORIGINAL expression!

eg3: Simplify $\dfrac{x-2}{x^2-4}$

$$= \frac{(x-2)}{(x-2)(x+2)}$$

$$\boxed{= \frac{1}{x+2}}$$

R: $x^2-4=0$

$x^2=4$

$x \neq \pm 2$

eg4: Simplify $\dfrac{x^2y + xy^2}{xy + y^2}$

R: $xy + y^2 = 0$

R: $y(x+y) = 0$

R: $y \neq 0$, $y \neq -x$ OR $x \neq -y$ SAME

$$= \frac{xy(x+y)}{y(x+y)}$$

$$= \frac{xy}{y} = \boxed{x}$$

eg5: Simplify $\dfrac{x^2 - x - 6}{x^2 - 4}$

R: $x^2 - 4 = 0$

$x^2 = 4$

$x \neq \pm 2$

$$= \frac{(x-3)(x+2)}{(x-2)(x+2)}$$

$$\boxed{= \frac{x-3}{x-2}}$$

eg6: Simplify $\dfrac{x^2 - 9}{12 - 7x + x^2}$

R: $x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x \neq 4, 3$

$$= \frac{(x+3)(x-3)}{(x-4)(x-3)}$$

$$\boxed{= \frac{x+3}{x-4}}$$

Hwk. p.52 - 55 # 1-9 (omit #7b)

#7 for challenge only!

3. Simplify and provide restrictions on x (2 marks ea.)

a) $$\frac{6x^2 - 11x + 3}{3x^2 + 11x - 4}$$

b) $$\frac{x^2 - 7x}{x^3 + 5x^2 - 14x}$$

Ch. 2.2 - Multiplication and Division of Rational Expressions

Multiplying: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational expressions,

then $\frac{a}{b} \cdot \frac{c}{d} = \boxed{\frac{ac}{bd}}$ ⇒ must simplify (and note restrictions when asked)

$\boxed{b, d \neq 0}$

eg1: $\frac{3}{10} \cdot \frac{5}{9} = \frac{15}{90} = \boxed{\frac{1}{6}}$

eg2: $\frac{x-y}{2x} \cdot \frac{x^2}{(x-y)^2}$

R: $2x = 0$ → $x \neq 0$; $(x-y)^2 = 0$ → $x - y = 0$ → $x \neq y$

$= \frac{x^2(x-y)}{2x(x-y)(x-y)}$

$= \frac{x^2}{2x(x-y)} = \boxed{\frac{x}{2x-2y}}$ or $\boxed{\frac{x}{2(x-y)}}$

eg3: $\frac{x^2-25}{x^2-4} \cdot \frac{4x-8}{3x-15}$; (*) assume all denominators do NOT equal 0.

$= \frac{(x+5)(x-5)(4)(x-2)}{(x+2)(x-2)(3)(x-5)} = \boxed{\frac{4(x+5)}{3(x+2)}}$

eg4: $\frac{x^2-3x}{x^2-3x-4} \cdot \frac{x^2-5x+4}{x^2-2x-3}$ (*)

$= \frac{(x)(x-3)(x-4)(x-1)}{(x-4)(x+1)(x-3)(x+1)} = \boxed{\frac{x(x-1)}{(x+1)^2}}$

eg5: $\dfrac{x^2-3x}{x^2-x-6} \cdot \dfrac{x^2+x-2}{x-x^2}$ (*)

$$= \frac{(x)(x-3)(x+2)(x-1)}{(x-3)(x+2)(x)(1-x)} = \frac{x-1}{1-x} = \frac{x-1}{-1(x-1)}$$

$$= \frac{1}{-1} = \boxed{-1}$$

eg6: $(x-7) \cdot \dfrac{x^2-x}{x^2-8x+7}$ (*)

$$= \frac{(x-7)(x)(x-1)}{(x-7)(x-1)} = \boxed{x}$$

eg7. $\dfrac{6x^2+xy-2y^2}{4x^2-8xy+3y^2} \cdot \dfrac{x-y}{3x+2y} \cdot \dfrac{8x-12y}{2y-2x}$ (*)

$\frac{x}{12}\ \frac{+}{-8}$

$-6,-2$

$4x^2-6xy-2xy+3y^2$

$2x(2x-3y)-y(2x-3y)$

$\frac{x}{-12}\ \frac{+}{1}$

$+4,-3$

$6x^2+4xy-3xy-2y^2$

$2x(3x+2y)-y(3x+2y)$

$$= \frac{(3x+2y)(2x-y)(x-y)(4)(2x-3y)}{(2x-3y)(2x-y)(3x+2y)(2)(y-x)}$$

$$= \frac{4(x-y)}{2(y-x)} = \frac{4(x-y)}{-2(x-y)} = \boxed{-2}$$

Dividing: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational expressions,

then $\dfrac{a}{b} \div \dfrac{c}{d} = \boxed{\dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}}$ (mult. by reciprocal!)

$\boxed{b, d \neq 0}$

eg8: $\frac{14}{35} \div \frac{77}{55} = \frac{14}{35} \times \frac{55}{77} = \frac{2}{5} \cdot \frac{5}{7}$

$= \boxed{\frac{2}{7}}$

eg9: $\frac{x-2}{x+3} \div \frac{x^2+x-2}{x^2-4}$ (*)

$= \frac{x-2}{x+3} \cdot \frac{x^2-4}{x^2+x-2}$

$= \frac{(x-2)(x-2)(x+2)}{(x+3)(x+2)(x-1)} = \boxed{\frac{(x-2)^2}{(x+3)(x-1)}}$

eg10: $\frac{2x^2+3x-2}{2x-1} \div (4-x^2)$; (*)

$\frac{x}{-4} \quad \frac{+}{3}$

$4, -1$

$2x^2+4x-1x-2$

$2x(x+2)-1(x+2)$

$= \frac{2x^2+3x-2}{2x-1} \cdot \frac{1}{4-x^2}$

$= \frac{(2x-1)(x+2)}{(2x-1)(2-x)(2+x)} = \boxed{\frac{1}{2-x}}$

Hwk. p. 59 - 62 # 1-3.

Ch. 2.3 - Sums and Differences of Rational Expressions

Adding and Subtracting with LIKE denominators:

If $\frac{a}{b}$ and $\frac{c}{b}$ are rational expressions, then:

$$\frac{a}{b} + \frac{c}{b} = \boxed{\frac{a+c}{b}} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \boxed{\frac{a-c}{b}} \quad \boxed{b \neq 0}$$

eg1: $$\frac{x}{6} + \frac{2x}{6} = \frac{3x}{6} = \boxed{\frac{x}{2}}$$

eg2: $$\frac{x}{x^2-1} - \frac{1}{x^2-1} = \frac{(x-1)}{x^2-1}$$

$$= \frac{x-1}{(x+1)(x-1)} = \boxed{\frac{1}{x+1}}$$

$x^2 - 1 = 0$

$x^2 = 1$

$\boxed{x \neq \pm 1}$

Adding and Subtracting with UNLIKE denominators:

- must find LOWEST COMMON DENOMINATOR (LCD)

(can refer to steps on p. 64 if you wish)

eg3: $$\frac{7}{6x^2} - \frac{3}{8x^3}$$

LCD = $24x^3$

(*) Assume all denominators $\neq 0$.

$$= \frac{7(4x) - 3(3)}{24x^3} \boxed{= \frac{28x-9}{24x^3}}$$

eg4: $\dfrac{1}{x^2-1} - \dfrac{2}{x^2+x}$ ⊛

$= \dfrac{1}{(x+1)(x-1)} - \dfrac{2}{x(x+1)}$ LCD $= x(x+1)(x-1)$

$= \dfrac{1(x) - 2(x-1)}{x(x+1)(x-1)}$

$= \dfrac{x - 2x + 2}{x(x+1)(x-1)} = \boxed{\dfrac{2-x}{x(x+1)(x-1)}}$

eg5: $\dfrac{3x}{3x+6} + \dfrac{1}{x+2}$ ⊛

$= \dfrac{3x}{3(x+2)} + \dfrac{1}{x+2}$ LCD $= 3(x+2)$

$= \dfrac{3x + 3(1)}{3(x+2)} = \dfrac{3x+3}{3(x+2)} = \dfrac{3(x+1)}{3(x+2)}$

$= \boxed{\dfrac{x+1}{x+2}}$

eg6: $\dfrac{7}{x-2} + \dfrac{4}{2-x}$ ⊛ multiply top / bottom by -1

$= \dfrac{7}{(x-2)} - \dfrac{4}{(x-2)}$

$= \boxed{\dfrac{3}{x-2}}$

eg 7. $\dfrac{3x+9}{x^2+7x+10} + \dfrac{14}{x^2+3x-10}$ (*)

$$= \frac{3x+9}{(x+5)(x+2)} + \frac{14}{(x+5)(x-2)}$$

LCD $= (x+5)(x+2)(x-2)$

$$= \frac{(3x+9)(x-2) + 14(x+2)}{(x+5)(x+2)(x-2)}$$

$$= \frac{3x^2+3x-18+14x+28}{(x+5)(x+2)(x-2)}$$

$$= \frac{3x^2+17x+10}{(x+5)(x+2)(x-2)}$$

$\dfrac{\times}{30}$ $\dfrac{+}{17}$

15, 2

$3x^2+15x+2x+10$

$3x(x+5)+2(x+5)$

$$= \frac{(x+5)(3x+2)}{(x+5)(x+2)(x-2)}$$

$$\boxed{= \frac{3x+2}{(x+2)(x-2)}}$$

Hwk. p. 66 - 70 # 1 - 6, 11, 12.

Ch. 2.4 – Mixed Operations

B E D M A S

- **B**RACKETS
- **E**XPONENTS
- **D**IVISION
- **M**ULTIPLICATN
- **A**DDITION
- **S**UBTRACTION

eg1: Simplify $\dfrac{x+5}{x+6} + \dfrac{1}{x+4} \div \dfrac{x+6}{x^2-x-20}$

$(x-5)(x+4)$

$\boxed{R: x \neq -6, -4, 5}$

$$= \frac{x+5}{x+6} + \left(\frac{1}{x+4} \cdot \frac{(x-5)(x+4)}{x+6}\right)$$

$$= \frac{x+5}{x+6} + \frac{(x-5)\cancel{(x+4)}}{(x+6)\cancel{(x+4)}}$$

$$= \frac{(x+5)+(x-5)}{x+6} = \boxed{\frac{2x}{x+6}}$$

eg2: Simplify $\left(\dfrac{x-3}{x^2-9} + \dfrac{x+3}{x^2+6x+9}\right)\left(\dfrac{x+3}{x+1}\right)$

$\boxed{R: x \neq 3, -3, -1}$

$$= \left(\frac{\cancel{x-3}}{\cancel{(x-3)}(x+3)} + \frac{\cancel{x+3}}{\cancel{(x+3)}(x+3)}\right)\left(\frac{x+3}{x+1}\right)$$

$$= \left(\frac{1}{x+3} + \frac{1}{x+3}\right)\left(\frac{x+3}{x+1}\right)$$

$$= \left(\frac{2}{x+3}\right)\left(\frac{x+3}{x+1}\right) = \boxed{\frac{2}{x+1}}$$

COMPLEX FRACTIONS

- when fractions exist in the <u>numerator</u> <u>and</u> the <u>denominator</u>.

eg3: Simplify

$$\frac{\frac{2}{5x} - \frac{3}{x^2}}{\frac{7}{2x} + \frac{3}{4x^2}}$$

TWO METHODS:

① Finding <u>LCD</u> of numerator and denominator and <u>simplifying</u> each. Then, divide.

② Finding <u>LCD</u> of all fractions, multiplying all terms by it, then <u>simplifying</u>.

Method ①: LCD(num.) = $5x^2$
LCD(den.) = $4x^2$

$$= \frac{\frac{2x}{5x^2} - \frac{15}{5x^2}}{\frac{14x}{4x^2} + \frac{3}{4x^2}}$$

$$= \frac{\frac{2x-15}{5x^2}}{\frac{14x+3}{4x^2}} = \left(\frac{2x-15}{5x^2}\right)\left(\frac{4x^2}{14x+3}\right)$$

$$= \frac{8x-60}{70x+15} \text{ OR } \boxed{\frac{4(2x-15)}{5(14x+3)}}$$

Method ② LCD of $5x$, x^2, $2x$, and $4x^2$
$= 20x^2$

$$= \frac{20x^2\left(\frac{2}{5x} - \frac{3}{x^2}\right)}{20x^2\left(\frac{7}{2x} + \frac{3}{4x^2}\right)}$$

$$= \frac{\frac{40x^2}{5x} - \frac{60x^2}{x^2}}{\frac{140x^2}{2x} + \frac{60x^2}{4x^2}}$$

$$= \frac{8x-60}{70x+15} \text{ OR } \frac{4(2x-15)}{5(14x+3)}$$

R: $x \neq -\frac{3}{14}, 0$ ⟶

eg4: Simplify $$\frac{\frac{1}{x-1}+\frac{2}{x+2}}{\frac{2}{x+2}-\frac{1}{x-3}}$$

R: $x \neq 1, -2, 3, 8$

① $$\frac{\frac{(x+2)+2(x-1)}{(x-1)(x+2)}}{\frac{2(x-3)-(x+2)}{(x+2)(x-3)}}$$

$$= \frac{3x}{(x-1)(x+2)} \cdot \frac{(x+2)(x-3)}{(x-8)}$$

$$\boxed{= \frac{3x(x-3)}{(x-1)(x-8)}}$$

② LCD $= (x-1)(x+2)(x-3)$

$$= \frac{(x-1)(x+2)(x-3)\left(\frac{1}{x-1}+\frac{2}{x+2}\right)}{(x-1)(x+2)(x-3)\left(\frac{2}{x+2}-\frac{1}{x-3}\right)}$$

$$= \frac{(x+2)(x-3)+2(x-1)(x-3)}{2(x-1)(x-3)-(x-1)(x+2)}$$

$$= \frac{x^2-x-6+2x^2-8x+6}{2x^2-8x+6-x^2-x+2}$$

$$= \frac{3x^2-9x}{x^2-9x+8} = \boxed{\frac{3x(x-3)}{(x-1)(x-8)}}$$

Hwk: p. 74–77 # 1–7

HINT: Remember $x^{-1} = \frac{1}{x^1} = \frac{1}{x}$

$$x^{-n} = \frac{1}{x^n}$$

Some solutions to Ch. 2.4 p 76-77

2s) $$\frac{\frac{x}{x+1}+3}{\frac{x}{x+1}+\frac{3}{\frac{x}{x+1}}} = \frac{\frac{x}{x+1}+\frac{3(x+1)}{x+1}}{\frac{x}{x+1}+\frac{3(x+1)}{x}}$$

$$= \frac{\frac{4x+3}{x+1}}{\frac{x^2+3(x+1)^2}{(x+1)(x)}}$$

$\Rightarrow x^2+2x+1$

$$= \left(\frac{4x+3}{x+1}\right)\cdot\left(\frac{x(x+1)}{4x^2+6x+3}\right)$$

$\frac{x}{12}$ $\frac{+}{6}$ cannot be factored

$$\boxed{= \frac{(4x+3)(x)}{4x^2+6x+3}}$$

t) $$3-\frac{x}{3-\frac{x}{3-x}} = 3-\frac{x}{\frac{3(3-x)}{3-x}-\frac{x}{3-x}}$$

$$= 3-\frac{x}{\frac{-4x+9}{3-x}} = 3-x\left(\frac{(3-x)}{(-4x+9)}\right)$$

$$= 3-\left(\frac{(-x^2+3x)}{(-4x+9)}\right)$$

$$= \frac{3(-4x+9)+x^2-3x}{-4x+9}$$

$$= \frac{-12x+27+x^2-3x}{-4x+9}$$

$$\boxed{= \frac{x^2-15x+27}{9-4x}}$$

3a) $$\frac{x^{-2}+x}{x} = \frac{\frac{1}{x^2}+x}{x} = \frac{\frac{1+x^3}{x^2}}{x}$$

$$= \left(\frac{1+x^3}{x^2}\right)\cdot\frac{1}{x}$$

$$\boxed{= \frac{1+x^3}{x^3}}$$

b) $$\frac{x^{-2}-3x^{-3}}{3x^{-2}-9x^{-3}} = \frac{\frac{1}{x^2}-\frac{3}{x^3}}{\frac{3}{x^2}-\frac{9}{x^3}} = \frac{\frac{x-3}{x^3}}{\frac{3x-9}{x^3}}$$

$$= \left(\frac{x-3}{x^3}\right)\left(\frac{x^3}{3x-9}\right) = \frac{x-3}{3(x-3)} = \boxed{\frac{1}{3}}$$

c) $$\left(x^{-1}+y^{-1}\right)^{-1} = \left(\frac{1}{x}+\frac{1}{y}\right)^{-1} = \left(\frac{y+x}{xy}\right)^{-1}$$

$$= \frac{1}{\frac{y+x}{xy}} = 1\left(\frac{xy}{y+x}\right) = \boxed{\frac{xy}{y+x}}$$

d) $$\left(x^{-1}-y^{-1}\right)^{-2} = \left(\frac{1}{x}-\frac{1}{y}\right)^{-2} = \left(\frac{y-x}{xy}\right)^{-2}$$

$$= \frac{1}{\frac{(y-x)^2}{(xy)^2}} = \boxed{\frac{x^2y^2}{x^2-2xy+y^2}}$$

4. $$T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} = \boxed{\frac{R_1 R_2}{R_2 + R_1}}$$

5. $$i = \frac{E}{R + \frac{r}{2}} \Rightarrow i = \frac{E}{\frac{2R + r}{2}} \Rightarrow i = \frac{2E}{2R + r}$$

$$(2R + r)\, i = 2E \Rightarrow 2R + r = \frac{2E}{i} \Rightarrow 2R = \frac{2E}{i} - r$$

$$2R = \frac{2E - ri}{i} \Rightarrow \boxed{R = \frac{2E - ri}{2i}}$$

$$r = \frac{2E}{i} - 2R$$

$$\boxed{r = \frac{2E - 2Ri}{i}}$$

6. $$f^{-1} = d_i^{-1} + d_o^{-1}$$

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$1 = \frac{f}{d_i} + \frac{f}{d_o}$$

$$1 = \frac{f d_o + f d_i}{d_i d_o}$$

$$1 = \frac{f(d_o + d_i)}{d_i d_o}$$

$$\Rightarrow d_i d_o = f(d_o + d_i)$$

$$\boxed{f = \frac{d_i d_o}{d_o + d_i}}$$

7) $$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}} \Rightarrow \frac{1}{\frac{x+1}{x}} = \frac{x}{x+1}$$

Rewrite: $$= \frac{1}{1+\frac{1}{1+\frac{x}{x+1}}} \Rightarrow \frac{1}{\frac{x+1+x}{x+1}} = \frac{1}{\frac{2x+1}{x+1}} = \frac{x+1}{2x+1}$$

Rewrite: $$= \frac{1}{1+\frac{x+1}{2x+1}} \Rightarrow \frac{1}{\frac{2x+1+x+1}{2x+1}} = \frac{1}{\frac{3x+2}{2x+1}}$$

$$\boxed{= \frac{2x+1}{3x+2}}$$

Ch. 2.5 - Rational Equations

Solving Rational Equations:

① Multiply both sides by the LCD to clear away the fractions.

② Solve

③ Check for extraneous roots / restrictions.
↳ that make denominator zero!

eg1: Solve $\frac{x}{2} + \frac{7}{3} = \frac{5}{6}$ LCD = 6

$$\frac{6x}{2} + \frac{6(7)}{3} = \frac{6(5)}{6}$$

no restrictions since no variables exist in a denominator!

$$3x + 14 = 5$$

$$3x = -9$$

$$\boxed{x = -3}$$ check... ✓

eg2: Solve $3 - \frac{6}{x} = x + 8$ $x \neq 0$ LCD = x

$$3x - 6 = x^2 + 8x$$

$$0 = x^2 + 5x + 6$$

$$0 = (x+3)(x+2)$$

$$\boxed{x = -3, -2}$$ Check!

eg3: Solve $\frac{2x}{x-4} = \frac{8}{x-4} + 1$ LCD $= x-4$ $\boxed{x \neq 4}$

$$2x = 8 + (x-4)$$
$$2x = 4 + x$$
$$x = 4 \quad \text{but}$$

so $\boxed{\begin{array}{c}\text{NO SOLUTION} \\ \text{or} \\ x = \varnothing\end{array}}$

eg4: Solve $\frac{1}{x-4} - \frac{1}{x-2} = \frac{2x}{x^2-6x+8}$ $x \neq 2, 4$

$$\frac{1}{x-4} - \frac{1}{x-2} = \frac{2x}{(x-4)(x-2)}$$ LCD $= (x-4)(x-2)$

$$(x-2) - (x-4) = 2x$$
$$2 = 2x$$
$$\boxed{x = 1}$$

eg5: Solve $\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{x^2-4x-5}$ $x \neq 5, -1$

$$\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{(x-5)(x+1)}$$ LCD $= (x-5)(x+1)$

$$x(x+1) - 3(x-5) = 30$$
$$x^2 - 2x + 15 = 30$$
$$x^2 - 2x - 15 = 0$$
$$(x-5)(x+3) = 0$$
$$x = \cancel{5}, \boxed{-3}$$

eg6: Solve $\dfrac{4}{2x-1} = \dfrac{2}{x+3}$ $\qquad x \neq \frac{1}{2}, -3$

$$4(x+3) = 2(2x-1)$$

$$4x+12 = 4x-2$$

$$0 = -14 \quad ??$$ NEVER TRUE!

NO SOLUTION

eg7: $\dfrac{5}{x-7} - \dfrac{1}{2x} = \dfrac{9x+7}{2x^2-14x}$

$$\frac{5}{x-7} - \frac{1}{2x} = \frac{9x+7}{2x(x-7)}$$

$x \neq 0, 7$

LCD $= 2x(x-7)$

$$5(2x) - 1(x-7) = 9x+7$$

$$10x - x + 7 = 9x+7$$

$$9x+7 = 9x+7$$

$$0 = 0$$ identity statement → ALWAYS true!

$$\boxed{x = \mathbb{R} \text{ except } 0 \text{ and } 7.}$$

eg8: Solve $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ for a

$a, b, c \neq 0$

LCD = abc

$bc + ac = ab \xrightarrow{\text{OR}}$

$ac - ab = -bc$

$a(c-b) = -bc$

$\boxed{a = \frac{-bc}{c-b}}$

$bc = ab - ac$

$bc = a(b-c)$

hmm... $\boxed{\frac{bc}{b-c} = a}$

Why? divide top/bottom by -1

Hwk: p. 81 - 85 # 1-6.

Chapter 2.6 – Applications of Rational Equations (Word Problems)

eg1: The sum of a number and twice its reciprocal is 9/2. Find the number.

let x = the number
then $\frac{1}{x}$ = reciprocal

$$x + \frac{2}{x} = \frac{9}{2}$$

$$2x^2 + 4 = 9x$$

$$2x^2 - 9x + 4 = 0$$

$$2x^2 - 8x - 1x + 4 = 0$$

$$2x(x-4) - 1(x-4) = 0$$

$$(x-4)(2x-1) = 0$$

$\boxed{x = 4}$, $\boxed{\frac{1}{2}}$

eg2: The cold water tap can fill a bathtub two hours faster than the hot water tap. The two taps running together can fill the bathtub in 80 minutes. How long does it take each tap to fill the container on its own?

rate $\frac{1}{x}$ → let x = time for cold water tap.

rate $\frac{1}{x+2}$ → then $x+2$ = time for hotwater tap.

together rate $= \frac{1}{80 \text{ mins}}$

$= 1.33$ hrs.

$= \frac{4}{3}$ hrs

$$\frac{1}{x} + \frac{1}{x+2} = \frac{1}{\left(\frac{4}{3}\right)}$$

$$\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4}$$

$$4x + 8 + 4x = 3x(x+2)$$

$$8x + 8 = 3x^2 + 6x$$

$$0 = 3x^2 - 2x - 8$$

$$0 = 3x^2 - 6x + 4x - 8$$

$$0 = (3x+4)(x-2)$$

$x = \cancel{-\frac{4}{3}}$ $\boxed{2 \text{ hrs}}$

$x + 2 = \boxed{4 \text{ hrs}}$

eg3: A speedboat can travel 108 km downstream in the same time it can travel 78 km upstream. If the current of the river is 10 km/h, what is the speed of the boat in still water?

let x = speed of boat in still water $\qquad t = \frac{d}{s}$

$$\frac{108}{x+10} = \frac{78}{x-10}$$

$$108x - 1080 = 78x + 780$$

$$30x = 1860$$

$$\boxed{x = 62 \text{ km/h}}$$

eg4: A car travels from home to work at an average speed of 80 km/h, and because of traffic, returns from work at an average speed of 50 km/h. What is the average speed of the entire trip if the distance to work is 20 km? Nearest tenth.

let x = avg. speed of entire trip. $\qquad t = \frac{d}{s}$

$$\underbrace{\left(\frac{20}{80}\right)}_{\text{(home to work)}} + \underbrace{\left(\frac{20}{50}\right)}_{\text{(work to home)}} = \frac{2(20)}{x}$$

$$\frac{1}{4} + \frac{2}{5} = \frac{40}{x}$$

$$5x + 8x = 800$$

$$13x = 800$$

$$\boxed{x = 61.5 \text{ km/h}}$$

eg5: Ray and Ann ride a bicycle a distance of 4 km each morning. They both finish at the same time but Ann starts 1 minute before Ray, and Ray travels 1 km/h faster than Ann. At what speed are they traveling?

let x = Ann's speed
then $x+1$ = Ray's speed

$$t = \frac{d}{s}$$

$$\frac{4}{x} - \frac{1}{60} = \frac{4}{x+1}$$

$$(60)(4)(x+1) - (x)(x+1) = (4)(x)(60)$$

$$\cancel{240x} + 240 - x^2 - x = \cancel{240x}$$

$$0 = x^2 + x - 240$$

$$0 = (x+16)(x-15)$$

$$x = \cancel{-16},\ 15$$

$$\boxed{\begin{array}{l} \text{Ann} = 15 \text{ km/hr.} \\ \text{Ray} = 16 \text{ km/hr} \end{array}}$$

Homework: p. 91 – 93 # 1-3 and p. 70 #7-9

Chapter Review: p. 94 – 97 #1-11

Solutions to Word Problems Ch. 2.6 p. 91–93

1 a) $x + \frac{1}{x} = \frac{13}{6}$

$6x^2 + 6 = 13x$

$6x^2 - 13x + 6 = 0$

$6x^2 - 9x - 4x + 6 = 0$

$3x(2x-3) - 2(2x-3) = 0$

$(2x-3)(3x-2) = 0$

$\boxed{x = \frac{3}{2}, \frac{2}{3}}$

b) $x + \frac{1}{x} = \frac{65}{8}$

$8x^2 + 8 = 65x$

$8x^2 - 65x + 8 = 0$

$8x^2 - 64x - 1x + 8 = 0$

$8x(x-8) - 1(x-8) = 0$

$(8x-1)(x-8) = 0$

$x = \cancel{\frac{1}{8}}, \boxed{8}$

not an INTEGER!

c) $\frac{1}{x} + \frac{1}{x+1} = \frac{7}{12}$

$12(x+1) + 12x = 7(x)(x+1)$

$24x + 12 = 7x^2 + 7x$

$\frac{x}{-84}\ \frac{+}{-17}$ $0 = 7x^2 - 17x - 12$

$0 = 7x^2 - 21x + 4x - 12$

$0 = 7x(x-3) + 4(x-3)$

$0 = (x-3)(7x+4)$

$x = 3, -\cancel{\frac{4}{7}}$

$\boxed{\text{3 and 4}}$

d) $\frac{1}{x} + \frac{1}{x+2} = \frac{8}{15}$

$15(x+2) + 15x = 8x(x+2)$

$30x + 30 = 8x^2 + 16x$

$0 = 8x^2 - 14x - 30$

$0 = 4x^2 - 7x - 15$

$0 = 4x^2 - 12x + 5x - 15$

$0 = 4x(x-3) + 5(x-3)$

$0 = (x-3)(4x+5)$

$x = 3, -\cancel{\frac{5}{4}}$

$\boxed{\text{3 and 5}}$

c) $\frac{1}{x} + \frac{1}{x+2} = \frac{7}{24}$

$24(x+2) + 24x = 7x(x+2)$

$48x + 48 = 7x^2 + 14x$

$0 = 7x^2 - 34x - 48$

$0 = 7x^2 - 42x + 8x - 48$

$0 = 7x(x-6) + 8(x-6)$

$0 = (x-6)(7x+8)$

$x = 6, \cancel{-\frac{8}{7}}$

$\boxed{\text{6 and 8}}$

f) $\frac{x}{5} - \frac{3}{2} = \frac{x}{2}$

$2x - 15 = 5x$

$-15 = 3x$

$\boxed{x = -5}$

Typo in text key!

g) $x + \frac{6}{x} = -5$

$x^2 + 6 = -5x$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$\boxed{x = -3, -2}$ another TYPO in text.

h) $\frac{12}{x} + \frac{7x-5}{x+1} = 8$

$12(x+1) + x(7x-5) = 8x(x+1)$

$12x + 12 + 7x^2 - 5x = 8x^2 + 8x$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$\boxed{x = -4, 3}$

2a) Let x = Sue's rate of painting

1 painted room = $4x$

$$4x = 1$$

$x = \frac{1}{4}$ room/h so... Bert's rate would be $\frac{1}{5}$ room/h

Let t = time it would take both of them.

Together: $$1 = \frac{1}{4}t + \frac{1}{5}t$$

$$1 = \frac{t}{4} + \frac{t}{5}$$

$$20 = 5t + 4t$$

$$20 = 9t$$

$t = \frac{20}{9}$ = 2 hrs. 13 mins 20 secs.

b) Worker 1's rate = $\frac{1}{20}$ job/h ; worker 2's rate = $\frac{1}{15}$ job/h

Let x = worker 3's rate   rate = $\frac{\text{JOB}}{h}$

$$\frac{1}{20} + \frac{1}{15} + x = \frac{1}{6}$$ LCD = 60   rate = $\frac{1}{6}$ with all 3!

$$3 + 4 + 60x = 10$$

$$60x = 3$$

$x = \frac{3}{60} = \boxed{\frac{1}{20} \text{ job/h}}$

job = rate × time

2 c) Let x = Anna's rate
then $2x$ = Jane's rate

$$1 \text{ clean kitchen} = (x + 2x)(15)$$

$$1 = 15x + 30x$$

$$1 = 45x$$

$$x = \frac{1}{45}$$ Anna's rate = $\frac{1}{45}$ kitchen/min

$\boxed{\text{45 mins. alone}}$

2 d) Let x = Hans' ~~time~~ time alone
then $x+3$ = Ken's time alone

time = $\frac{\text{job}}{\text{rate}}$

So... Hans' rate = $\frac{1}{x}$
Ken's rate = $\frac{1}{x+3}$

together rate = $\frac{1}{2}$ motor/h

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2}$$

$$2(x+3) + 2x = x(x+3)$$

$$4x + 6 = x^2 + 3x$$

$$0 = x^2 - x - 6$$

$$(x-3)(x+2) = 0$$

$$x = 3, \cancel{-2}$$

$$\boxed{\begin{array}{l}\text{Hans} = 3 \text{ hrs.} \\ \text{Ken} = 6 \text{ hrs}\end{array}}$$

2e) Cold water rate = $\frac{1}{6}$ tub/min.

Hot water rate = $\frac{1}{8}$ tub/min $\qquad$ Rate = $\frac{\text{job}}{\text{time}}$

Drain rate = $\frac{1}{10}$ tub/min

let x = time to fill tub $\qquad$ Job = (Rate)(time)

$$\frac{1}{6}x + \frac{1}{8}x - \frac{1}{10}x = 1$$

$$\frac{x}{6} + \frac{x}{8} - \frac{x}{10} = 1$$

$$20x + 15x - 12x = 120$$

$$23x = 120$$

$x = \frac{120}{23}$ minutes (* ANOTHER book typo!)

f) Brick layer rate = $\frac{1}{6}$ wall/hr.

Apprentice rate = $\frac{1}{16}$ wall/hr.

let x = # hrs. worked by brick layer
then $x+5$ = # hrs worked by apprentice.

$$1 = \frac{x}{6} + \frac{x+5}{16}$$

$$48 = 8x + 3x + 15$$

$$33 = 11x$$

$\boxed{\begin{array}{l} x = 3 \text{ hrs for brick layer} \\ x+5 = 8 \text{ hrs. for apprentice} \end{array}}$

3.a) Let s = speed in still water $\qquad t = \frac{d}{s}$

$$\frac{40}{s+6} = \frac{30}{s-6}$$

$$40(s-6) = 30(s+6)$$

$$40s - 240 = 30s + 180$$

$$10s = 420$$

$$\boxed{s = 42 \text{ km/h}}$$

b) Let c = speed of current $\qquad t = \frac{d}{s}$

$$5 = \frac{24}{10+c} + \frac{24}{10-c}$$

$$5(10+c)(10-c) = 24(10-c) + 24(10+c)$$

$$-5c^2 + 500 = 480$$

$$0 = 5c^2 - 20$$

$$20 = 5c^2$$

$$c^2 = 4 \qquad \boxed{c = +2 \text{ mph}}$$

c) Let x = avg. speed for round trip.

$\boxed{\text{distances are the same.}}$

$t = \frac{d}{s}$ $\qquad$ TOTAL TIME = time to work + time to home

$$\frac{2d}{x} = \frac{d}{50} + \frac{d}{30}$$

$$3000d = 30xd + 50xd$$

$$3000d = 80xd$$

$$3000 = 80x \qquad \boxed{x = 37.5 \text{ mph}}$$

3d) Let x = avg. speed for entire trip. $\qquad t = \frac{d}{s}$

TOTAL TIME = time there + time back

$$\frac{100}{x} = \frac{50}{40} + \frac{50}{60}$$

$$\frac{100}{x} = \frac{5}{4} + \frac{5}{6}$$

$$1200 = 15x + 10x$$

$$1200 = 25x$$

$$\boxed{x = 48 \text{ km/h.}}$$

e) Let x = # of km walked

TOTAL TIME = time @ faster rate + time @ slower rate $\qquad t = \frac{d}{s}$

$$2 = \frac{8-x}{7} + \frac{x}{3}$$

$$42 = 3(8-x) + 7(x)$$

$$42 = 24 - 3x + 7x$$

$$18 = 4x$$

$$\boxed{x = 4.5 \text{ km walked}}$$

f) Let x = driver A's speed
then $x+10$ = driver B's speed

Slower time (A) − faster time (B) = $\frac{1}{6}$

10 mins = $\frac{1}{6}$ h.

$t = \frac{d}{s}$

$$\frac{80}{x} - \frac{100}{x+10} = \frac{1}{6}$$

$$(6)(x+10)(80) - (6)(x)(100) = x(x+10)$$

$$480x + 4800 - 600x = x^2 + 10x$$

$$0 = x^2 + 130x - 4800$$

$$0 = (x-30)(x+160)$$

$$x = 30,\ \cancel{-160}$$

Driver A = 30 km/h.
Driver B = 40 km/h.

p. 70 # 7–9

7) Rate of cold = $\frac{1}{t}$ tub/min

Rate of hot = $\frac{1}{t+2}$ tub/min

Let x = amount filled in 4 mins.

$$\frac{1}{t} + \frac{1}{t+2} = \frac{x}{4}$$

$$4(t+2) + 4t = x(t)(t+2)$$

$$4t + 8 + 4t = x(t^2 + 2t)$$

$$\boxed{\frac{8t+8}{t^2+2t}} = x$$

8) Let y = # of rooms painted in 8 hrs.

$$\frac{1}{x} + \frac{1}{x+1} = \frac{y}{8}$$

$$8x + 8 + 8x = y(x)(x+1)$$

$$16x + 8 = y(x^2 + x)$$

$$y = \frac{16x+8}{x^2+x}$$

9) Let T = total time
Let x = speed

$t = \frac{d}{s}$

$$T = \frac{100}{x} + \frac{200}{x+10}$$

$$T = \frac{100x + 1000 + 200x}{x(x+10)}$$

$$\boxed{T = \frac{300x + 1000}{x(x+10)} \text{ hours}}$$