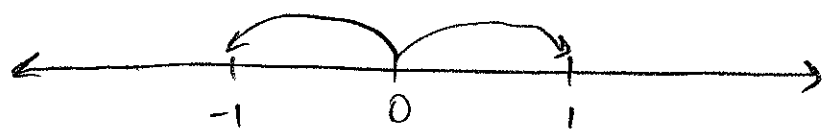
Ch. 1.1 - Absolute Value

On a one-dimensional number line, the numbers 1 and -1 are the same distance from 0. Regardless of direction on the number line, the distance a number is from 0 is known as the ABSOLUTE VALUE (magnitude) of the number.



Defn: <u>Absolute Value</u> - the number of units that a number is from zero on a number line.

"The absolute value of x" is depicted by $|x|$ ← STRAIGHT brackets.

eg: a) $|5| = 5$ b) $|0| = 0$

c) $|-3| = 3$ d) $\left|\frac{5}{2}\right| = \frac{5}{2}$

eg2: Solve $|-7| + |2|$

$= 7 + 2$

$= 9$

eg3: Solve $|6 - (-3)|$

$= |6 + 3|$ Bedmas

$= |9|$

$= 9$

eg4: Insert $||$ symbol to make statement true:

$-4 + (-5) - 10 = -1$

$-4 - 5 - 10 = -1$ glance at all scenarios

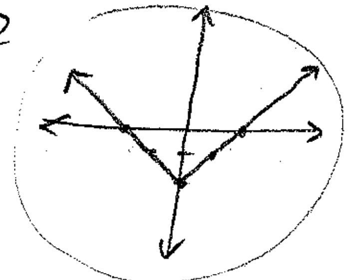
✓ $|-4-5| - 10 = -1$

Do Qs 1-3 p. 6

Ch. 4.4 – Solving Absolute Value Equations

- can be used to find the <u>x-intercepts</u> of an absolute value <u>function</u> (when y is set to be <u>0</u>)

eg: $y = |x| - 2$



* for any real number x:

If $x \geq 0$, then $|x| = \boxed{x}$;

If $x \leq 0$, then $|x| = \boxed{-x}$.

So, when solving for a variable that is being '<u>absolute valued</u>', we must account for both the <u>positive</u> and <u>negative</u> versions.

eg1: Solve for x:

$$|x| = 5$$

$\xleftarrow{+}$ $\xrightarrow{-}$

$(x) = 5$ $\quad$ $-(x) = 5$

$\boxed{x = 5}$ ALWAYS Check! $\boxed{x = -5}$

<u>eg2:</u> Solve $|x-2| = x-2$ * takes some thought

$(x-2) = x-2$ $\quad$ $-(x-2) = x-2$

$0 = 0$ $\quad$ $-x+2 = x-2$

$x \in \mathbb{R}$ $\quad$ $4 = 2x$

$x = 2$ OK, but... $|x| = x$ when $x \geq 0$

so $|x-2| = x-2$ when $x-2 \geq 0$

$\boxed{x \geq 2}$

eg3: Solve $|x-3| = -(x-3)$

$|x| = -x$ when $x \leq 0$

$|x-3| = -(x-3)$ when $x-3 \leq 0$

$\boxed{x \leq 3}$

Solutions of Absolute Value Equations:

If $|ax+b| = c$, $a \neq 0$, then:

i) If $c > 0 \Rightarrow$ two solutions

ii) If $c = 0 \Rightarrow$ one solution

iii) If $c < 0 \Rightarrow$ no solutions

eg4: Solve $|x-1| = 4$

$(x-1) = 4$

$\boxed{x = 5}$

$-(x-1) = 4$

$x - 1 = -4$

$\boxed{x = -3}$

Check!

eg5: Solve $|2x-1| = 5$

$2x - 1 = 5$

$2x = 6$

$\boxed{x = 3}$

$-(2x-1) = 5$

$2x - 1 = -5$

$2x = -4$

$\boxed{x = -2}$

eg6: $|1-3x| = 0$

$(1-3x) = 0$

$3x = 1$

$x = \frac{1}{3}$

$-(1-3x) = 0$

$3x = 1$

$\boxed{x = \frac{1}{3}}$

eg7: $|2x-3| = -5$ $\boxed{\text{NO SOLUTION}}$

$2x-3=-5$

$2x=-2$

$x = \cancel{-1}$

check.

$-(2x-3)=-5$

$2x-3=5$

$2x=8$

$x=\cancel{4}$

eg8: Solve $|3x-2| = 1-x$

$(3x-2) = 1-x$

$4x = 3$

$\boxed{x = \frac{3}{4}}$

$-(3x-2) = 1-x$

$-3x+2 = 1-x$

$1 = 2x$

$\boxed{x = \frac{1}{2}}$

eg9: Solve $|x^2-7x+2| = 10$

$(x^2-7x+2) = 10$

$x^2-7x-8=0$

$(x-8)(x+1)=0$

$\boxed{x = 8, -1}$

$-(x^2-7x+2) = 10$

$x^2-7x+2=-10$

$x^2-7x+12=0$

$(x-4)(x-3)=0$

$\boxed{x=4, 3}$

eg10: Write an absolute value equation for the statement: x is 2 units from 5

x could be 7 or 3

$3 = x = 7$

$3-5 = x-5 = 7-5$

$-2 = x-5 = 2$

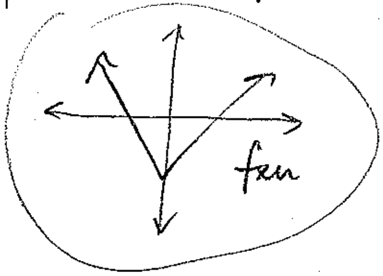
$\boxed{|x-5| = 2}$

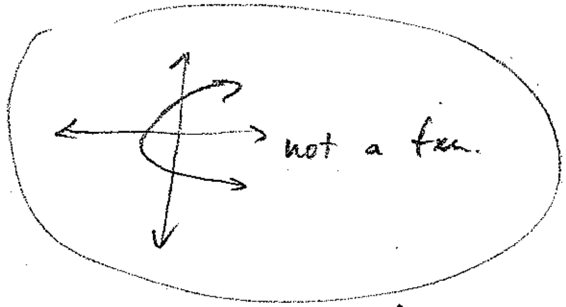
Homework:
p. 191-195
1, 2 (intercepts only),
3 - 14.

Ch. 4.3 - Absolute Value Functions (Day 1)

* for a function to actually be called a FUNCTION, it must pass the vertical line test (must be only one y-value per x-value)

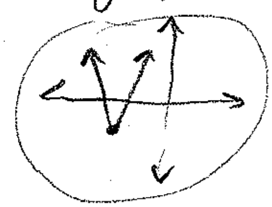
e.g.





Graphing Linear Absolute Value functions: a V-shaped graph!

Two methods: ① Vertex / Translation Method
② Piecewise Method.



Standard form = y or $f(x)$ = $a|bx+c|+d$

read as opposite (c) — read as is. (d)

To graph: ① a) Find vertex $\Rightarrow \left(\frac{-c}{b}, d\right)$

b) Graph $y = \boxed{a|bx| \text{ from vertex}}$

(graph is symmetrical about the vertex) (axis of symmetry)

* if $a > 0$, opens UP
if $a < 0$, opens DOWN.

② Graph two lines

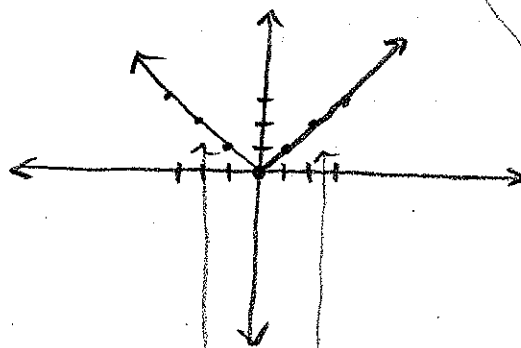
$\hookrightarrow y = a(bx+c)+d$, when $bx+c \geq 0$

$\hookrightarrow y = -a(bx+c)+d$, when $bx+c < 0$

eg1: Graph $y = |x|$. Find i) Domain / Range
ii) x/y intercepts.

$y = 1\,|\,1x + 0\,| + 0$

① Vertex = $(0, 0)$
then graph $y = 1\,|1x|$ from vertex



ⓘ D: $x \in \mathbb{R}$
R: $y \geq 0$

ⓘⓘ x-ints: set $y = 0$

$0 = |x|$
$x = 0$ $\boxed{(0,0)}$

y-int: set $x = 0$

$y = |0|$
$y = 0$ $\boxed{(0,0)}$

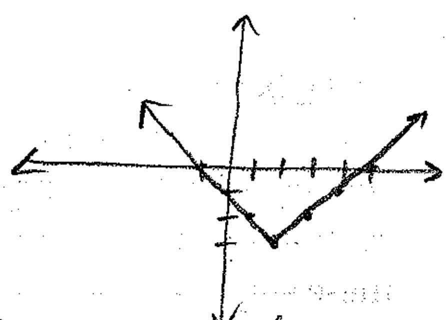
② Piecewise: Vertex = $(0, 0)$

$y = 1x$ when $x \geq 0$

$y = -1x$ when $x < 0$

eg2: Graph $y = |x - 2| - 3$ Find ⓘ and ⓘⓘ

① Vertex = $(2, -3)$
Graph $y = |x|$ from $(2, -3)$



ⓘ D: $x \in \mathbb{R}$ R: $y \geq -3$

ⓘⓘ x ints:

$0 = |x - 2| - 3$

$x - 2 = 3$
$\boxed{x = 5}$
$(5, 0)$

$-x + 2 = 3$
$\boxed{x = -1}$
$(-1, 0)$

y-int:

$y = |0 - 2| - 3$
$y = 2 - 3$
$y = -1$ $\boxed{(0, -1)}$

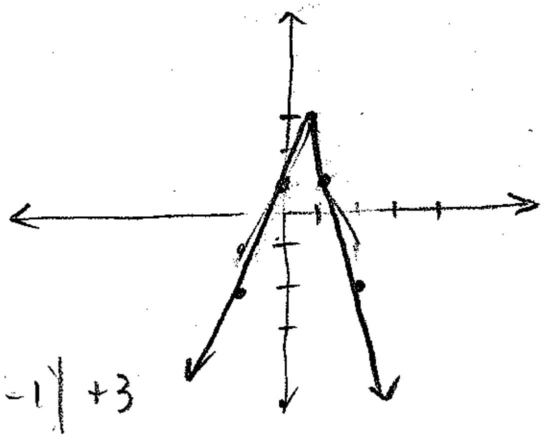
② Piecewise (Vertex = $(2, -3)$)

$y = x$ when $x - 2 \geq 0$
$x \geq 2$

$y = -x$ when $x - 2 < 0$
$x < 2$

eg3: Graph $y=-2|2x-1|+3$ with method ①. Find i/ii.

Vertex $= \left(\frac{1}{2}, 3\right)$; then graph $y=-2|2x|$ from vertex



(i) D: $x \in \mathbb{R}$ R: $y \le 3$

(ii) x-ints.

$$0 = -2|2x-1|+3$$
$$2|2x-1| = 3$$
$$|2x-1| = \frac{3}{2}$$

$2x-1 = \frac{3}{2}$ $\qquad$ $-2x+1 = \frac{3}{2}$

$2x = \frac{5}{2}$ $\qquad$ $-2x = \frac{1}{2}$

$x = \frac{5}{4}$ $\qquad$ $x = -\frac{1}{4}$

$\boxed{\left(\frac{5}{4}, 0\right)}$ $\qquad$ $\boxed{\left(-\frac{1}{4}, 0\right)}$

y-ints: $y = -2|2(0)-1|+3$

$$y = -2+3$$
$$y = 1 \quad \boxed{(0,1)}$$

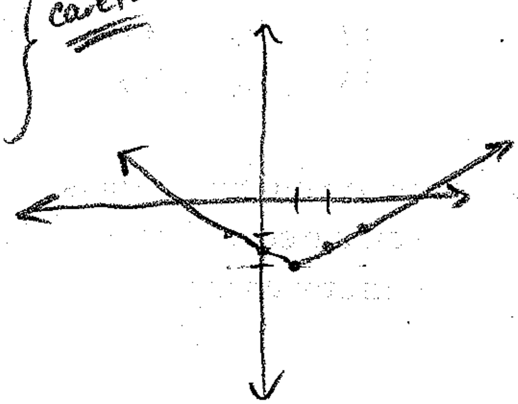
eg4: Graph $y=\frac{1}{2}|1-x|-2$ as a piecewise fxn.

$y = \frac{1}{2}|-1x+1|-2$ $\qquad$ Vertex $= (1,-2)$

$$\begin{cases} y = \frac{1}{2}(1-x)-2 & \text{when } 1-x \ge 0 \\ y = -\frac{1}{2}x - \frac{3}{2} & x \le 1 \end{cases}$$

$$\begin{cases} y = -\frac{1}{2}(1-x)-2 & \text{when } 1-x < 0 \\ y = \frac{1}{2}x - \frac{5}{2} & x > 1 \end{cases}$$

careful!



~~Homework p.191 #2 p.184 #2&3~~

eg5: Write each absolute value function as a piecewise function:

a) $f(x) = -\frac{1}{3}|3x+2| - 1$

$f(x) = -\frac{1}{3}(3x+2) - 1$ when $3x+2 \geq 0$

$\boxed{f(x) = -x - \frac{5}{3} \text{ when } x \geq -\frac{2}{3}}$

$f(x) = \frac{1}{3}(3x+2) - 1$ when $3x+2 < 0$

$\boxed{f(x) = x - \frac{1}{3} \text{ when } x < -\frac{2}{3}}$

b) $f(x) = |x^2 - 4|$

$f(x) = x^2 - 4$ when $\underbrace{x^2-4 \geq 0}$

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$x \geq 2$; $-x \geq 2$

$x \leq -2$

$\boxed{f(x) = x^2 - 4 \text{ when } x \geq 2 \text{ and } x \leq -2}$

$f(x) = -(x^2 - 4)$ when $x^2 - 4 < 0$

$\boxed{f(x) = 4 - x^2 \text{ when } x < 2,\ x > -2}$

Hwk: p. 191 # 2

p. 184 # 2-3

+ Worksheet

Absolute Value Functions (Day Two)

Another way of graphing absolute value functions – utilizing Transformation Theories:

Translation:

Vertical Translation of a Graph:

Given any function with the graph $y = f(x)$, and $k > 0$:

The graph of $y = f(x) + k$ is the graph $f(x)$ shifted UPWARDS k units;

The graph of $y = f(x) - k$ is the graph $f(x)$ shifted DOWNWARDS k units.

Horizontal Translation of a Graph:

Given any function with the graph $y = f(x)$, and $h > 0$:

The graph of $y = f(x + h)$ is the graph of $f(x)$ shifted LEFT h units;

The graph of $y = f(x - h)$ is the graph of $f(x)$ shifted RIGHT h units.

Reflection:

Vertical Reflection of a Graph:

Given any function with the graph $y = f(x)$, the graph of $y = -f(x)$ is the graph of $f(x)$ REFLECTED in the x-axis.

Compression and Expansion:

Given any function with the graph $y = f(x)$:

The graph of $y = a \cdot f(x)$ is a vertical expansion (horizontal compression) if $|a| > 1$;

The graph of $y = a \cdot f(x)$ is a vertical compression (horizontal expansion) if $0 < |a| < 1$;

Ch. 4.3 - Absolute Value Functions (Day Two)

eg 1: Graph each of the following:

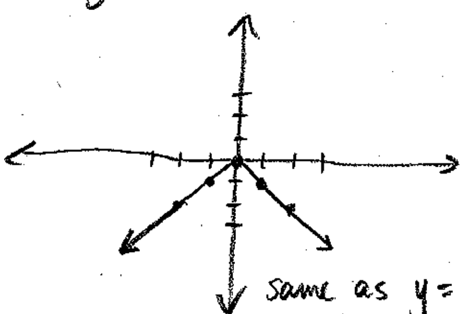
a) $y = -|x|$ b) $y = |x-2|$ c) $y = |x| - 2$

d) $y = 2|x|$ e) $y = \frac{1}{2}|x|$ f) $y = -\frac{1}{2}|x+1| + 3$

a) $y = -x$ when $x \geq 0$

$y = -(-x)$ when $x < 0$

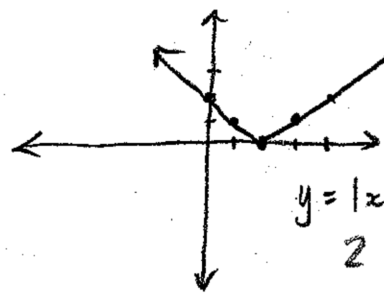
$y = x$ when $x < 0$



same as $y = |x|$ reflected about x-axis

b) $y = x - 2$ when $x \geq 2$

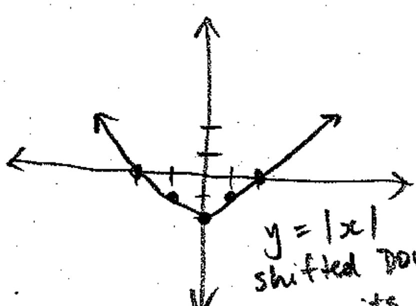
$y = 2 - x$ when $x < 2$



$y = |x|$ shifted RIGHT 2 units.

c) $y = x - 2$ when $x \geq 0$

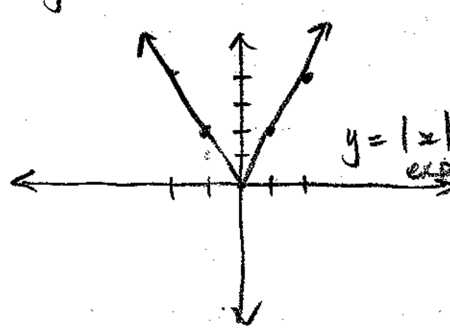
$y = -x - 2$ when $x < 0$



$y = |x|$ shifted DOWN 2 units.

d) $y = 2x$ when $x \geq 0$

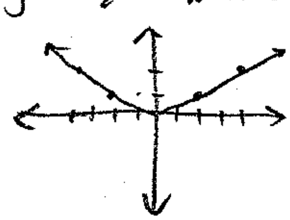
$y = -2x$ when $x < 0$



$y = |x|$ vertically expanded by factor of 2.

e) $y = \frac{1}{2}x$ when $x \geq 0$

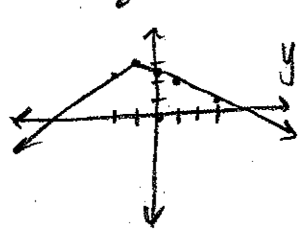
$y = -\frac{1}{2}x$ when $x < 0$



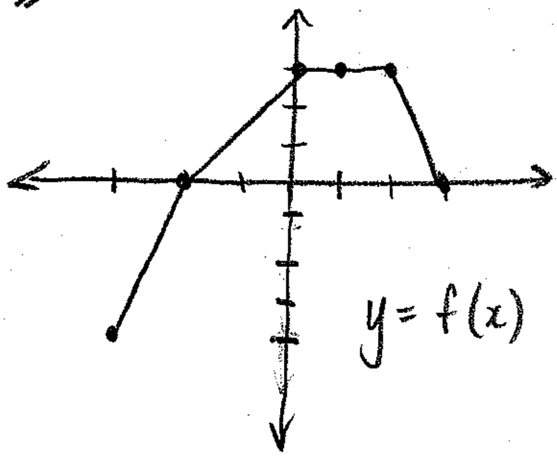
$y = |x|$ vert. compressed by factor of 2

f) $y = -\frac{1}{2}x + \frac{5}{2}$ when $x \geq -1$

$y = \frac{1}{2}x + \frac{7}{2}$ when $x < -1$

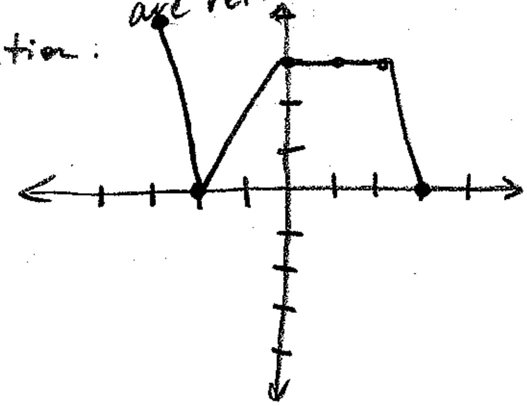


$y = |x|$ shifted LEFT 1, UP 3 and vert. compressed by factor of 2. Also reflected about x-axis eg5: Graph $y = |f(x)|$



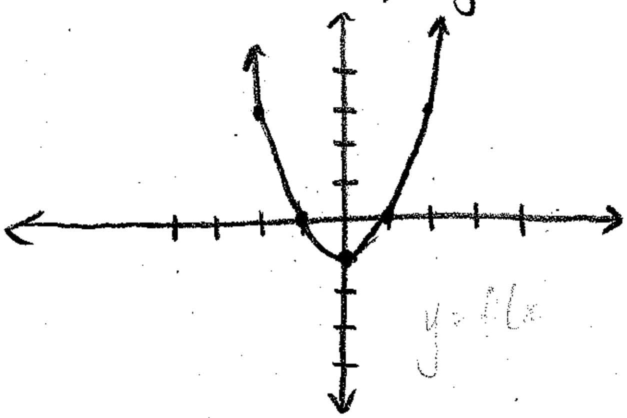
all neg. y values of $y = f(x)$ are reflected about the x-axis

Solution:



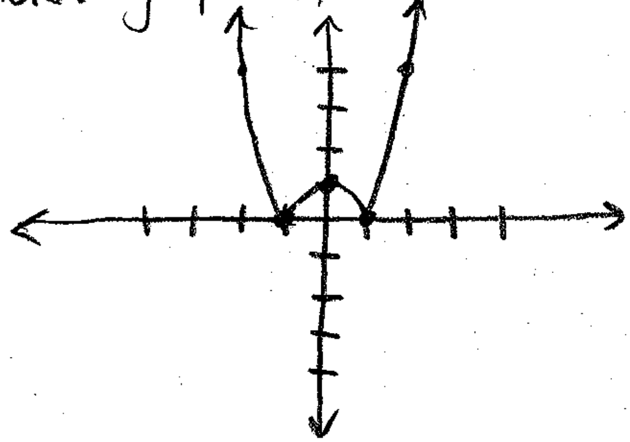
eg6: Graph $f(x) = |x^2 - 1|$

FIRST: Graph $y = x^2 - 1$



THEN: $y = |x^2 - 1|$

all neg. y values reflected!



Hwk: p. 182 # 1, 4 – 13.

Ch. 4.5 - Rational Functions (Day 1)

A function f is a RATIONAL FUNCTION if $f(x) = \boxed{\dfrac{g(x)}{h(x)}}$, where $g(x)$ and $h(x)$ are polynomials and where $h(x) \neq 0$.

Domain: $x \in \mathbb{R}$ except where $h(x) = 0$

eg: (i) $t(x) = \dfrac{1}{x+2}$; Domain: $x \neq -2$

(ii) $s(x) = \dfrac{x}{x^2-9} + 1$; Domain: $x \neq \pm 3$

(iii) $q(x) = \dfrac{x-1}{x^2+1}$; $x \in \mathbb{R}$ (domain)

Graphing Simple Rational Functions

$y = \dfrac{a}{kx - p} + q$ where a is an integer ($a \neq 0$)

read as opposite

read as is

Vertex of Asymptotes $= \left(\boxed{\dfrac{p}{k}}, \boxed{q} \right)$, then graph $\boxed{y = \dfrac{a}{kx}}$ from vertex

Asymptote - a dotted line (representing an x or y value) that a curve approaches, but does not touch.

Greek: ASYMPTŌTOS ⇒ "not meeting"

Two types of Asymptotes:

① Vertical asymptote: $x = \boxed{\dfrac{p}{k}}$ (can have > 1) if quadratic or greater.

② Horizontal asymptote: $y = \boxed{q}$ (only one)

Look back at 1st eg and state equations of asymptotes:

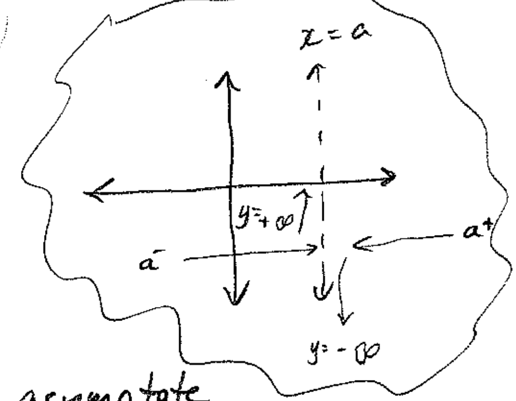
$\boxed{\text{(i) V: } x = -2 \quad \text{H: } y = 0 \quad \text{(ii) V: } x = 3,\ x = -3 \quad \text{H: } y = 1 \quad \text{(iii) No V ; H: } y = 0}$

Other terminology

The line $x = a$ is a vertical asymptote of the function $y = f(x)$ if $y \to \infty$ or as $y \to -\infty$, as $x \to a^+$ or $x \to a^-$

as x approaches 'a' from the RIGHT

as x approaches 'a' from the LEFT.

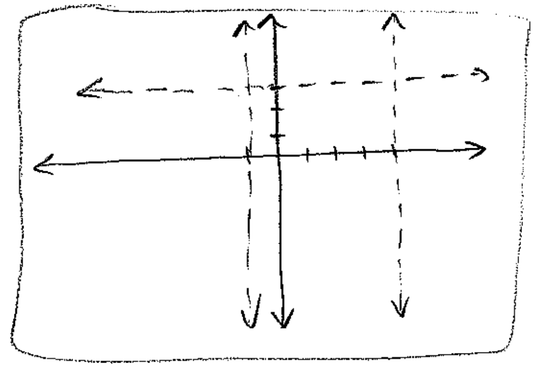


The line $y = b$ is a horizontal asymptote of the fxn $y = f(x)$ if $y \to b$ as $x \to \infty$ or $x \to -\infty$

eg1: Find the equation of, and graph all asymptotes for:

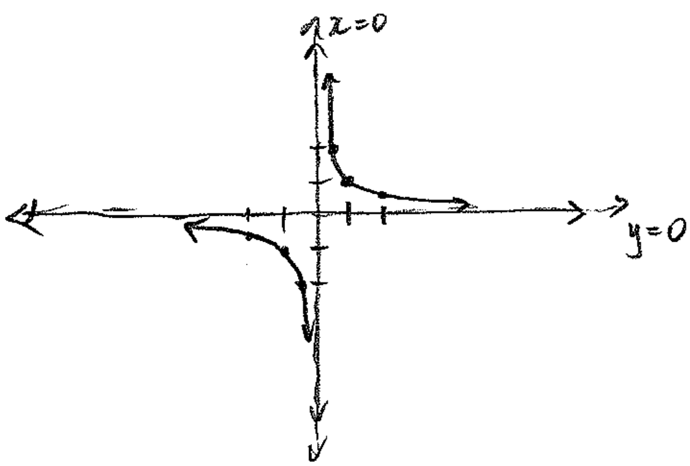
$$y = \frac{6}{(x-4)(x+1)} + 3$$

Verticals: $x = 4$ $x = -1$
Horizontal: $y = 3$



eg2: Graph $y = \frac{1}{x}$

$$y = \frac{1}{1x - 0} + 0$$



Find x & y intercepts.

Vert: $x = 0$ → domain $x \neq 0$
Horiz: $y = 0$
Range: $y \neq 0$

Asymptote Vertex = $(0,0)$

then graph $y = \frac{1}{x}$ from $(0,0)$

x-int: set $y = 0$

$0 = \frac{1}{x}$

$0 = 1$ untrue → no x-int.

y-int: set $x = 0$

$y = \frac{1}{0}$ → no y-int.

eg3: Graph $y = \dfrac{2}{x-3} - 1$

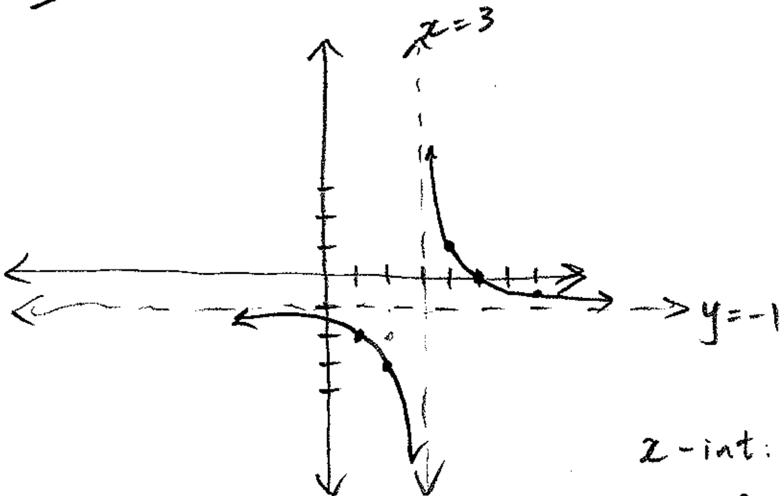
Find x & y ints.

Vert: $x = 3$ (Domain: $x \neq 3$)

Horiz.: $y = -1$ (Range: $y \neq -1$)

Asymp. vertex $= (3, -1)$

then graph $y = \dfrac{2}{x}$ from $(3,-1)$



x-int: $y = 0$

$$0 = \frac{2}{x-3} - 1$$

$$1 = \frac{2}{x-3}$$

$$x - 3 = 2$$

$$x = 5$$

$$\boxed{(5,0)}$$

y-int: $x = 0$

$$y = \frac{2}{0-3} - 1$$

$$y = -\frac{2}{3} - 1$$

$$y = -\frac{5}{3} \quad \boxed{\left(0, -\frac{5}{3}\right)}$$

eg4: Graph $y = \dfrac{-1}{2x+5}$

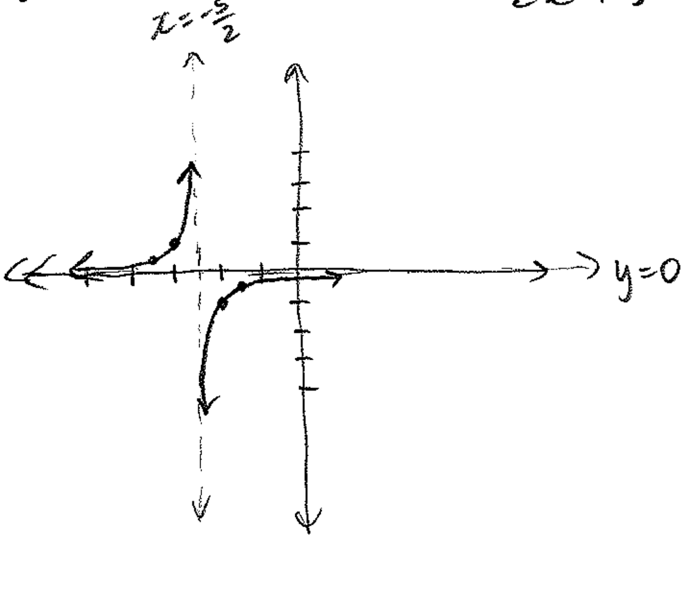
Find x & y ints.

Vert: $x = -\frac{5}{2}$ D: $x \neq -\frac{5}{2}$

Horiz: $y = 0$ R: $y \neq 0$

Asymp. Vertex $= \left(-\frac{5}{2}, 0\right)$

then graph $y = \dfrac{-1}{2x}$ from $\left(-\frac{5}{2}, 0\right)$



Hwk: Worksheet + p. 201 – #1, 3, 4, 5.

Ch. 4.5 Rational Functions (Day 2)

Another Strategy : useful when numerator has a variable and/or when denominator is quadratic.

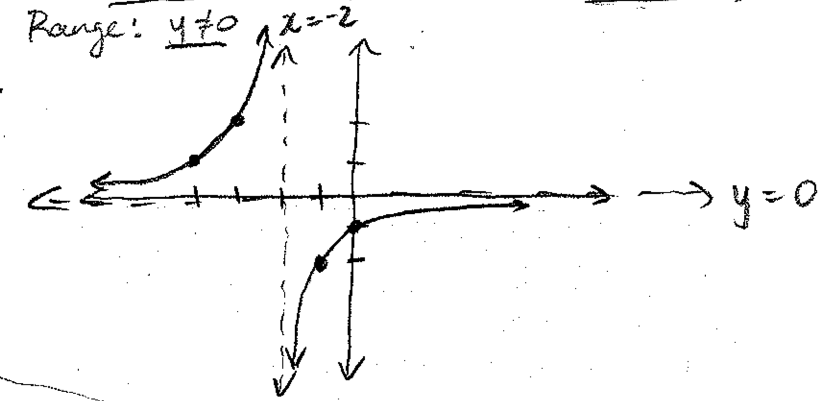
eg1: Using a simple fxn : Graph $f(x) = \frac{-2}{x+2}$

Vertical asymptote: $x = -2$ (Domain : $x \neq -2$)
Horiz. : $y = 0$ Range: $y \neq 0$

x	$f(x)$ or y
0	−1
−3	2
−1	−2
−4	1
-2^-	∞
-2^+	$-\infty$
−100	0^+ ← just higher than 0
100	0^- ← just lower than 0

approaching −2 from left → -2^-
approaching −2 fr. (R) → -2^+

so horiz : $y = 0$



More on Horizontal Asymptote:

- the horizontal asymptote is the value y approaches as x approaches $\pm\infty$.

Consider: $f(x) = \frac{g(x)}{h(x)}$

① If $h(x)$ has a higher power than $g(x)$, then the horiz. asymptote is $y = 0$.

② If $g(x)$ has a higher power than $h(x)$, then there is no horiz. asymptote.

③ If $g(x)$ and $h(x)$ have the same power, then the horizontal asymptote is $y = \boxed{\frac{\text{leading coeff. of numerator}}{\text{leading coeff. of denom.}}}$

eg2: find the horiz. asymptote of each of the following:

a) $f(x) = \frac{3x+1}{x-2}$

$\boxed{y = 3}$

b) $g(x) = \frac{2x}{x^2-4}$

$\boxed{y = 0}$

c) $h(x) = \frac{(2x-1)(3x+2)}{4x^2-1}$

$\boxed{y = \frac{3}{2}}$

d) $i(x) = \frac{2}{x} + 3$

$i(x) = \frac{6+3x}{x}$

$\boxed{y = 3}$

e) $j(x) = \frac{x^2}{2x+1}$

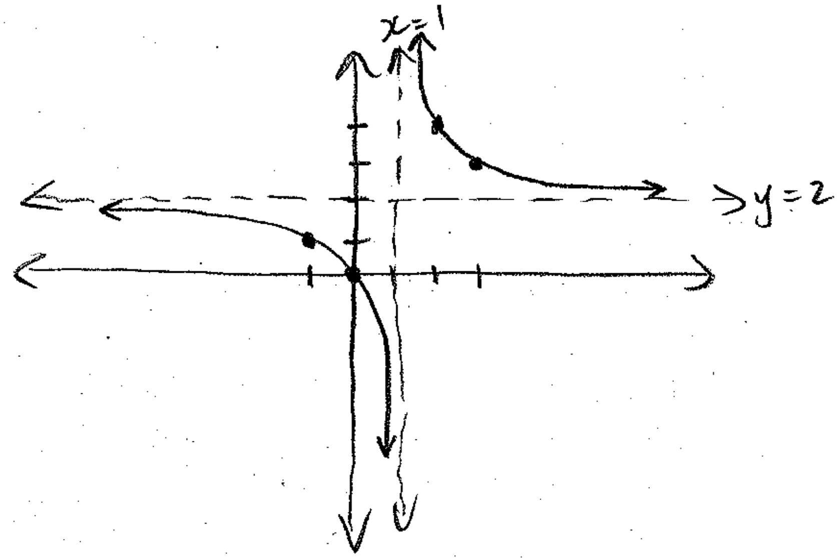
$\boxed{\text{no horiz. asymptote}}$

eg3: Graph $f(x) = \frac{2x}{x-1}$

Vert: $x = 1$ D: $x \neq 1$

Horiz: $y = 2$ R: $y \neq 2$

x	$f(x)$ or y
0	0
2	4
-1	1
3	3
1^-	$-\infty$
1^+	∞
100	2.02 (2^+)
-100	1.98 (2^-)

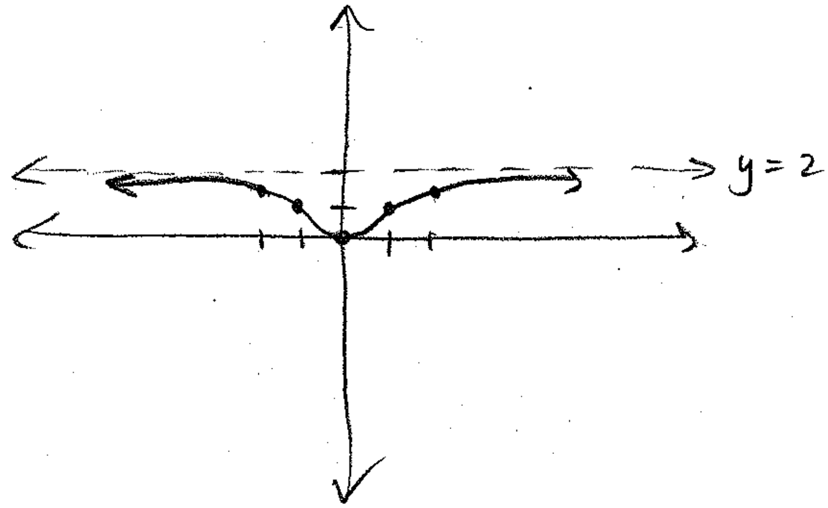


eg4: Graph $f(x) = \dfrac{2x^2}{x^2+1}$

Vert: $\boxed{\begin{array}{c}\text{NONE} \\ x^2 = -1\ ??\end{array}}$ D: $x \in \mathbb{R}$

Horiz: $y = 2$ R: $y \neq 2$

x	$f(x)$ or y
0	0
1	1
−1	1
2	$8/5$
−2	$8/5$
100	$1.99\ldots(2^-)$
−100	$1.99\ldots(2^-)$



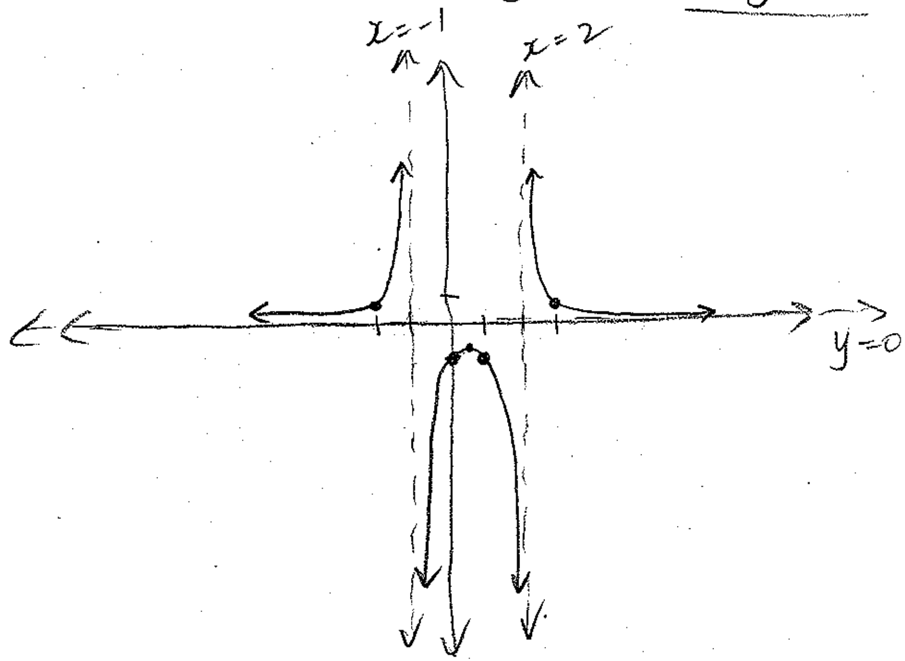
eg5: Graph $g(x) = \dfrac{2}{x^2-x-2}$

Vert: $\boxed{\begin{array}{l} x^2 - x - 2 = 0 \\ (x-2)(x+1) = 0 \\ x = 2 \quad x = -1 \end{array}}$

D: $x \neq 2 \quad x \neq -1$

Horiz: $y = 0$ R: $y \neq 0$

x	$g(x)$ or y
0	−1
1	−1
0.5	−0.9
2^-	$-\infty$
-1^+	$-\infty$
3	$\frac{1}{2}$
2^+	∞
100	0^+
−2	$\frac{1}{2}$
-1^-	∞
−100	0^+

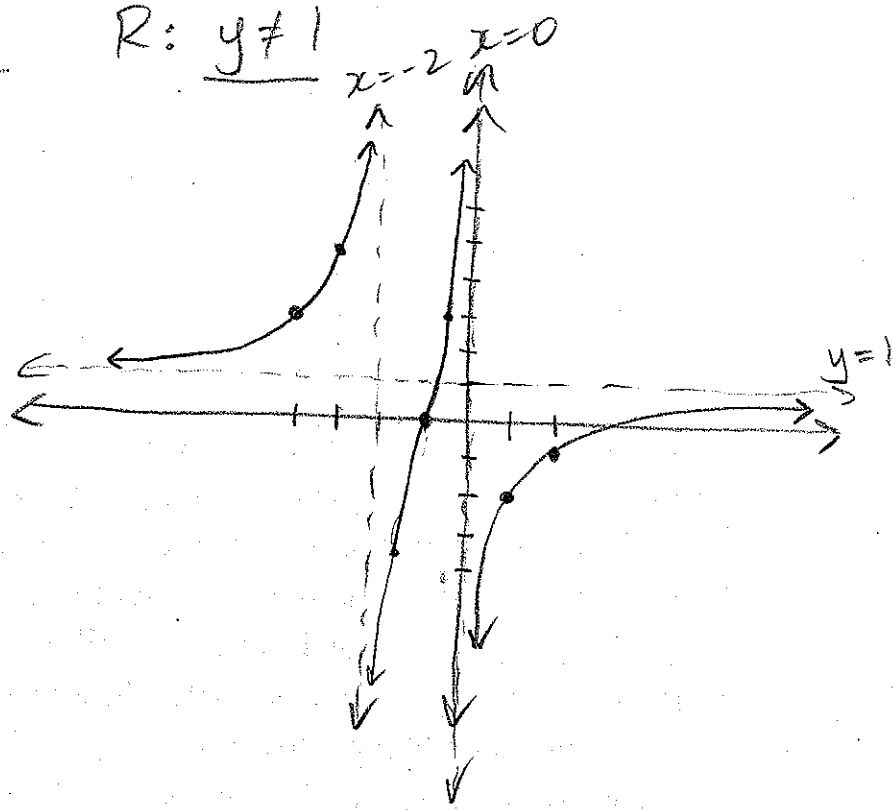


eg6: Graph $h(x) = \dfrac{x^2 - 3x - 4}{x^2 + 2x}$

Vert: $\boxed{\begin{array}{l} x^2 + 2x = 0 \\ x(x+2) = 0 \\ x = 0 \quad x = -2 \end{array}}$ D: $\boxed{\begin{array}{l} x \neq 0 \\ x \neq -2 \end{array}}$

Horiz: $y = 1$ R: $y \neq 1$

x	$h(x)$ or y
1	-2
2	$-\frac{3}{4}$
100	0.95 (1^-)
0^+	$-\infty$
-3	$14/3$
-4	3
-2^-	∞
-100	1^+
-1	0
-0.5	-3.2
-1.5	-3.7
0^-	∞
-2^+	$-\infty$



Homework:

p. 201 – 204

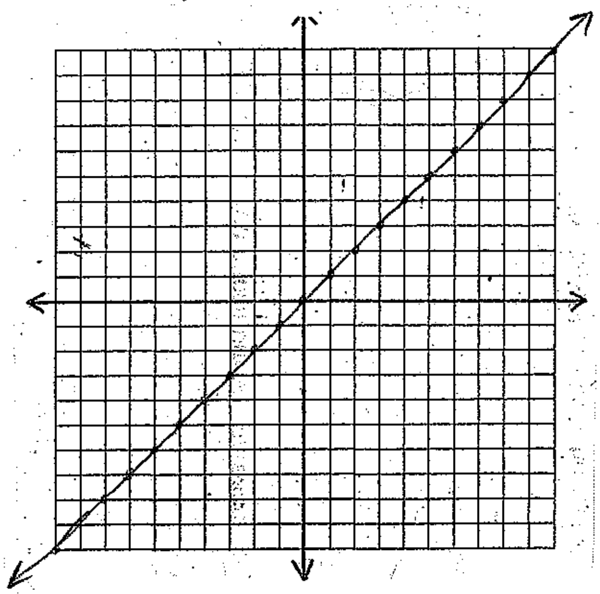
2, 3 - again, 6, 7

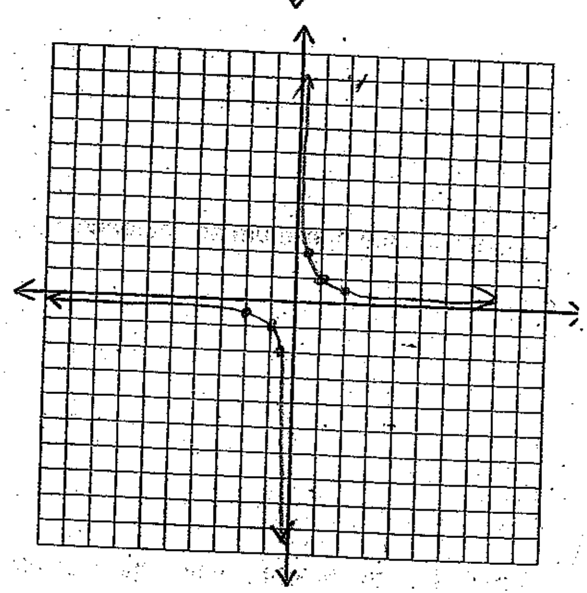
7h not testable

denominator cannot be larger than quadratic.

Ch. 4.6 - Reciprocal Functions

Consider the relationship between the function $f(x) = x$, and its reciprocal $f(x) = \frac{1}{x}$:





Notes: - as x moves from (L) to (R), the graph of $f(x) = x$ is increasing in value, but, the graph of $f(x) = \frac{1}{x}$ is decreasing in value.

- also, as x moves (R) to (L), $f(x) = x$ is decreasing while $f(x) = \frac{1}{x}$ is increasing.

The Relationship Between $f(x)$ and $\frac{1}{f(x)}$:

$f(x)$	$\frac{1}{f(x)}$	Examples
$f(x) \leq -1$	$-1 \leq \frac{1}{f(x)} < 0$	$f(x) = -2 \rightarrow \frac{1}{f(x)} = -\frac{1}{2}$
$-1 \leq f(x) < 0$	$\frac{1}{f(x)} \leq -1$	$f(x) = -\frac{1}{3} \rightarrow \frac{1}{f(x)} = -3$
0	∞ (undefined)	$f(x) = 0 \rightarrow \frac{1}{f(x)} = \frac{1}{0} = \infty$
∞	0	$f(x) = \frac{5}{0} = \infty \rightarrow \frac{1}{f(x)} = \frac{0}{5} = 0$
$0 < f(x) \leq 1$	$\frac{1}{f(x)} \geq 1$	$f(x) = \frac{1}{2} \rightarrow \frac{1}{f(x)} = 2$
$f(x) \geq 1$	$0 < \frac{1}{f(x)} \leq 1$	$f(x) = \frac{7}{3} \rightarrow \frac{1}{f(x)} = \frac{3}{7}$
$f(x)$ INCREASING	$\frac{1}{f(x)}$ DECREASING	$f(x) = 1, 2, 3 \ldots \rightarrow \frac{1}{f(x)} = 1, \frac{1}{2}, \frac{1}{3} \ldots$
$f(x)$ DECREASING	$\frac{1}{f(x)}$ INCREASING	$f(x) = -1, -2, -3 \ldots \rightarrow \frac{1}{f(x)} = -1, \frac{-1}{2}, \frac{-1}{3} \ldots$

eg1: If the point (m, n) is on the graph $y = f(x)$, then which point must be on the graph of $y = \frac{1}{f(x)}$?

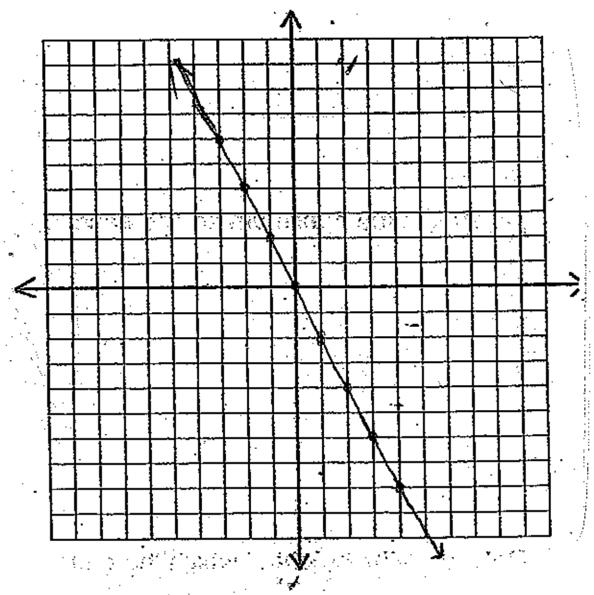
The input (m) does not change, but the output y value is reciprocated $\Rightarrow$ $\boxed{\left(m, \frac{1}{n}\right)}$.

eg2: Given the graph $y = f(x)$, graph $y = \frac{1}{f(x)}$:

a) $f(x) = -2x$

b) $f(x) = \frac{1+x}{x} = \boxed{\frac{1}{x} + 1}$

a) $y = -2x$

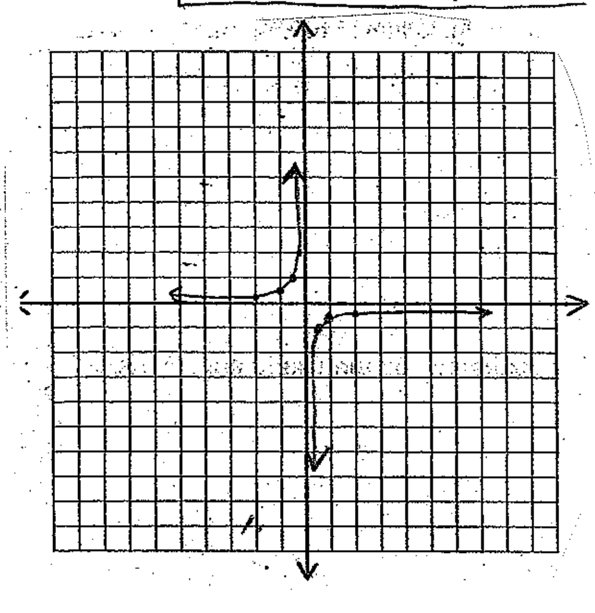


vert. asymp. horiz. asymp.

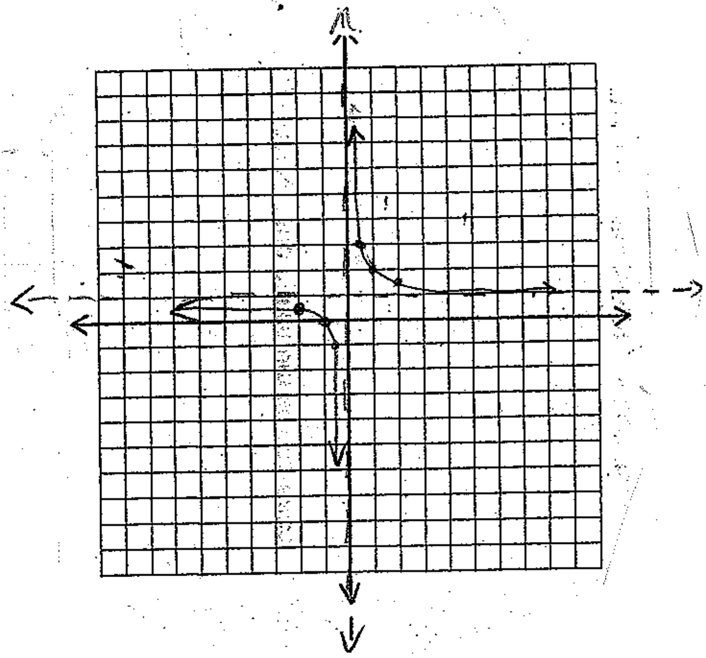
x	$f(x)$	$\frac{1}{f(x)}$
0	0	∞
$\frac{1}{2}$	-1	-1
1	-2	$-\frac{1}{2}$
2	-4	$-\frac{1}{4}$
3	-6	$-\frac{1}{6}$
$-\frac{1}{2}$	1	1
-1	2	$\frac{1}{2}$
-2	4	$\frac{1}{4}$
-3	6	$\frac{1}{6}$

fill in

$y = \boxed{\frac{-1}{2x}}$



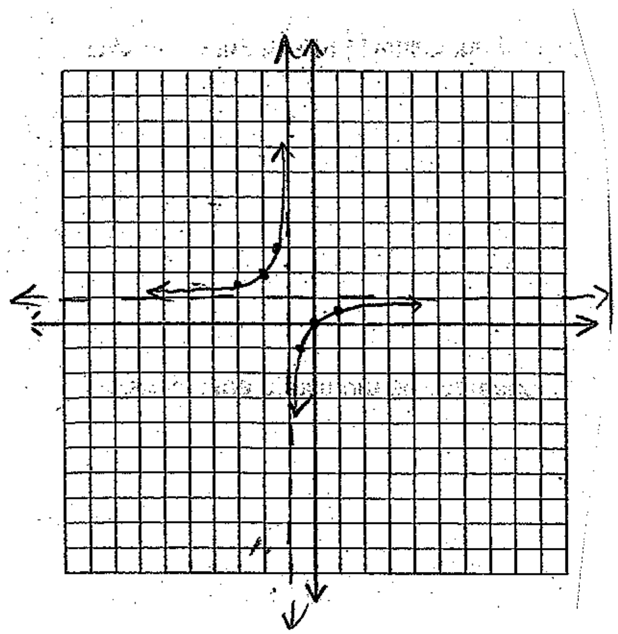
b) $y = \frac{1+x}{x} = \boxed{\frac{1}{x}+1}$



x	$f(x)$	$\frac{1}{f(x)}$
0	∞	0
$\frac{1}{2}$	3	$\frac{1}{3}$
5	$\frac{6}{5}$	$\frac{5}{6}$
$-\frac{1}{2}$	-1	-1
3	$\frac{4}{3}$	$\frac{3}{4}$
vert asymp -1	0	∞
-2	$\frac{1}{2}$	2
-5	$\frac{4}{5}$	$\frac{5}{4}$

fill in

$y = \frac{1}{f(x)} = \boxed{\frac{x}{x+1}}$



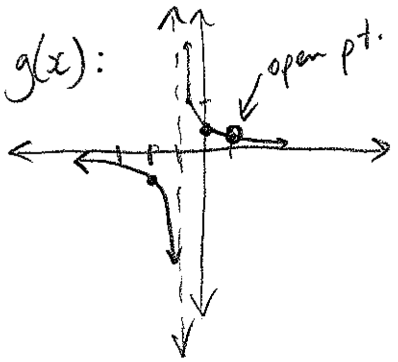
eg3: How do the graphs of $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{x-1}{(x+1)(x-1)}$ differ?

$g(x)$ simplifies to $\frac{1}{x+1}$, so they're the same $\Rightarrow$ EXCEPT...

in $g(x)$, $x \neq 1$, so the point $\left(1, \frac{1}{2}\right)$ is omitted.

see next pg. →

$g(x)$:



eg 4: The product of two positive numbers is 8.

a) Express the sum of the two numbers as a function of one variable.

b) Sketch a graph and estimate:

i) for what values the sum will be as small as possible

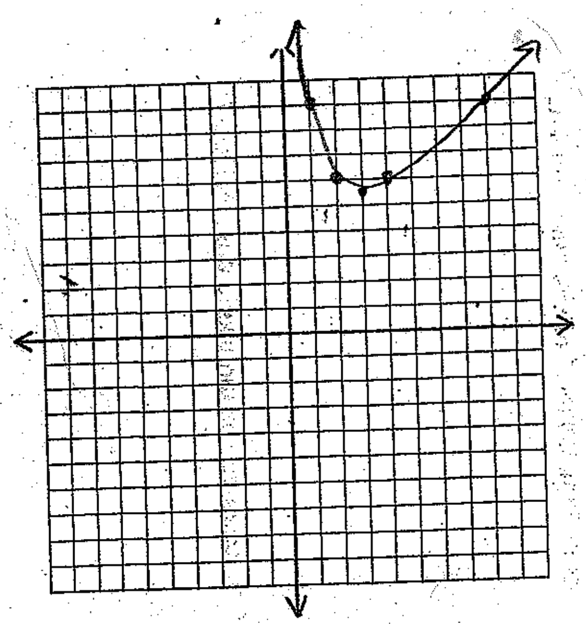
ii) the minimum sum.

S = Sum

a) $xy = 8$ $\qquad x + y = S$

$y = \frac{8}{x}$ $\qquad x + \frac{8}{x} = S$

graph!



x	S
0	∞
1	9
2	6
4	6
3	$\frac{17}{3}$
8	9

good estimate!

$x = 3$

$y = \frac{8}{3}$ $\qquad$ Sum $= \frac{17}{3}$

Hwk: p. 209 # 1–11.

Chapter Review pp. 213–216 # 5–14

Absolute Value Functions (worksheet)

1. $f(x) = |x+2| - 1$

Vertex = $(-2, -1)$

a) D: $x \in \mathbb{R}$

R: $y \geq -1$

b) x-ints:

$0 = |x+2| - 1$

$1 = |x+2|$

$x + 2 = 1$

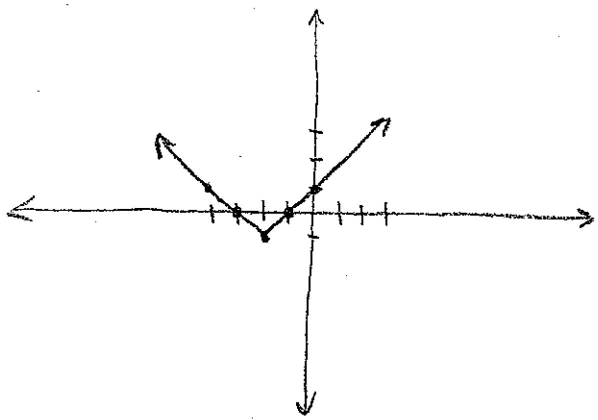
$x = -1$

$\boxed{(-1, 0)}$

$-(x+2) = 1$

$-x - 2 = 1$

$x = -3$ $\boxed{(-3, 0)}$



y-int: $y = |0+2| - 1$

$y = 1$ $\boxed{(0, 1)}$

2. $f(x) = |x-4| - 3$

Vertex = $(4, -3)$

a) D: $x \in \mathbb{R}$

R: $y \geq -3$

b) x-ints:

$0 = |x-4| - 3$

$3 = |x-4|$

$x - 4 = 3$

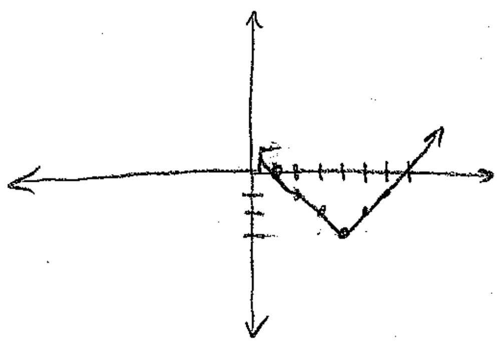
$x = 7$

$\boxed{(7, 0)}$

$-(x-4) = 3$

$-x + 4 = 3$

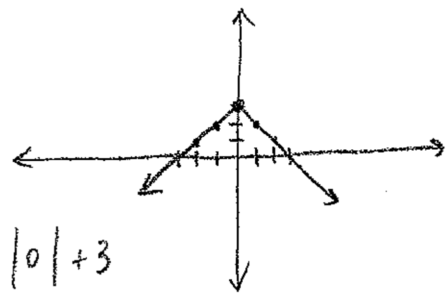
$x = 1$ $\boxed{(1, 0)}$



y-int: $y = |0-4| - 3$

$y = 4 - 3 = 1$ $\boxed{(0, 1)}$

3. $f(x) = -|x| + 3$ $\quad V = (0,3)$



a) D: $x \in \mathbb{R}$

R: $y \leq 3$

b) x-ints:

$|x| = 3$

$x = 3$ $\quad -x = 3$

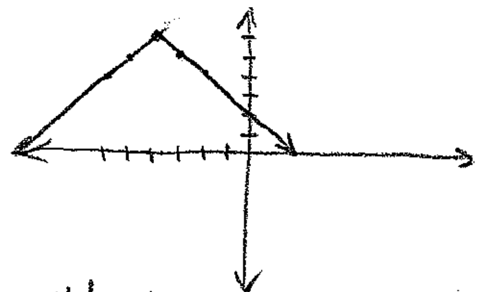
$\boxed{(3,0)}$ $\quad x = -3$

$\boxed{-3,0}$

y-int: $y = -|0| + 3$

$y = 3$ $\boxed{(0,3)}$

4. $y = -|x+4| + 6$ $\quad V = (-4,6)$



a) D: $x \in \mathbb{R}$

R: $y \leq 6$

b) x-ints:

$|x+4| = 6$

$x + 4 = 6$ $\quad -(x+4) = 6$

$x = 2$ $\quad -x - 4 = 6$

$\boxed{(2,0)}$ $\quad x = -10$

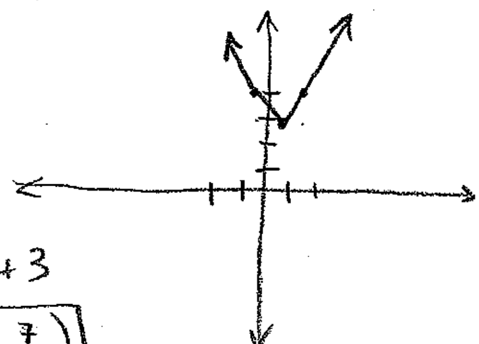
$\boxed{(-10,0)}$

y-int:

$y = -|0+4| + 6$

$y = 2$ $\boxed{(0,2)}$

5. $y = \left|x - \frac{1}{2}\right| + 3$ $\quad V = \left(\frac{1}{2}, 3\right)$



a) D: $x \in \mathbb{R}$

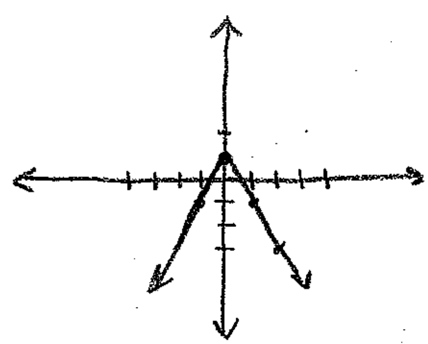
R: $y \geq 3$

b) No x-ints!

y-int: $y = \left|0 - \frac{1}{2}\right| + 3$

$y = \frac{7}{2}$ $\boxed{\left(0, \frac{7}{2}\right)}$

6. $f(x) = -2|x| + 1$ $\quad V = (0,1)$



a) D: $x \in \mathbb{R}$

R: $y \leq 1$

b) x-ints:

$2|x| = 1$

$|x| = \frac{1}{2}$

$x = \frac{1}{2}$ $\quad x = -\frac{1}{2}$

$\boxed{\frac{1}{2}, 0}$ $\quad \boxed{-\frac{1}{2}, 0}$

y-int: $y = -2|0| + 1$

$y = 1$ $\boxed{(0,1)}$

7. $y = -3|x+2| - 3 \quad V = (-2,-3)$

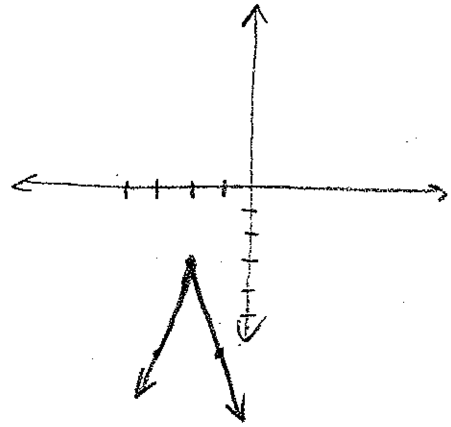
a) D: $x \in \mathbb{R}$

R: $y \leq -3$

b) NO x-ints! y-int: $y = -3|0+2| - 3$

$y = -9$

$\boxed{(0,-9)}$



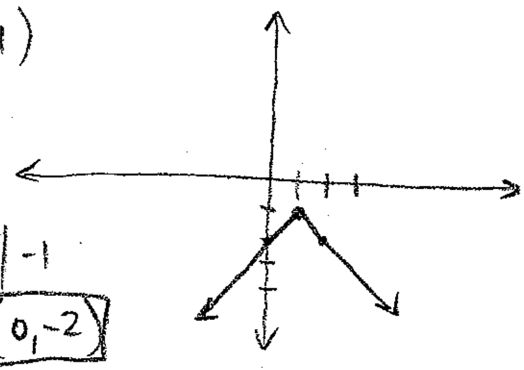
8. $f(x) = -|x-1| - 1 \quad V = (1,-1)$

a) D: $x \in \mathbb{R}$

R: $y \leq -1$

b) NO x-ints! y-int: $y = -|0-1| - 1$

$y = -2 \ \boxed{(0,-2)}$



9. $f(x) = 3 - |x+5|$

$f(x) = -|x+5| + 3 \quad V = (-5,3)$

a) D: $x \in \mathbb{R}$

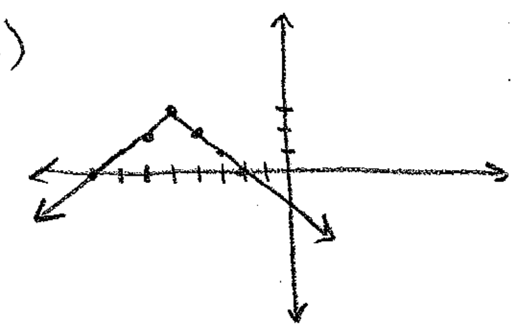
R: $y \leq 3$

b) x-ints:

$|x+5| = 3$

$x+5 = 3 \qquad -(x+5) = 3$

$x = -2 \ \boxed{(-2,0)} \qquad x = -8 \ \boxed{(-8,0)}$



10. $y = -\frac{1}{4}|x-3| + \frac{1}{2} \quad V = \left(3, \frac{1}{2}\right)$

a) D: $x \in \mathbb{R}$

R: $y \leq \frac{1}{2}$

b) x-ints:

$\frac{1}{4}|x-3| = \frac{1}{2}$

$|x-3| = 2$

$x-3 = 2 \qquad -x+3 = 2$

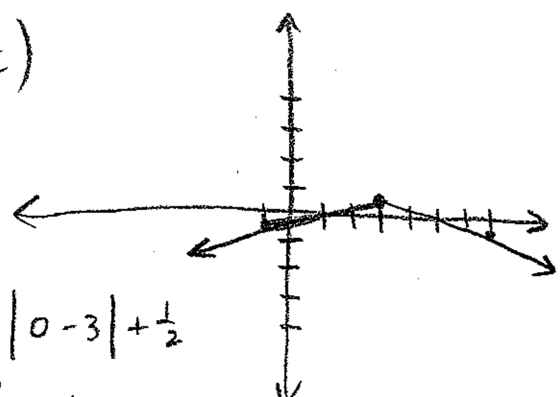
$x = 5 \qquad x = 1$

$\boxed{(5,0)} \qquad \boxed{(1,0)}$

y-int: $y = -\frac{1}{4}|0-3| + \frac{1}{2}$

$y = -\frac{3}{4} + \frac{1}{2}$

$y = -\frac{1}{4} \ \boxed{\left(0,-\frac{1}{4}\right)}$



Rational Functions (worksheet)

1. $f(x) = \dfrac{1}{(x+3)(x-2)}$; D: $x \in \mathbb{R}$; $x \neq -3, 2$
   R: $y \in \mathbb{R}$; $y \neq 0$

Vertical Asymptotes $\begin{cases} x = -3 \\ x = 2 \end{cases}$ Horizontal Asymptotes: $y = 0$

2. $f(x) = \dfrac{-3}{x^2+6x+8} = \dfrac{-3}{(x+4)(x+2)}$; D: $x \in \mathbb{R}$; $x \neq -2, -4$
   R: $y \in \mathbb{R}$; $y \neq 0$

Vertical: $x = -2$, $x = -4$ Horizontal: $y = 0$

3. $f(x) = \dfrac{2}{(2x+3)(2x-3)}$; D: $x \in \mathbb{R}$; $x \neq -\frac{3}{2}, \frac{3}{2}$
   R: $y \in \mathbb{R}$; $y \neq 0$

Vertical: $x = -\frac{3}{2}$, $x = \frac{3}{2}$ Horizontal: $y = 0$

4. $y = \dfrac{-1}{(x+4)(x-1)} + 2$; D: $x \in \mathbb{R}$; $x \neq -4, 1$
   R: $y \in \mathbb{R}$; $y \neq 2$

Vertical: $x = -4$, $x = 1$ Horizontal: $y = 2$

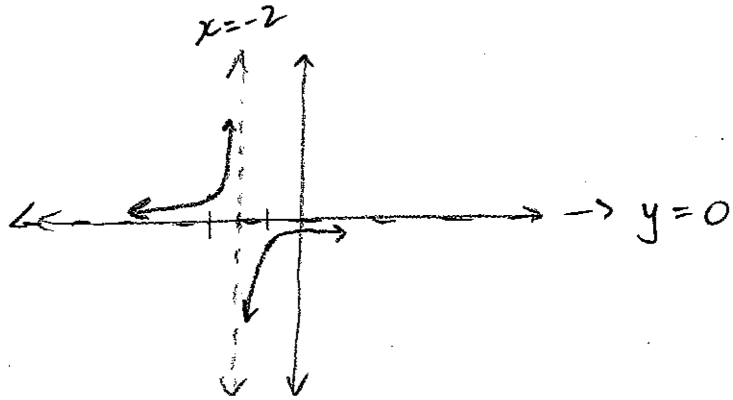
5. $y = \dfrac{1}{(x-3)(2x-1)} - 1$; D: $x \in \mathbb{R}$; $x \neq 3, \frac{1}{2}$
   R: $y \in \mathbb{R}$; $y \neq -1$

Vertical: $x = 3$, $x = \frac{1}{2}$ Horizontal: $y = -1$

B) 1. $f(x) = \dfrac{-1}{x+2}$

$\boxed{\text{No } x\text{-ints!}}$

y-int $= \boxed{\left(0, -\frac{1}{2}\right)}$

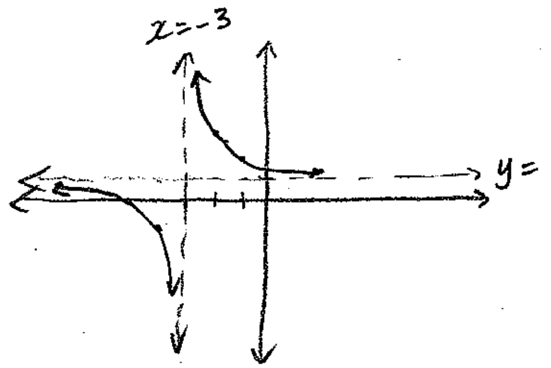


2. $f(x) = \dfrac{2}{x+3} + 1$

x-int: $0 = \dfrac{2}{x+3} + 1$

$\boxed{(-5, 0)}$

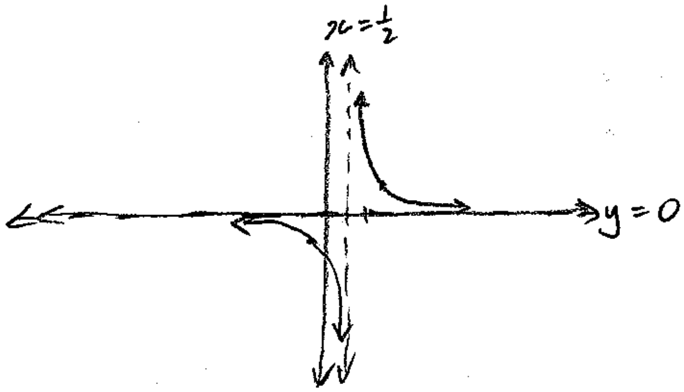
y-int: $\boxed{\left(0, \frac{5}{3}\right)}$



3. $y = \dfrac{2}{2x-1}$

$\boxed{\text{No } x\text{-ints!}}$

y-int $= \boxed{(0, -2)}$

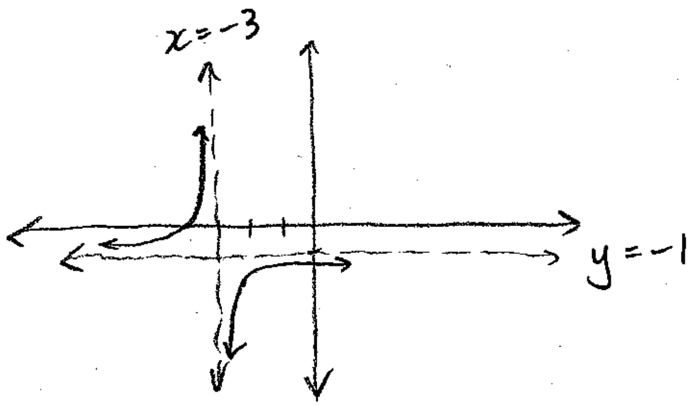


4. $f(x) = \dfrac{-1}{x+3} - 1$

x-int: $0 = \dfrac{-1}{x+3} - 1$

$\boxed{(-4, 0)}$

y-int: $\boxed{\left(0, -\frac{4}{3}\right)}$



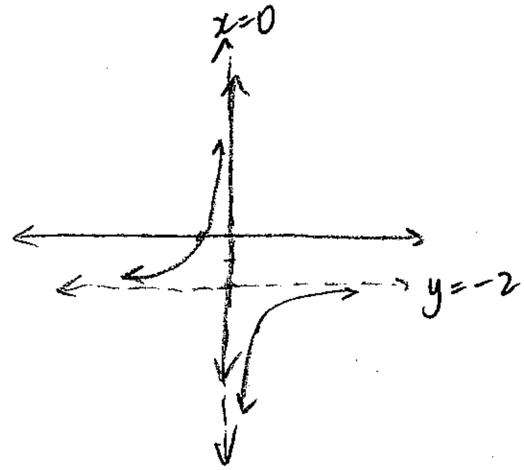
5. $y = \frac{2}{-x} - 2$

$y = \frac{-2}{x} - 2$

x-int: $0 = \frac{-2}{x} - 2$

$x = -1$ $\boxed{(-1,0)}$

$\boxed{\text{No y-int!}}$

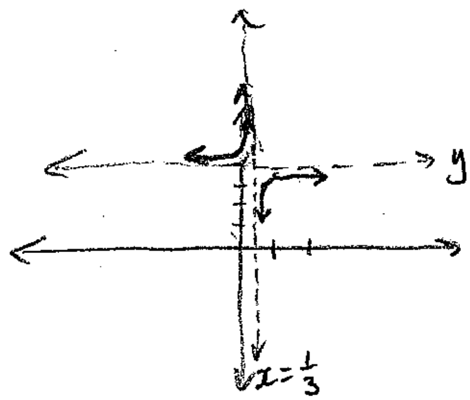


6. $f(x) = \frac{-1}{3x-1} + 4$

x-int: $0 = \frac{-1}{3x-1} + 4$

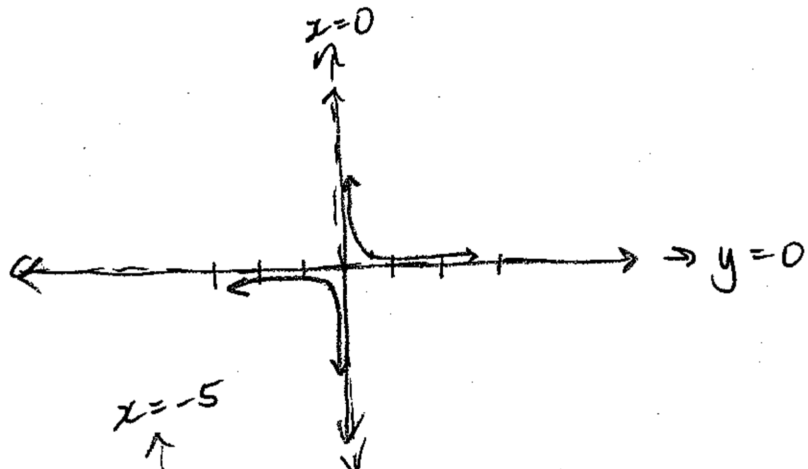
$\boxed{\left(\frac{5}{12}, 0\right)}$

y-int: $\boxed{(0,5)}$



7. $y = \frac{0.25}{2x}$

$\boxed{\text{No x-ints!}}$
$\boxed{\text{No y-ints!}}$



8. $f(x) = \frac{-3}{x+5} - 4$

x-int: $0 = \frac{-3}{x+5} - 4$

$\boxed{\left(-\frac{23}{4}, 0\right)}$

y-int: $\boxed{\left(0, -\frac{23}{5}\right)}$