Enrichment - Finding the equation of a line given the slope and one point.

We already know how to graph a line given that info.

But, can we create an equation without a diagram? YES!

How?

eg: Find the equation of a line with a slope of 2 and a point at $(4, -1)$.

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{2}{1} = \frac{y-(-1)}{x-4}$$

$$2(x-4) = y+1$$

$$2x-8 = y+1$$

$$\boxed{y = 2x-9}$$

- regardless of the information, we use the same process.

Thus, a generalized formula can be created:

POINT-SLOPE FORM:

$$y - y_1 = m(x - x_1)$$

where: m = slope

(x_1, y_1) = given point on line

(x, y) = all other points on line

eg2: Find equation of line with slope $-\frac{1}{2}$ and passing through point $(-3, -2)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{1}{2}(x - (-3))$$

$$y + 2 = -\frac{1}{2}x - \frac{3}{2}$$

$$\boxed{y = -\frac{1}{2}x - \frac{7}{2}}$$

Handout # 7 - 9

7. Match each description with an equation.

a) Slope $= -3$, passing through $(-1, 2)$ _____

b) Slope $= 3$, y-intercept $(0, -6)$ _____

c) Passing through $(0, 0)$ and $(3, -1)$ _____

d) Passing through $(0, 0)$ and $(-1, 3)$ _____

e) Passing through $(2, 0)$ and $(0, -6)$ _____

i) $y = 3x$

ii) $y = -\frac{1}{3}x$

iii) $y = -3x$

iv) $x - 3y = 6$

v) $3x - y = 6$

vi) $y - 2 = -3(x + 1)$

vii) $y + 2 = -3(x - 1)$

8. Match each equation with the graph it most closely resembles.

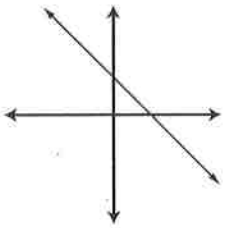
a) $y = x - 2$ _____

b) $y = -x - 2$ _____

c) $y = -x + 2$ _____

d) $y = x + 2$ _____

i)



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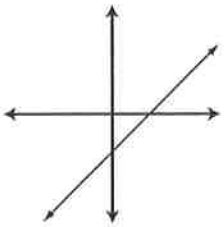

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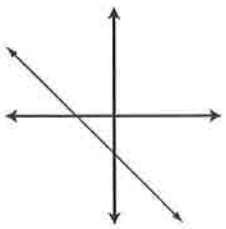

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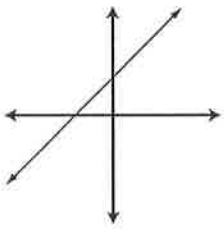


9. Write the equation of each line in slope-intercept form.

a) $(0, 2)$, $m = 2$ _____

b) $(0, -3)$, $m = \frac{1}{2}$ _____

c) $(0, 3)$, $m = 0$ _____

d) $(0, -2)$, $m = -\frac{2}{3}$ _____

e) $\left(0, -\frac{1}{2}\right)$, $m = -\frac{3}{4}$ _____

f) $(0, 2.3)$, $m = 0.4$ _____

7. **a)** *vi* **b)** *v* **c)** *ii* **d)** *iii* **e)** *v*

Answers!

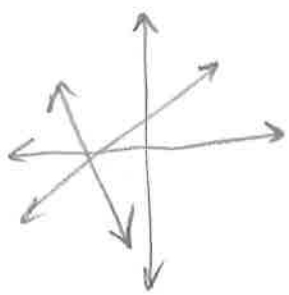
8. **a)** *ii* **b)** *iii* **c)** *i* **d)** *iv*

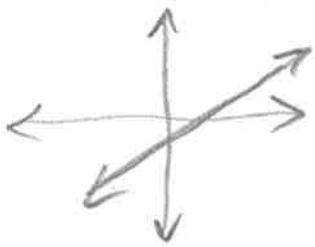
9. **a)** $y = 2x + 2$ **b)** $y = \frac{1}{2}x - 3$ **c)** $y = 3$ **d)** $y = -\frac{2}{3}x - 2$ **e)** $y = -\frac{3}{4}x - \frac{1}{2}$

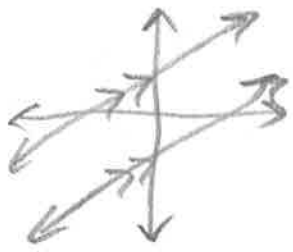
Intersecting Lines vs. Non-Intersecting Lines

- lines that intersect ONCE (at 1 point) must have different slopes.
- lines that intersect INFINITELY (at an infinite number of points) must be the same.
- lines that do NOT intersect must be PARALLEL.

Type of Arrangement	Number of Solutions	Slope	y-int
Intersecting	1	different	who cares?
Coincidental	∞	same	same
Parallel	0	same	different

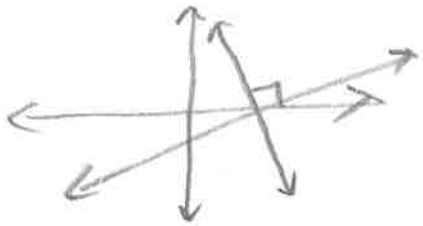






Perpendicular Lines

Perpendicular lines intersect at <u>one</u> point, AND, their intersection creates a 90° angle



Their slopes are different, but related ⇒ their slopes are NEGATIVE RECIPROCALS of one another.

eg: What is the negative reciprocal of:

a) 2 — $-\frac{1}{2}$

b) 0 — undef.

c) $-\frac{1}{3}$ — 3

d) $\frac{4}{3}$ — $\frac{-3}{4}$

e) $-\frac{7}{2}$ — $\frac{2}{7}$

f) -6 — $\frac{1}{6}$

eg 2: Determine if the line through the first pair of points is paralled to, perpendicular to, or neither parallel nor perpendicular to the line through the second pair of points.

a) $(-4,1)(3,5)$; $(1,-3)(15,-11)$

$m = \frac{5-1}{3+4} = \frac{4}{7}$ $\quad$ $m = \frac{-11+3}{15-1} = \frac{-8}{14} = -\frac{4}{7}$

NEITHER

b) $(-4,1)(3,5)$; $(1,-11)(15,-3)$

$m = \frac{4}{7}$ $\quad$ $m = \frac{-3+11}{15-1} = \frac{8}{14} = \frac{4}{7}$

Parallel

c) $(-4,1)(3,5)$; $(-13,10)(-9,3)$

$m = \frac{4}{7}$ $\quad$ $m = \frac{3-10}{-9+13} = \frac{-7}{4}$

Perpendicular

d) $(x, 3)(3, x)$; $(7, x)(x, 7)$

$$m = \frac{x-3}{3-x} = \frac{x-3}{-1(x-3)} = -1$$

$$m = \frac{7-x}{x-7} = \frac{-1(x-7)}{x-7} = -1$$

Parallel $(\parallel)$

e) $(-x, -2)(2, x)$; $(x, 8)(8, x)$

$$m = \frac{x+2}{2+x} = 1 \qquad m = \frac{x-8}{8-x} = -1$$

Perpendicular $(\perp)$

Handout # 1, 2, 3

5.4 Exercise Set

Determine if the following slopes are parallel, perpendicular, or neither.

a) $m_1 = \frac{2}{3},\ m_2 = \frac{3}{2}$ _____

b) $m_1 = 5,\ m_2 = -\frac{1}{5}$ _____

c) $m_1 = \frac{6}{3},\ m_2 = 2$ _____

d) $m_1 = 0,\ m_2 =$ undefined _____

e) $m_1 = -\frac{9}{6},\ m_2 = \frac{3}{2}$ _____

f) $m_1 = 0,\ m_2 = 0$ _____

g) $m_1 = \frac{15}{12},\ m_2 = \frac{4}{5}$ _____

h) $m_1 = 4,\ m_2 = \frac{12}{3}$ _____

i) $m_1 = \sqrt{8},\ m_2 = 2\sqrt{2}$ _____

j) $m_1 = 0.125,\ m_2 = \frac{1}{8}$ _____

k) $m_1 = -0.125,\ m_2 = 8$ _____

l) $m_1 = ab^{-1},\ m_2 = -a^{-1}b$ _____

Answers:

1. **a)** neither **b)** perpendicular **c)** parallel **d)** perpendicular **e)** neither **f)** parallel **g)** neither **h)** parallel **i)** parallel **j)** parallel **k)** perpendicular **l)** perpendicular

Complete the table.

Line l_1 has slope m_1, line l_2 has slope m_2, line l_3 has slope m_3, with $l_1 \parallel l_2$, and $l_1 \perp l_3$

m_1	m_2	m_3
$\frac{2}{3}$		
	$-\frac{3}{4}$	
		-4
undefined		
	undefined	
		undefined
0		
	0	
		0

Answers:

2.

m_1	m_2	m_3
$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{3}{2}$
$-\frac{3}{4}$	$-\frac{3}{4}$	$\frac{4}{3}$
$\frac{1}{4}$	$\frac{1}{4}$	-4
undefined	undefined	0
undefined	undefined	0
0	0	undefined
0	0	undefined
0	0	undefined
undefined	undefined	0

3. Determine whether the line passing through the first pair of points is parallel, perpendicular, or neither to the line passing through the second pair of points.

a) (3, 2) and (1, 4); (− 1, − 2) and (− 3, − 4)

b) (5, 6) and (7, 8); (− 5, − 6) and (− 7, − 8)

c) (0, 4) and (− 1, 2); (− 3, 5) and (1, 7)

d) (2, 3) and (3, 0); (− 2, − 5) and (1, − 6)

e) (3, 5) and (− 2, 5); (1, 4) and (1, − 2)

f) (4, − 3) and (− 2, − 1); (10, − 1) and (1, − 4)

g) (a, b) and (b, a); (c, d) and (d, c)

h) (a, b) and (b, a); (c, d) and $\left(\frac{d}{2}, \frac{c}{2}\right)$

i) (a, b) and (b, a); $(-c, d)$ and $(-d, c)$

j) (a, b) and (b, a); (c, d) and $(-d, -c)$

Answers:

3. **a)** perpendicular **b)** parallel **c)** neither **d)** neither **e)** perpendicular **f)** neither **g)** parallel **h)** neither **i)** parallel **j)** perpendicular