Chapter 7.1-7.2 – Non-Linear Systems of Equations

A *system of equations* consists of two or more equations.

A *linear system* possesses all variables of the 1^{st} degree (no higher than power 1).

A *non-linear system* possesses at least one variable with a power other than 1.

The key to solving Non-Linear Systems (and Linear Systems, for that matter) is the process of *SUBSTITUTION.*

Steps:

1. Solve one equation for a particular variable (HINT: Look for a variable with a coefficient of 1 or -1.

2. Substitute the computed expression into the other equation given.

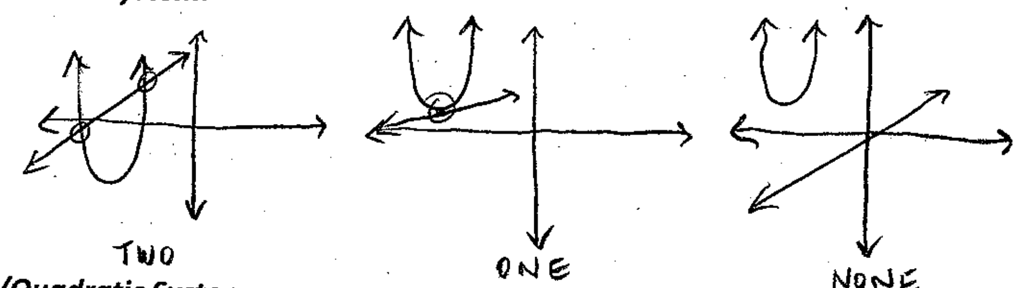
3. Solve for the remaining variable in the equation.

4. Substitute the solution into either of the given equations in order to solve for the other variable.

5. Check the solution(s) → Answer(s) must satisfy ("work in") both equations!

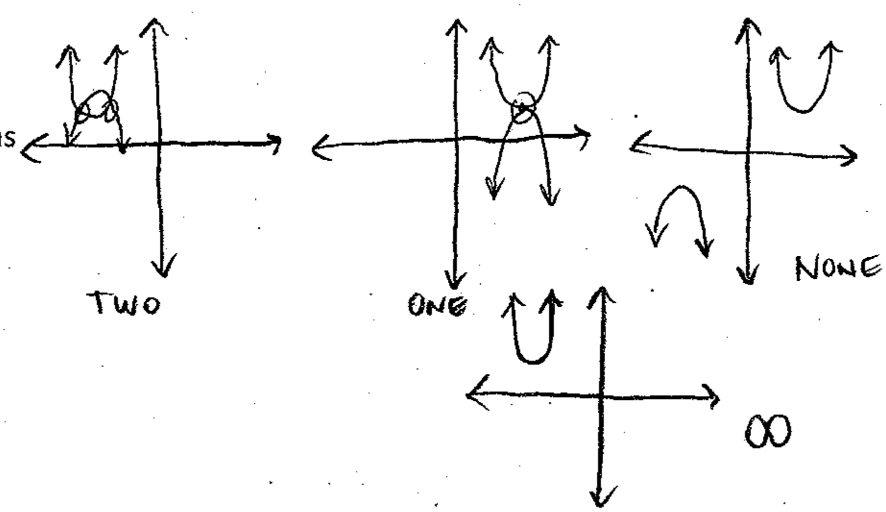
***Possibilities for a Linear/Quadratic System*:**

- Two real solutions
- One real solution
- No solution



***Possibilities for a Quadratic/Quadratic System*:**

- Two real solutions
- One real solution
- No solution
- Infinitely many real solutions



Ch. 7.2 - Solving Non-Linear Systems Algebraically

- PROVIDE typed HANDOUT:
  - the key? Using SUBSTITUTION:

* A system of eq'ns consists of ≥ 2 eq'ns.

LINEAR = all 1^{st} degree (power 1)

NON-LINEAR = at least one power > 1.

on handout!!

Steps:
1. Solve one equation for a variable (look for a variable w/ coefficient 1 or -1)
2. Substitute expression into other equation
3. Solve ~~for~~ the existing variable.
4. Substitute into either equation to solve for other variable.
5. Check! Answer(s) must work in both equations!

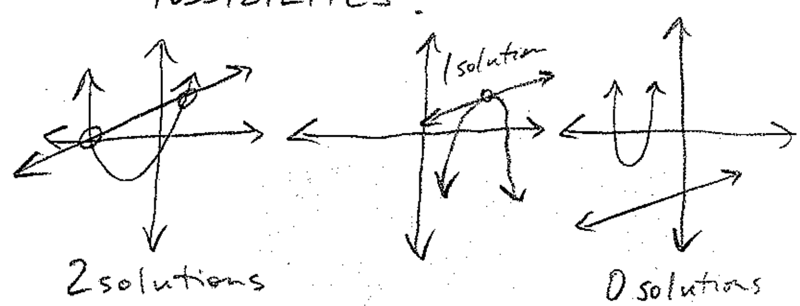
eg 1: Solve the system:

① $y = x^2 - 3x - 4$

② $2x - y = 4$

* essentially, this question is asking where a line intersects a parabola

POSSIBILITES?



② $y = 2x - 4$

① $(2x-4) = x^2 - 3x - 4$

$0 = x^2 - 5x$

$0 = x(x-5)$

$x = 0$ and 5

SUB into ②

$y = 2(0) - 4$ $\qquad$ $y = 2(5) - 4$

$y = -4$ $\qquad$ $y = 6$

$\boxed{(0,-4)}$ $\qquad$ $\boxed{(5,6)}$

eg2: $y = -\frac{1}{2}x^2 + 2x - 3$
$y = x - 2$

so, $x - 2 = -\frac{1}{2}x^2 + 2x - 3$

$$2x - 4 = -x^2 + 4x - 6$$

$$x^2 - 2x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2} = \boxed{\text{no solution}}$$

line & parabola do not intersect.

eg3: $y = \frac{1}{2}x^2 - 2x + 3$
$2x - 2y = 3$

$$2x - 2\left(\tfrac{1}{2}x^2 - 2x + 3\right) = 3$$

$$2x - x^2 + 4x - 6 = 3$$

$$0 = x^2 - 6x + 9$$

$$(x-3)(x-3) = 0$$

$$x = 3$$

$$2(3) - 2y = 3$$

$$-2y = -3$$

$$y = \frac{3}{2}$$

$$\boxed{\left(3, \tfrac{3}{2}\right)}$$

line is tangent to parabola.

eg4: $y = x^2 - 3x - 4$
$2x - y = 3$

$$y = 2x - 3$$

$$2x - 3 = x^2 - 3x - 4$$

$$0 = x^2 - 5x - 1$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2}$$

$$x = \frac{5 \pm \sqrt{29}}{2}$$

(i) $y = 2\left(\frac{5 + \sqrt{29}}{2}\right) - 3$

$y = 2 + \sqrt{29}$ ✓

(ii) $y = 2\left(\frac{5 - \sqrt{29}}{2}\right) - 3$

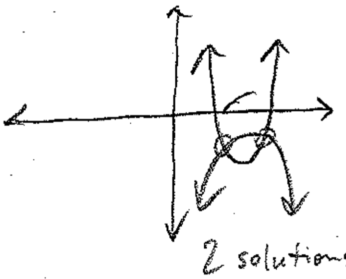
$y = 2 - \sqrt{29}$

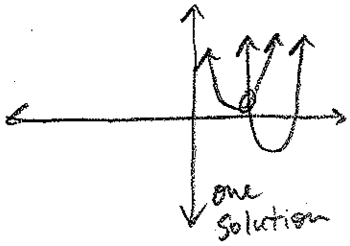
$$\boxed{\left(\frac{5+\sqrt{29}}{2},\ 2+\sqrt{29}\right),\quad \left(\frac{5-\sqrt{29}}{2},\ 2-\sqrt{29}\right)}$$

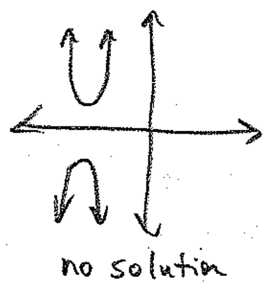
Check w/ decimals

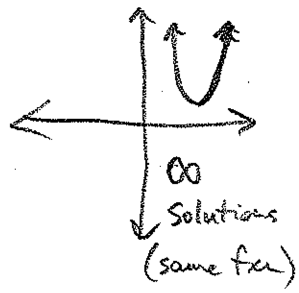
Solving System of Two Quadratics:

4 possibilities:









eg 5: Solve the system:

$$x^2 - y = 0 \Rightarrow y = x^2$$

$$x^2 - 2x + y = 4$$

$$x^2 - 2x + x^2 = 4$$

$$2x^2 - 2x - 4 = 0$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, \quad x = -1$$

$$y = 2^2 \qquad y = (-1)^2$$

$$y = 4 \qquad y = 1$$

$$\boxed{(0,4)} \qquad \boxed{(-1,1)}$$

eg6: Solve the system:

$$y = x^2 - x - 3$$
$$y = 2x^2 - x + 7$$

$$x^2 - x - 3 = 2x^2 - x + 7$$

$$0 = x^2 + 10$$

$$x^2 = -10$$

$\boxed{\text{no solutions}}$

* Do example 6 p. 311 and example 8 p. 312

Homework:

p. 313 # 1–10

hint x-axis $\Rightarrow$ $y = 0$

y-axis $\Rightarrow$ $x = 0$

horizontal line $\Rightarrow$ $y = \#$

Vertical line $\Rightarrow$ $x = \#$

Ch. 7.1 Graphing Non-Linear Systems of Equations

eg1: Solve the system by graphing:

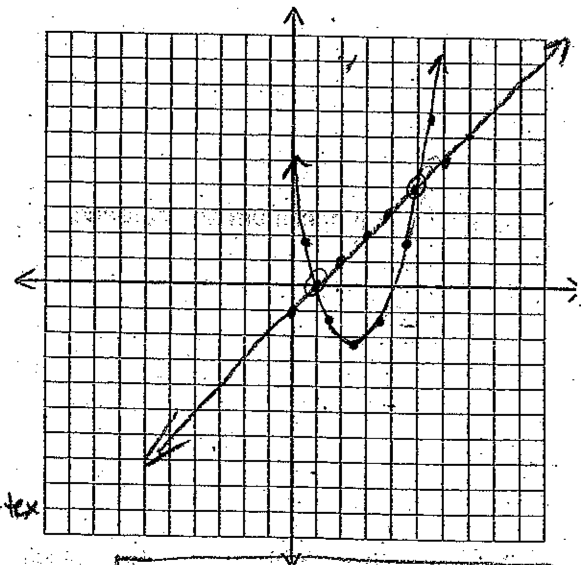
$$y = x^2 - 5x + 4$$
$$x - y = 1$$

parabola:

$y - 4 = x^2 - 5x$ $\boxed{\frac{-5}{2}}$ $\frac{25}{4}$

$y - 4 + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2$

$y = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$ $\quad V = \left(\frac{5}{2}, -\frac{9}{4}\right)$ then graph $y = x^2$ from vertex



line: $y = x - 1$

$\boxed{(1,0) \text{ \& } (5,4)}$

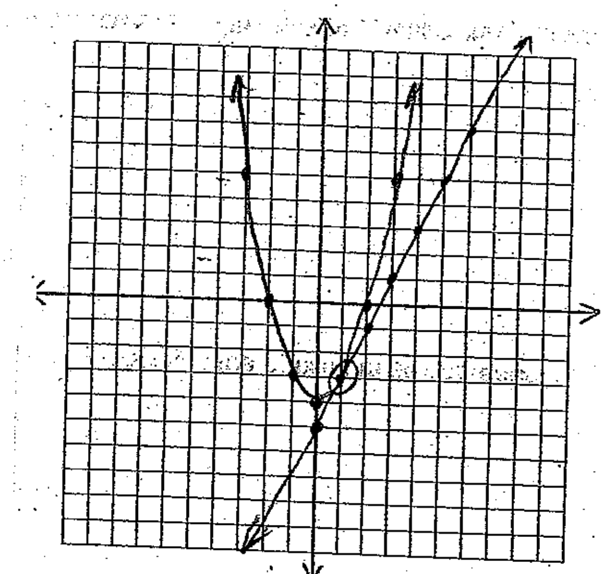
Check!

eg2: Solve by graphing:

$$y = x^2 - 4$$
$$2x - y = 5$$

Parabola:

$y = x^2 - 4$ $\quad (V = (0,-4))$ then graph $y = x^2$ from vertex



line:

$y = 2x - 5$

$\boxed{(1,-3)}$

Check!

eg3: Solve by graphing:

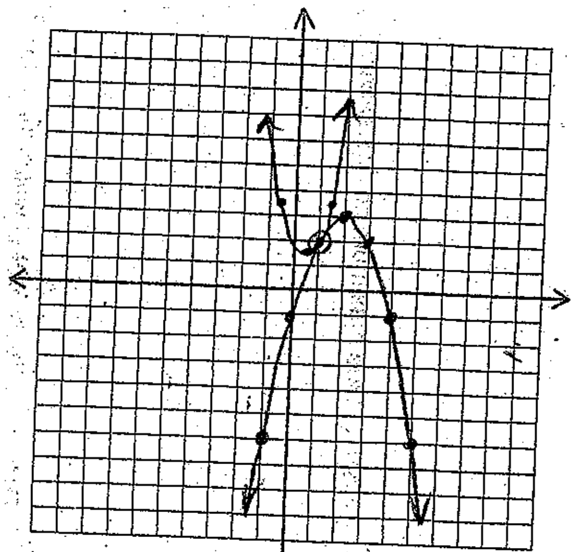
① $y = -(x-2)^2 + 3$

② $y = 2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$

Two parabolas!

① $V = (2, 3)$ → then graph $y = -x^2$ from vertex.

② $V = \left(\frac{1}{2}, \frac{3}{2}\right)$ → then graph $y = 2x^2$ from vertex.



$\boxed{(1, 2)}$ Check!

eg4: Solve ① $y - x^2 + 4 = 0$

② $-2y - 8 = -2x^2$

Two parabolas!

① $y = x^2 - 4$

② $2y = 2x^2 - 8$

$y = x^2 - 4$

Coincident parabolas!

$\boxed{\infty \text{ solutions}}$

Homework: p.305–308 #1–5 (estimate for 5)

* no graphing calc.

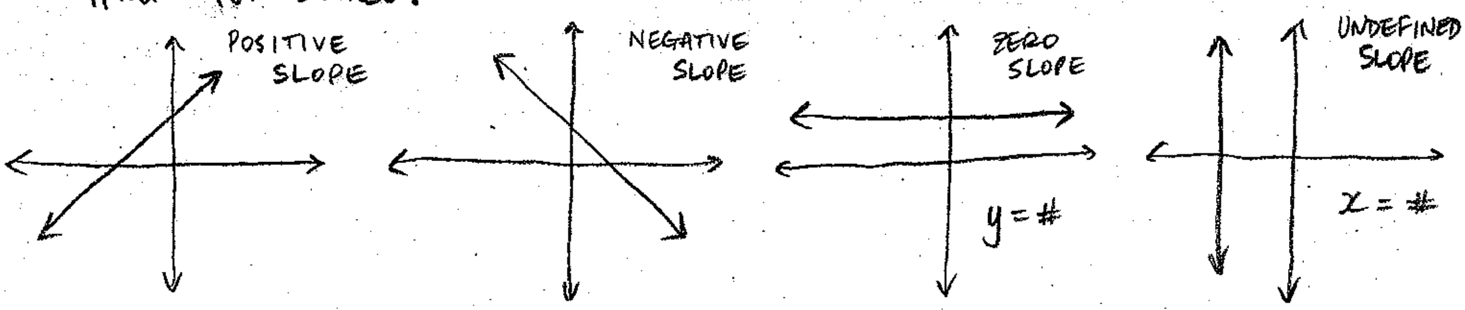
Ch. 7.3 - Graphing Linear Inequalities in Two Variables

- two dimensional (x/y axes graph).
- when $y = mx + b$ is graphed, the result is a <u>LINE</u>. If the '=' sign is replaced by an inequality, the line is known as a <u>BOUNDARY</u> <u>LINE</u> and the solution to the inequality is a set of points (x, y) that satisfy (make true) the inequality (involves SHADING the graph).
- to solve a linear inequality in two variables, you MUST <u>graph</u>! (ie. <u>CANNOT</u> be done algebraically)

* When inequality is INCLUSIVE: ($\leq, \geq$)
i) POINTS are <u>closed</u>;
ii) BOUNDARY LINE is <u>solid</u>.
* both are part of the solution.

* when inequality is EXCLUSIVE: ($<, >$)
i) POINTS are <u>open</u>;
ii) BOUNDARY LINE is <u>broken</u> (<u>dotted</u>).
* both are <u>not</u> part of the solution.

Hints for lines:



eg 1: Solve $y \geq 2x + 1$

* must GRAPH to solve!

↳ includes SHADING one side of the boundary line.

Shading - shading a portion of the graph includes all those shaded points as solutions.

How to know where to shade?

TWO METHODS:

① Vertical Line Rule:

Line equation must be in $y = mx + b$ form.

(i) If $y > mx + b$ or $y \geq mx + b$, draw a vertical line UPWARDS from boundary line and shade entire region.

(ii) If $y < mx + b$ or $y \leq mx + b$, draw a vertical line DOWNWARDS from boundary line and shade entire region.

* exception: if line is vertical (ie. $x = \#$), then "greater than" means shade to the RIGHT, and "less than" means shade to the LEFT.

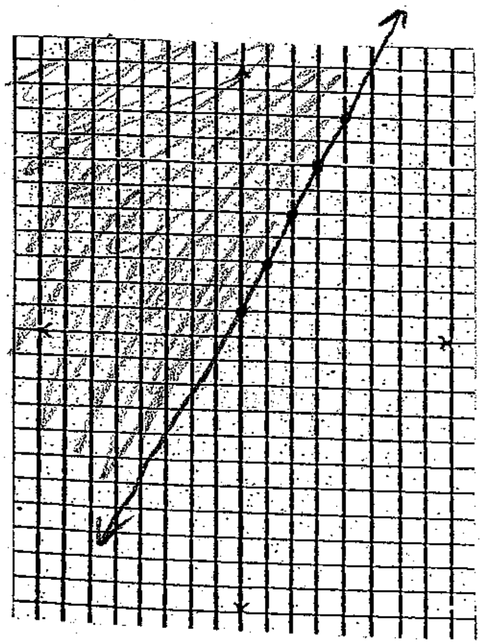
$\boxed{\text{OR}}$

② Use TEST POINTS.

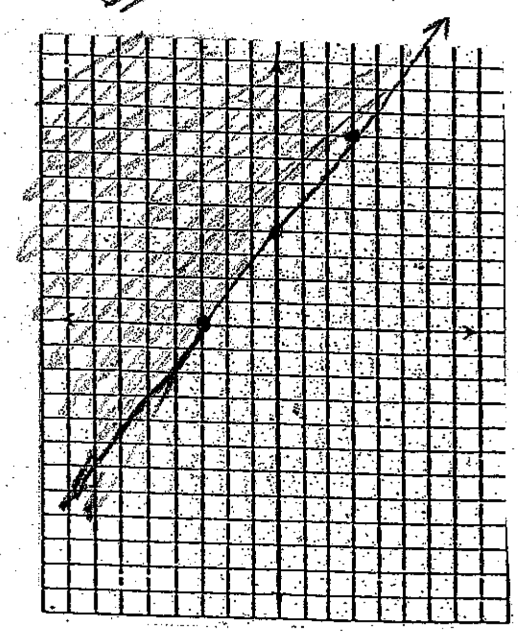
- choose a point from each region formed by boundary line and shade the region that contains the point satisfying the inequality.

* your choice!

So... Solve $y \geq 2x+1$



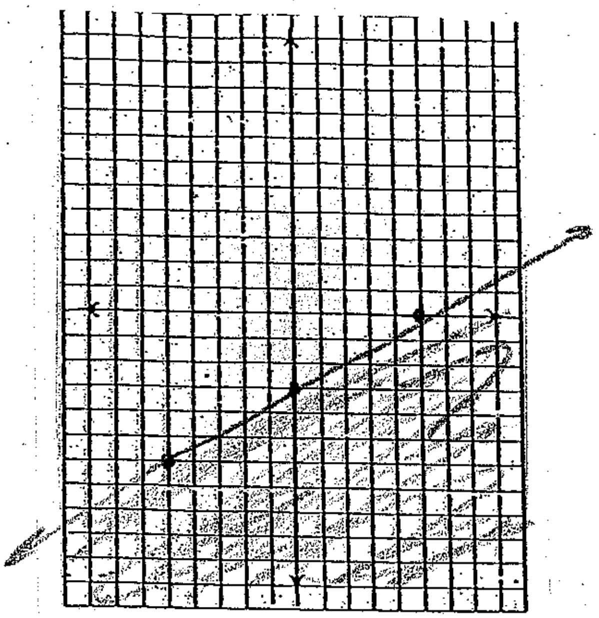
eg2: Solve $4x - 3y < -12$



$3y \geq 4x + 12$

$y \geq \frac{4}{3}x + 4$

eg3: Solve $y \le \frac{3}{5}x - 3$



eg4: Determine whether $(2,-1)$, $(3,-4)$ and $(-3,-2)$ are solutions to the following inequalities:

a) $2x - 3y > 7$ — a) NO ; YES ; NO

b) $2x - 3y \ge 7$ — b) YES; YES; NO

c) $2x - 3y < 7$ — c) NO, NO ; YES

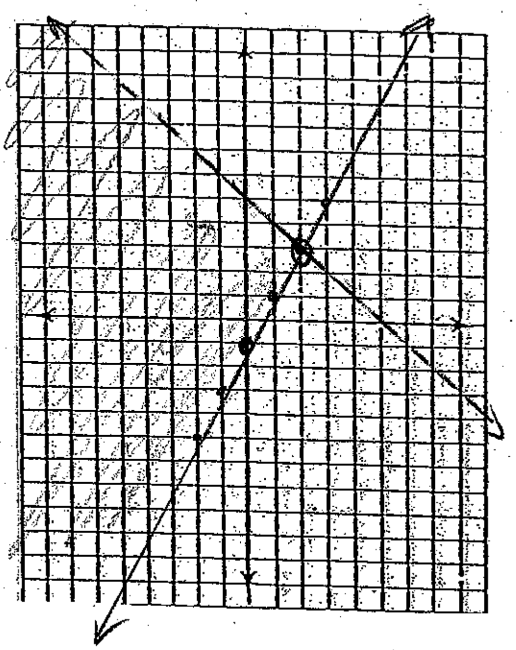
d) $2x - 3y \le 7$ — d) YES; NO ; YES

Hwk: p. 320 - 321 # 1-3

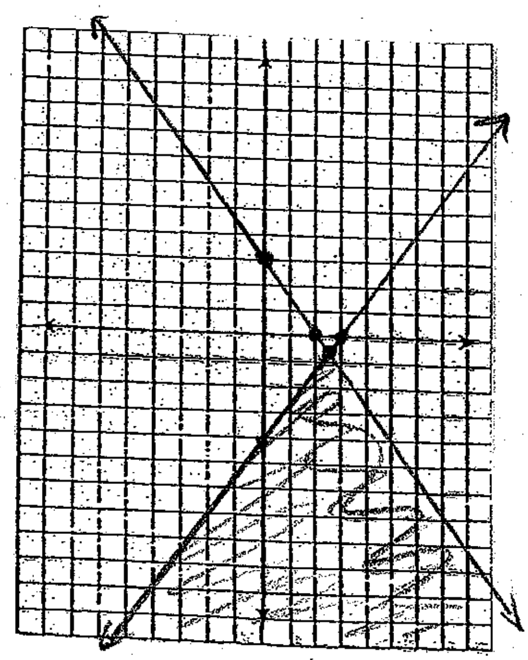
Ch. 7.3 cont'd – Solving Systems of Linear Inequalities

<u>Note</u>: When a broken line intersects a solid line, you <u>must</u> indicate this intersecting point with an <u>OPEN</u> point.

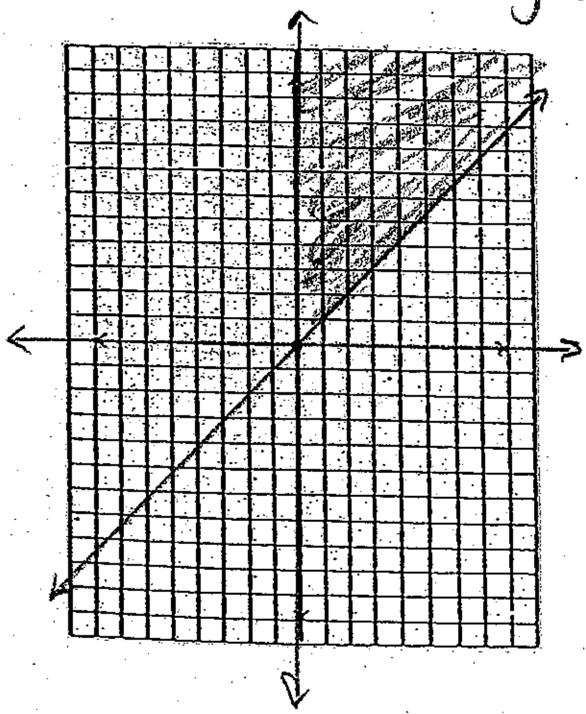
<u>eg1:</u> Solve $y \geq 2x - 1$
$y < -x + 5$



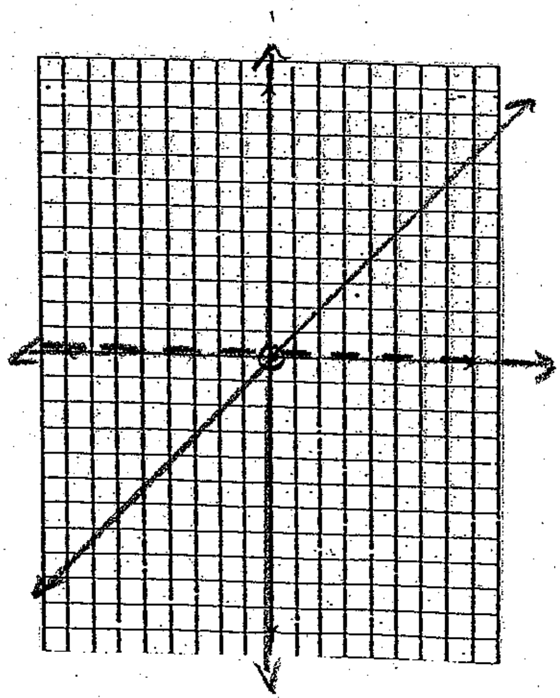
<u>eg2:</u> Solve $3x + 2y \leq 6$
$4x - 3y \geq 12$



eg3: Solve $y \geq x$
$x \geq 0$
$y \geq 0$



eg4: Solve $y \geq x$
$x \geq 0$
$y < 0$



no solution

Homework: p322-324
#4-7

Ch. 7.4 - Graphing Non-Linear Inequalities in Two Variables

Steps: (1) Graph $y = f(x)$ (serves as <u>boundary parabola</u>)

(2) Solid line for inclusive ($\geq, \leq$); dashed line for exclusive ($>, <$).

(3) a) If $y \geq f(x)$ or $y > f(x) \Rightarrow$ shade <u>ABOVE</u> boundary parabola.

b) If $y \leq f(x)$ or $y < f(x) \Rightarrow$ shade <u>BELOW</u> boundary parabola.

<u>eg1:</u> Graph $y \geq 2x - x^2$

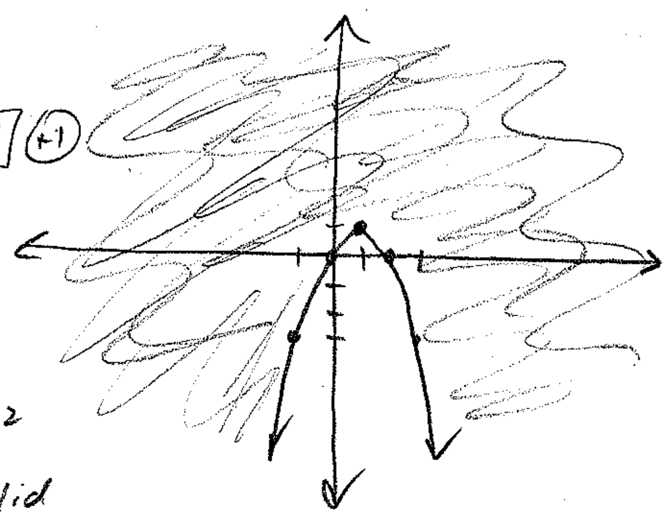
$y \geq -x^2 + 2x$

$\boxed{-1}$ (+1)

$y - 1 \geq -1(x^2 - 2x + 1)$

$y \geq -(x-1)^2 + 1$

$V = (1, 1)$ (then graph $y = x^2$ from vertex (solid line))



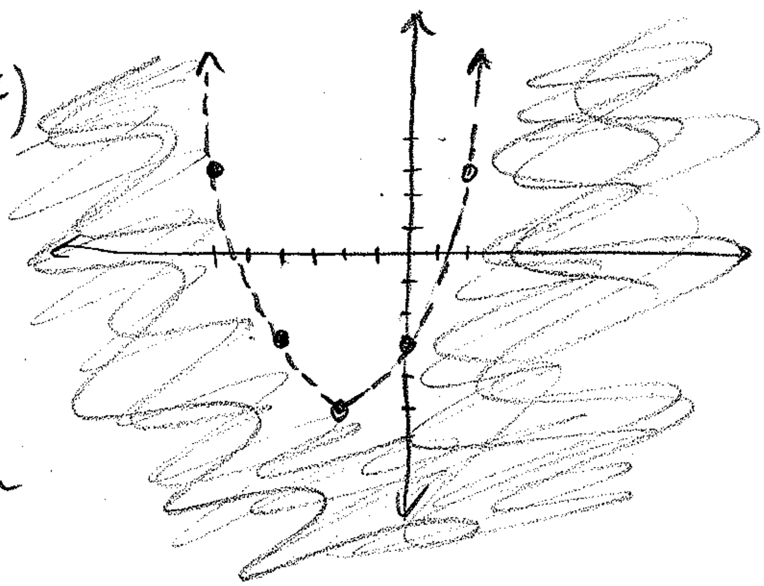
eg2: Graph $y < \frac{1}{2}x^2 + 2x - 3$

$y + 3 < \frac{1}{2}(x^2 + 4x)$ $\boxed{2}$ 4

$y + 3 + 2 < \frac{1}{2}(x^2 + 4x + 4)$

$y + 5 < \frac{1}{2}(x+2)^2$

$y < \frac{1}{2}(x+2)^2 - 5$



$V = (-2, -5)$; then graph

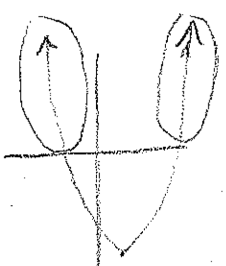
$y = \frac{1}{2}x^2$ from vertex

$\Rightarrow$ DA<u>SHED</u> line!

<u>Quadratic Inequalities in One Variable</u>

ie. $ax^2 + bx + c \begin{Bmatrix} \geq \\ > \\ \leq \\ < \end{Bmatrix} 0 \quad a, b, c \in \mathbb{R}, \ a \neq 0$

eg1: Solve $x^2 - x - 6 > 0$ ← y has been set $= 0$ so... question is asking when the parabola is above the x-axis. * need to factor to find x-intercepts.

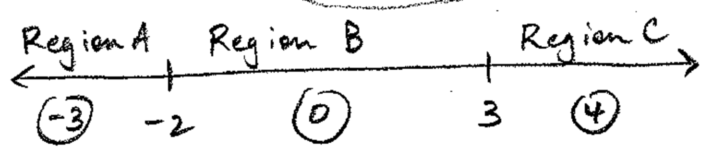


opens <u>up</u>

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, \ -2$



Use TEST points from each region

Region A: $(-3)^2 - (-3) - 6 = 6 > 0$ ✓

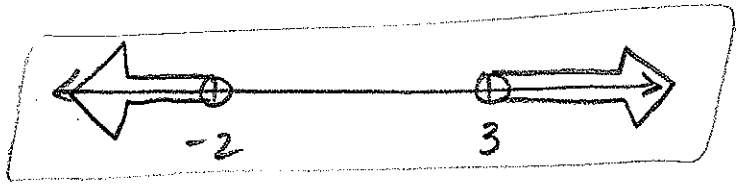
Region B: $(0)^2 - (0) - 6 = -6 < 0$ ✗

Region C: $(4)^2 - (4) - 6 = 6 > 0$ ✓

over →

Solution:

graphically:



algebraically: $\left\{ \begin{array}{l} x < -2 \\ x > 3 \end{array} \right\}$

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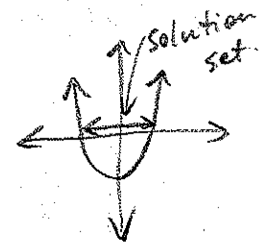
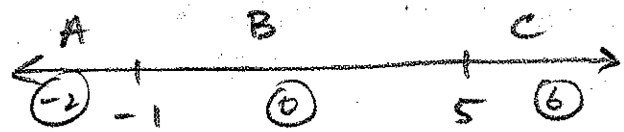
eg2: Solve $x^2 - 4x - 5 \leq 0$ opens up



$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = 5, \ -1$$

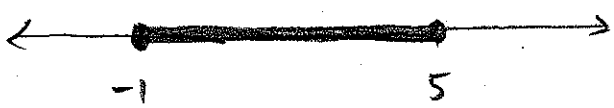


A: $(-2)^2 - 4(-2) - 5 = 7 > 0$ ✗

B: $(0)^2 - 4(0) - 5 = -5 < 0$ ✓

C: $(6)^2 - 4(6) - 5 = 7 > 0$ ✗

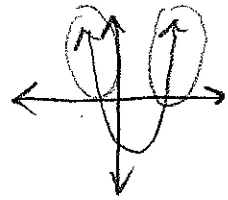
Graphically:



Algebraically: $\left. \begin{array}{l} x \geq -1 \\ x \leq 5 \end{array} \right\}$ $\underline{\underline{-1 \leq x \leq 5}}$

eg3: Solve $x^2 - 2x > 2$

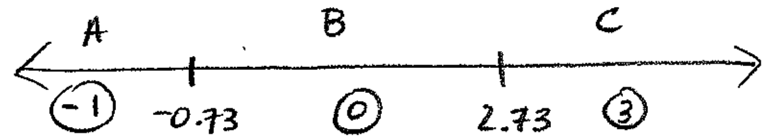
$x^2 - 2x - 2 > 0$ opens up



$x^2 - 2x - 2 = 0$ cannot be factored... use Q. Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4+8}}{2}$$



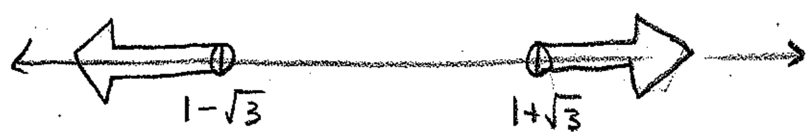
$$x = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

A: $(-1)^2 - 2(-1) - 2 = 1 > 0$ ✓

B: $(0)^2 - 2(0) - 2 = -2 < 0$ ✗

C: $(3)^2 - 2(3) - 2 = 1 > 0$ ✓

G:



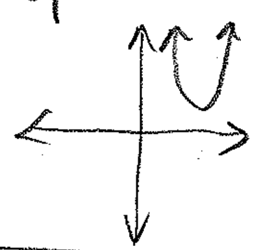
A: $x < 1 - \sqrt{3}$; $x > 1 + \sqrt{3}$

eg4: Solve $x^2 - x + 3 \leq 0$

$x^2 - x + 3 = 0$ cannot factor

$$x = \frac{1 \pm \sqrt{1 - 12}}{2} = \frac{1 \pm \sqrt{-11}}{2}$$

no x-intercepts opens up



function never ≤ 0

$\boxed{\text{NO SOLUTION}}$

$\boxed{\text{Hwk: p.329-331 \#1-3}}$

Chapter 7.5 – Word Problems and Non-linear functions/Inequalities

eg1: Manuel is a college student with two part-time jobs. He earns \$10/h at a music store and \$15/h at a warehouse. He needs to earn at least \$180/week to survive, but he cannot work more than 14 h/week (so that he can study). Write and solve (by graphing) a system of inequalities to determine how many hours Manuel could work at each job during a week.

Let x = hours @ music store

Let y = hours @ warehouse

$x + y \leq 14$

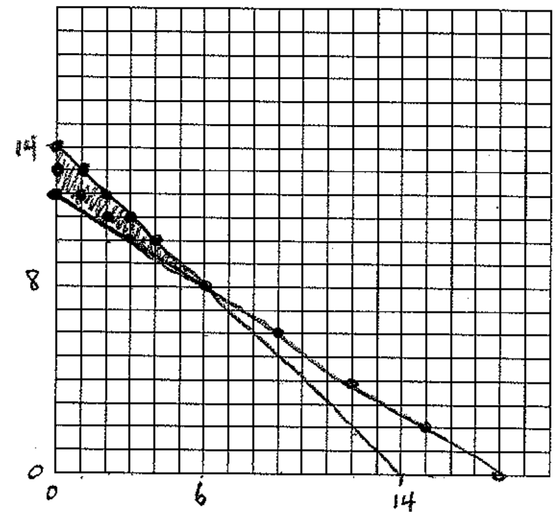
$10x + 15y \geq 180$

$x \geq 0$

$y \geq 0$

$15y \geq -10x + 180$

$y \geq -\frac{2}{3}x + 12$

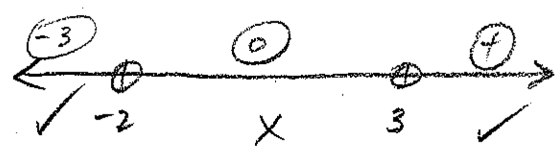


eg2: The total resistance of two electric circuits is given by $R^2 - R + 1$, where R is the resistance in ohms. When is the resistance more than 7 ohms?

$R^2 - R + 1 > 7$

$R^2 - R - 6 > 0$

$(R-3)(R+2) > 0$



$\boxed{R < -2 \qquad R > 3}$

eg3: The height in meters of a projectile shot from the top of a building is given by $h(t) = -16t^2 + 60t + 25$, where t represents the time in seconds the projectile is in the air. Find the time interval that the projectile is above 25m.

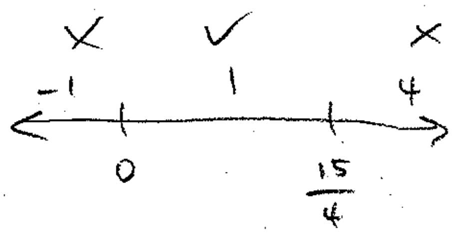
$$-16t^2 + 60t + 25 > 25$$

$$-16t^2 + 60t > 0$$

$$16t^2 - 60t < 0$$

$$4t^2 - 15t < 0$$

$$t(4t - 15) < 0$$

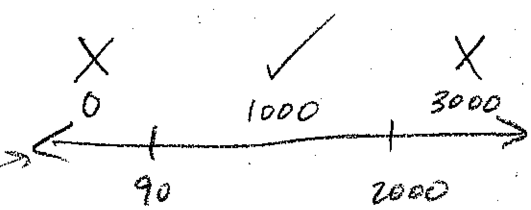


$$\boxed{0 < t < \frac{15}{4}}$$

eg4: The price of stereos is given by $S(x) = 224 - 0.1x$, $0 \le x \le 2240$, where x is the number of stereos produced each day. It costs \$18000 per day to operate the factory and \$15 for materials to produce each stereo. A) Find the daily revenue; B) Find the daily cost; C) Find the interval that produces a profit.

A) $R = \underbrace{x}_{\text{\# sold}}\underbrace{(224 - 0.1x)}_{\text{price}} = -0.1x^2 + 224x$

B) $C = 15x + 18000$



C) $R > C$

$$-0.1x^2 + 224x > 15x + 18000$$

$$0 > 0.1x^2 - 209x + 18000$$

$$x^2 - 2090x + 180000 < 0$$

$$(x - 2000)(x - 90) < 0$$

$$\boxed{90 < x < 2000}$$

eg5: A bullet is fired from a gun by a person standing at the top of a seemingly endless, yet gently sloping hill. The trajectory of the bullet is given by $y = -0.00002x^2 + 3x$, where y is the height of the bullet off the ground and x is the horizontal distance from the starting point (in centimeters). The hill has a slope of -1/25. How far below the person's feet does the bullet land, and how far (horizontally) from the person does that landing take place? (ie. What are the landing coordinates?)

$$y = \frac{-1}{25}x$$

$$y = -0.00002x^2 + 3x$$

$$\frac{-1}{25}x = -0.00002x^2 + 3x$$

$$0 = -0.00002x^2 + 3.04x$$

$$0 = x^2 - 152000x$$

$$0 = x(x - 152000)$$

$$\underset{\text{start}}{x = 0}, \quad \underset{\text{end}}{152000}$$

$$y = -\frac{1}{25}(152000) = -6080$$

$$\boxed{(152000, -6080)}$$

Ch. 7.5 Word Problem Solutions

1. a) Expenses = \$1125

   b) Profit interval ⇒ in terms of units sold:

   $10 < \text{units sold} < 90$

   c) Max profit = \$2000 (when 50 units sold)

2. $$\frac{n(n+1)}{2} \geq 78$$

$$n(n+1) \geq 156$$

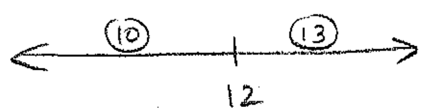
$$n^2 + n - 156 \geq 0$$

$$n^2 + n - 156 = 0$$

$$(n+13)(n-12) = 0$$

$$n = \cancel{-13},\ 12$$

Test: 10: X
13: ✓



$$\boxed{n \geq 12}$$

3. $2y = x \quad y = \frac{1}{2}x$

$y \geq 10$

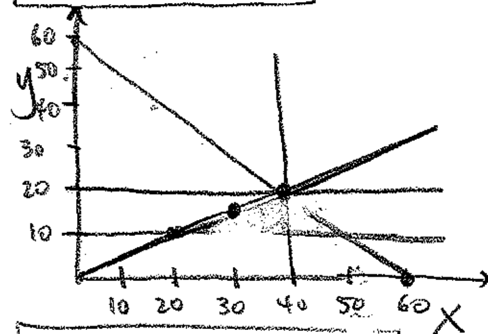
$x + y \leq 60$

$2y + y \leq 60$

$3y \leq 60$

$y \leq 20$

⟹ $\boxed{10 \leq y \leq 20}$



$y \leq -x + 60$

$y = \frac{1}{2}x$

$y \geq 10$

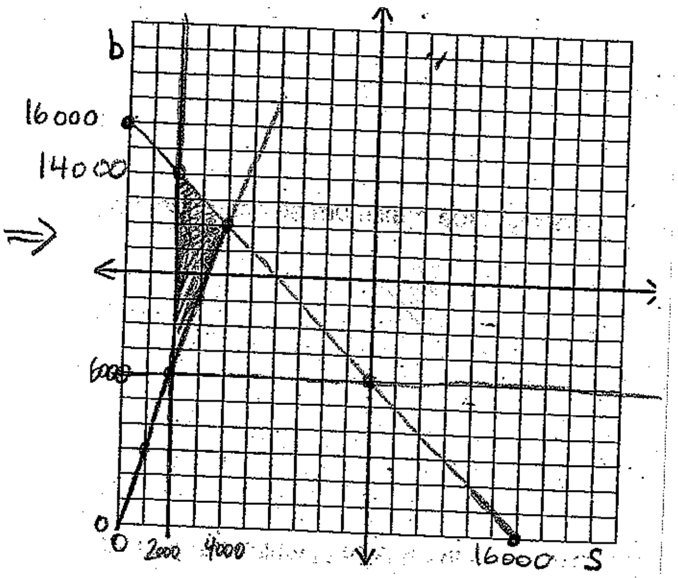
$y \leq 20$

So ...

$$\boxed{20 \leq x \leq 40}$$

4. Let s = amt. invested in stocks
Let b = amt. invested in bonds

$$\left.\begin{aligned} s + b &\le 16000 \\ s &\ge 2000 \\ b &\ge 3s \\ b &\ge 6000 \end{aligned}\right\} \Rightarrow$$



$2000 \le s \le 4000$
$6000 \le b \le 14000$

5. Let t = # tables
Let c = # chairs

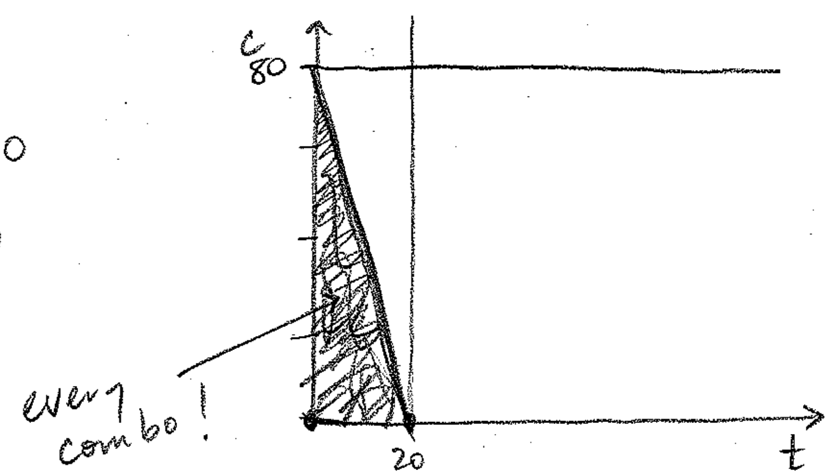
10 workers @ 40 hrs / week = 400 hours max. / week

$\dfrac{400 \text{ hours}}{20 \text{ hours / table}}$ = 20 tables max. → $t \le 20$

$\dfrac{400 \text{ hours}}{5 \text{ hours/chair}}$ = 80 chairs max → $c \le 80$

$20t + 5c \le 400$

$c \le -4t + 80$



6. Let c = # of cars

Let t = # of trucks

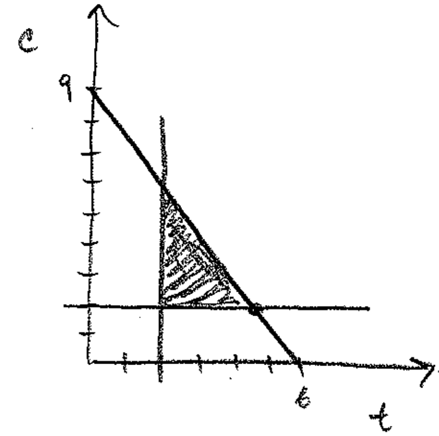
$c \ge 2$

$t \ge 2$

$20000c + 30000t \le 180000$

$20000c \le -30000t + 180000$

$c \le -\frac{3}{2}t + 9$



7. $\boxed{400x + 500y \le 48000}$

$x + y = 100$

Let $x = 0$

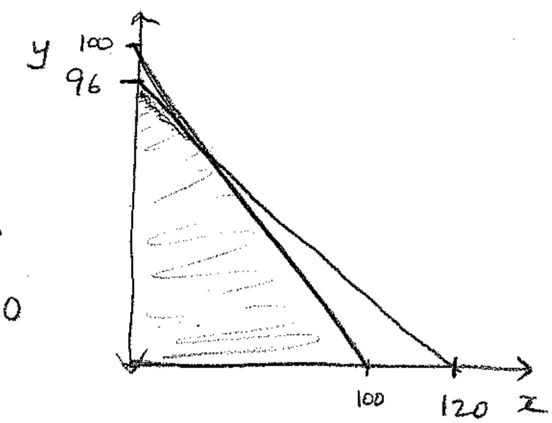
$y \le 96$

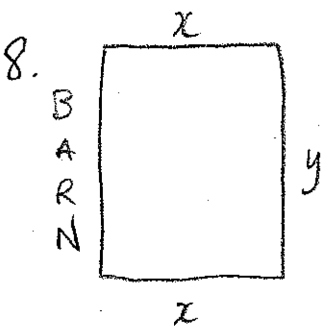
Let $y = 0$

$x \le 120$ (but, cannot be > 100)

$0 \le y \le 96$

$0 \le x \le 100$

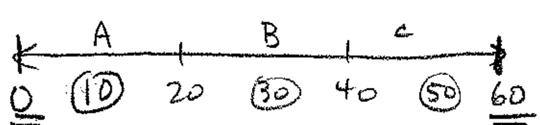


8. 

Let x = width

$2x + y = 120$

$y = -2x + 120$



$xy \le 1600$

$x(-2x + 120) \le 1600$

$-2x^2 + 120x - 1600 \le 0$

$x^2 - 60x + 800 \ge 0$

$(x - 20)(x - 40) = 0$

$x = 20, 40$

Test:

A: ✓

B: X

C: ✓

$0 < x \le 20$

$40 \le x < 60$

9. $N = 60T - T^2$

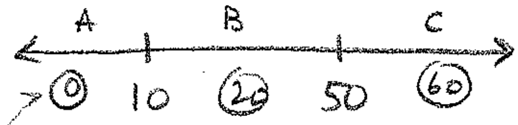
$60T - T^2 > 500$

$0 > T^2 - 60T + 500$

$T^2 - 60T + 500 < 0$

$(T-50)(T-10) = 0$

$T = 50 \ , \ 10$



A: X

B: ✓

C: X

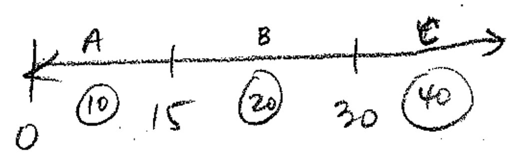
$\boxed{10° < T < 50°}$

10. $-x^2 + 45x - 450 > 0$

$x^2 - 45x + 450 < 0$

$(x-30)(x-15) = 0$

$x = 30 \ , \ 15$



A: X

B: ✓ $\boxed{15 < x < 30}$

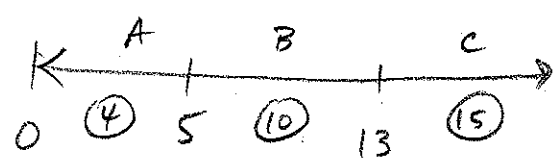
C: X

11. $x^2 - 18x + 140 < 75$

$x^2 - 18x - 65 < 0$

$(x-13)(x-5) = 0$

$x = 13 \ , \ 5$



A: X

B: ✓

C: X

$\boxed{5 < x < 13}$

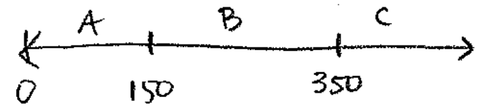
12) $-0.1x^2 + 50x - 5250 > 0$

$0.1x^2 - 50x + 5250 < 0$

$x^2 - 500x + 52500 < 0$

$(x-350)(x-150) = 0$

$x = 350, 150$



A: X

B: ✓

C: X

$\boxed{150 < x < 350}$

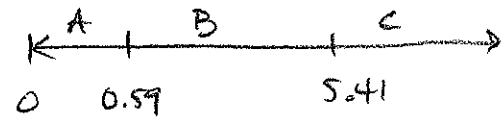
13. $-4.9t^2 + 29.4t + 24.3 > 40$

$-4.9t^2 + 29.4t - 15.7 > 0$

$4.9t^2 - 29.4t + 15.7 < 0$

$t^2 - 6t + 3.2 < 0$

$t = \dfrac{6 \pm \sqrt{36 - 12.8}}{2} = 0.59, 5.41$



A: X

B: ✓

C: X

$\boxed{0.59 < t < 5.41}$

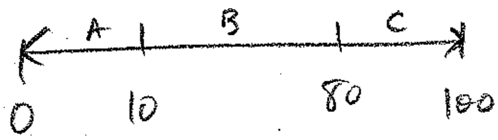
14. $\frac{1}{20}v^2 + \frac{9}{2}v \geq 40$

$\frac{1}{2}v^2 - \frac{9}{2}v + 40 \leq 0$

$v^2 - 90v + 800 \leq 0$

$(v-80)(v-10) = 0$

$v = 80, 10$



A: X

B: ✓

C: X

$\boxed{10 \leq v \leq 80}$

15. $y = -0.0015x^2 + 0.5x \qquad y = \frac{1}{8}x$

SUBSTITUTE

parabola / line intersection.

$$-0.0015x^2 + 0.5x = \frac{1}{8}x$$

$$0 = 0.0015x^2 - 0.375x$$

$$0 = x^2 - 250x$$

$$x(x-250) = 0$$

$x = 0$, $250 \Rightarrow y = 31.25$

starting pt.   end point

$$\boxed{(250, 31.25)}$$

16. a) $R = \text{Sales} \times \text{Cost}$

$$R = n(36 - 0.4n)$$

$$R = -0.4n^2 + 36n$$

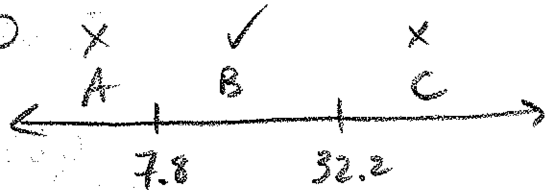
b) $C = 100 + 20n$

c) $P = -0.4n^2 + 36n - (20n + 100)$

$$P = -0.4n^2 + 16n - 100$$

$$-0.4n^2 + 16n - 100 \geq 0$$

$$n^2 - 40n + 250 \leq 0$$



using Q. Formula:

$n = 7.8,\ 32.2$

$$\boxed{8 \leq n \leq 32}$$