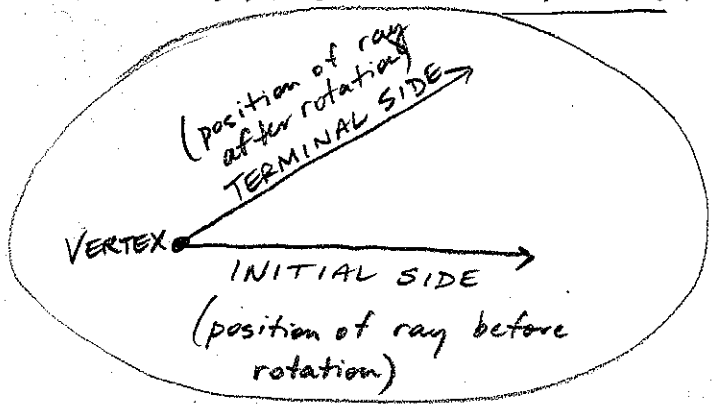
Ch. 3.1 - Angles and their Measure

- the study of TRIGONOMETRY depends upon the concept of ANGLES.

→ an angle is determined by rotating a ray about its ENDPOINT or VERTEX.

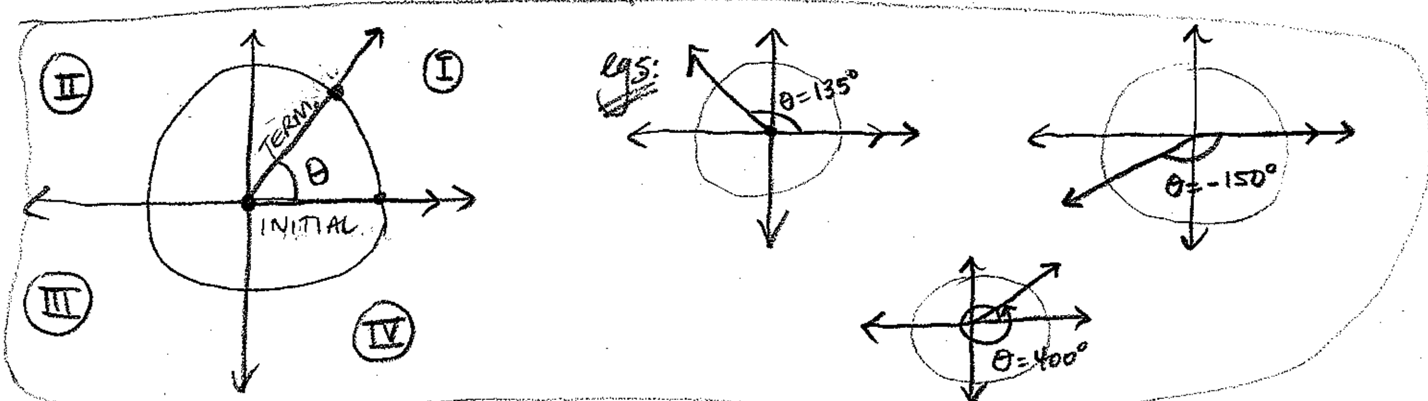


If the ray is rotated COUNTER-CLOCKWISE, the formed angle is POSITIVE.

If the ray is rotated CLOCKWISE, the formed angle is NEGATIVE.

Angles in Standard Position

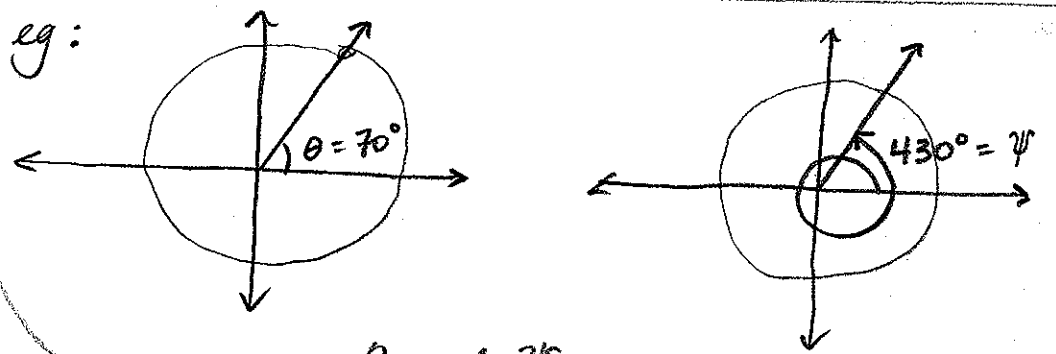
- on a coordinate x/y plane (graph), an angle θ is said to be in STANDARD POSITION if its vertex is the ORIGIN and its initial side coincides with the positive x-axis. (draw circle too!)



Coterminal Angles

- angles in standard position that share the same TERMINAL SIDE.

eg:



θ and ψ are COTERMINAL ANGLES.

eg1: If $\theta = 120°$ and is in standard position, find two positive coterminal and two negative coterminal ∠s

POSITIVE:

$120° + 360° = \boxed{480°}$

$480° + 360° = \boxed{840°}$

NEGATIVE:

$120° - 360° = \boxed{-240°}$

$-240° - 360° = \boxed{-600°}$

Generally: $\theta \pm (n)(360°)$ $\quad n \in$ INTEGERS

eg2: Find the smallest possible coterminal angle for:

a) $2692°$ $\qquad$ b) $-1940°$

$\frac{2692°}{360°} = 7.5$, so seven full circles in $2692°$

$2692 - 7(360°) = \boxed{172°}$

$\frac{-1940°}{360°} = 5.4 \Rightarrow$ need 6 complete circles.

$-1940° + 6(360°)$

$\boxed{= 220°}$

so far... p. 105-107 #2-6

Reference Angles

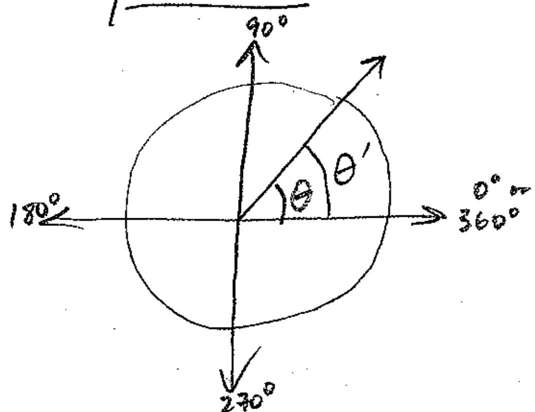
For angle θ in standard position, the reference angle is the <u>positive acute angle θ'</u> that is formed with the terminal side of θ and the <u>x-axis.</u>
(<u>never</u> the <u>y</u>!)

$$\boxed{0 \leq \theta' \leq 90^\circ}$$

Scenarios $\Rightarrow$ 4 QUADRANTS = 4 scenarios!

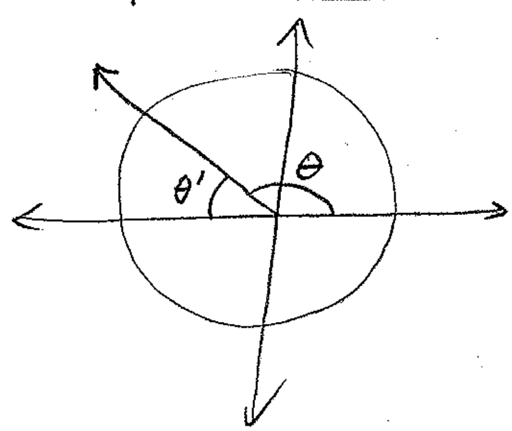
$\boxed{\text{RESTRICTION: } 0 \leq \theta \leq 360^\circ}$

Quadrant I



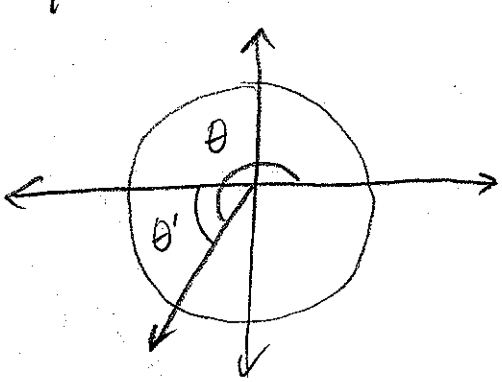
$\theta = \theta'$

Quadrant II



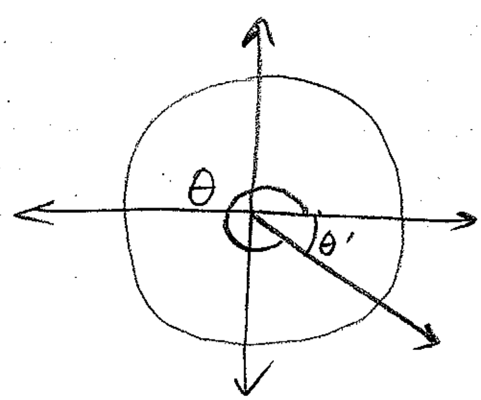
$\theta' = 180 - \theta$

Quadrant III



$\theta' = \theta - 180^\circ$

Quadrant IV



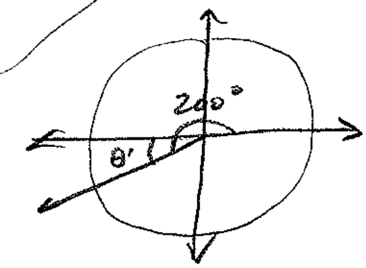
$\theta' = 360^\circ - \theta$

eg3: Find the reference angle for:

a) $85°$

$85°$ in Quadrant 1,
so ref $\angle = \boxed{85°}$

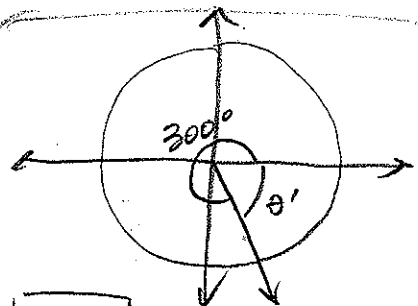
b) $200°$



Q III

$\theta' = 200 - 180 = \boxed{20°}$

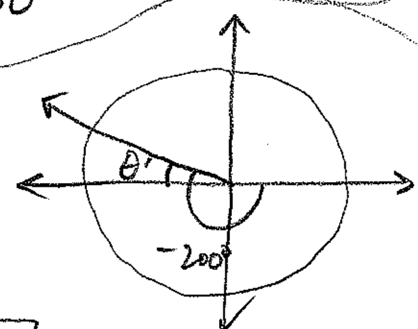
c) $300°$



Q IV

$\theta' = 360° - 300°$
$\boxed{= 60°}$

d) $-200°$

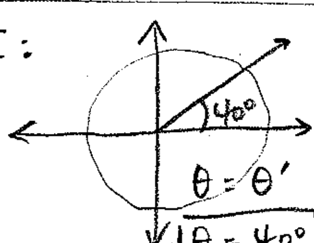


Q II but $-200°$ coterminal with $160°$

$\theta' = 180° - 160° = \boxed{20°}$

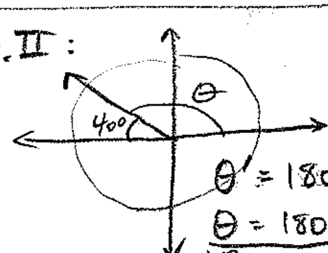
eg4: Determine the smallest possible angle in quadrants I, II, III, and IV that has a ref. $\angle$ of $40°$.

Q.I:



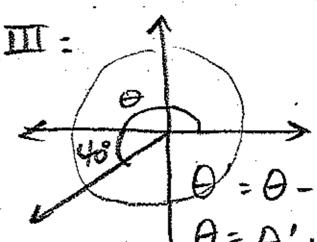
$\theta = \theta'$
$\boxed{\theta = 40°}$

Q.II:



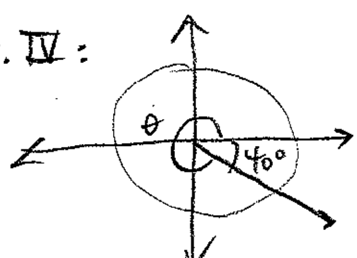
$\theta' = 180 - \theta$
$\theta = 180 - \theta'$
$\boxed{\theta = 140°}$

Q.III:



$\theta' = \theta - 180°$
$\theta = \theta' + 180°$
$\boxed{\theta = 220°}$

Q.IV:



$\theta' = 360° - \theta$
$\theta = 360° - \theta'$
$\boxed{\theta = 320°}$

eg5: Determine the ref. ∠ for:

a) $612°$

$612° - 360° = 252°$

QIII: $\theta' = 252° - 180°$
$\boxed{= 72°}$

b) $-420°$

$-420° + 360° + 360° = 300°$

QIV: $\theta' = 360° - 300°$
$\boxed{= 60°}$

c) $844°$

$844° - 360° - 360° = 124°$

QII: $\theta' = 180° - 124°$
$\boxed{= 56°}$

d) $6425°$

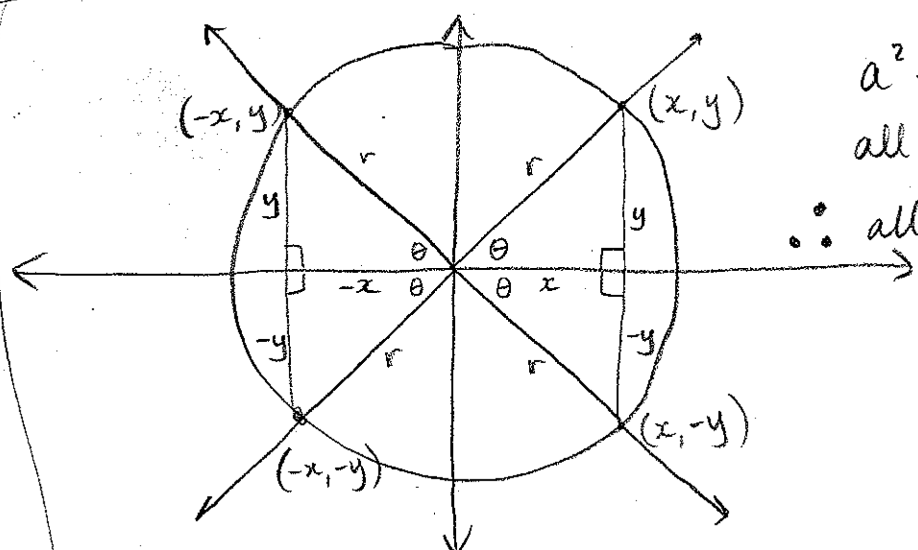
$\frac{6425°}{360°} = 17.8 \Rightarrow 17$

$6425° - 17(360°) = 305°$

QIV: $\theta' = 360° - 305°$
$\boxed{= 55°}$

The Coordinate System

- an x/y graph with a circle of radius r $(r > 0)$



$a^2 + b^2 = c^2$, so all 4 Δs are congruent.

∴ all θs shown are equal!

eg6: Find all angles $0° \leq \theta \leq 360°$, that have a reference angle of $20°$

$QI: \theta = 20°$

$QII: \theta = 180° - 20° = 160°$

$QIII: \theta = 180° + 20° = 200°$

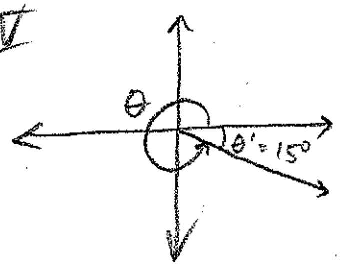
$QIV: \theta = 360° - 20° = 340°$

eg7: If Quadrants II and IV have the same reference ∠, and the standard position angle θ in quadrant II is $165°$, what is the standard position angle in quadrant IV?

$QII \quad \theta = 165° \quad \therefore \quad \theta' = 180° - 165°$

$= \boxed{15°}$

QIV



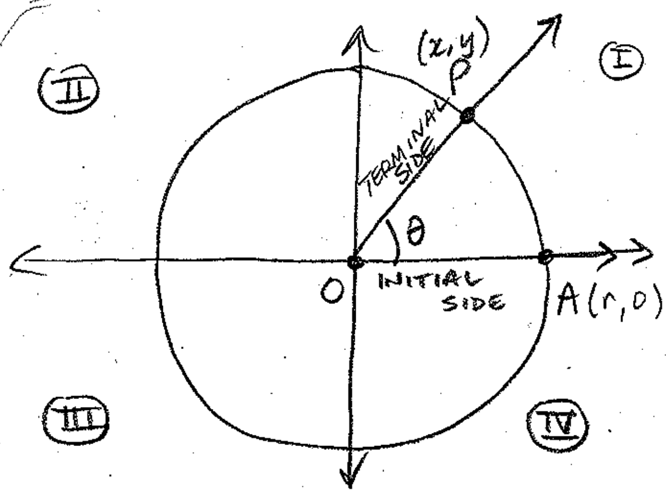
$\theta = 360° - 15°$

$\boxed{= 345°}$

p.105 - 108 # 1, 7-12

Ch. 3.2 – The Three Trigonometric Functions

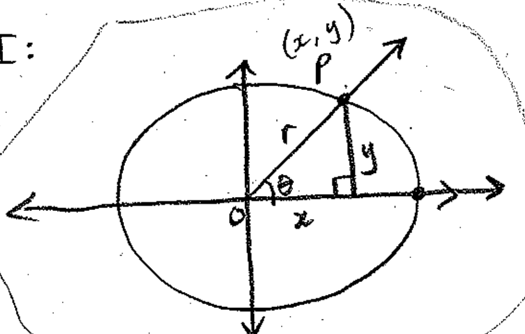
Let P $(\pm x, \pm y)$ be a point that moves around a circle with radius $\underline{r}$. P always starts on the positive $\underline{x\text{-axis}}$ at point A $(\underline{r}, \underline{0})$. The movement of P creates angle θ; the measure of θ depends upon the amount of rotation of P (the terminal side).



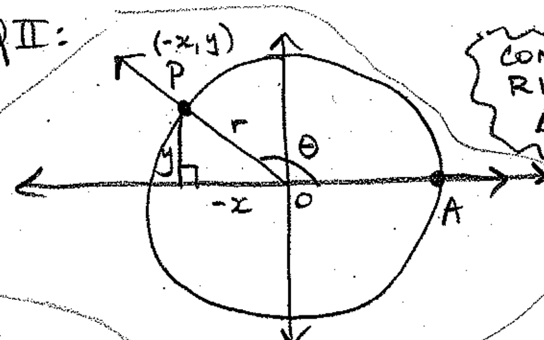
* θ is in STANDARD POSITION.

4 QUADRANTS = 4 SCENARIOS ... but $\underline{r}$ is always positive!

QI:

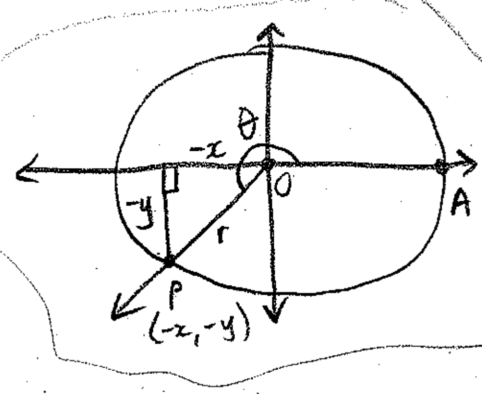


QII:

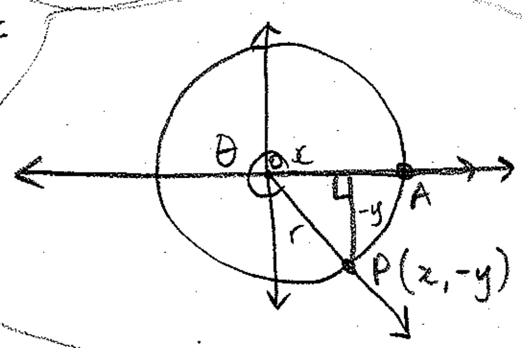


CONSTRUCT RIGHT Δs

QIII:



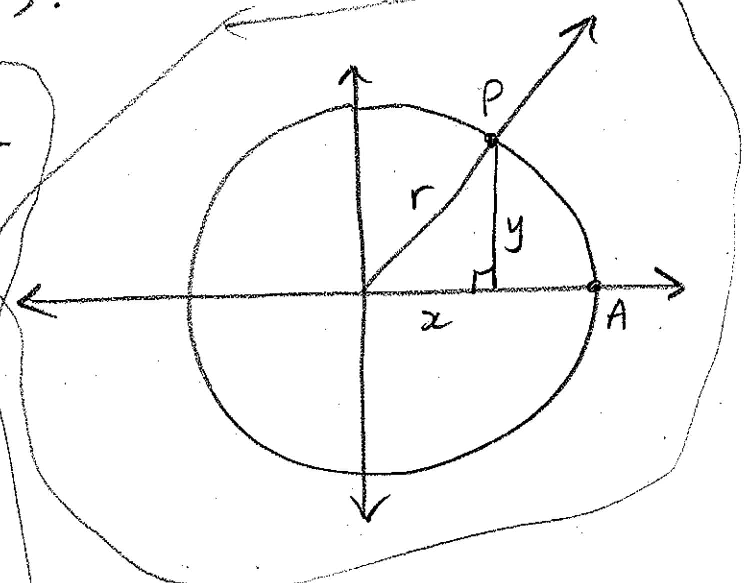
QIV:



By Pythagoras, $x^2 + y^2 = r^2$ (again, r is always POSITIVE)

$$r = \sqrt{x^2 + y^2}$$

Knowing the coordinates of P for an angle θ in standard position, one can find the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ (the three trig. ratios).



$\sin\theta = \dfrac{\text{OPP.}}{\text{HYP.}} = \dfrac{\pm y}{r}$

(SOH)

$\cos\theta = \dfrac{\text{ADJ.}}{\text{HYP.}} = \dfrac{\pm x}{r}$

(CAH)

$\tan\theta = \dfrac{\text{OPP.}}{\text{ADJ.}} = \dfrac{\pm y}{\pm x}$

(TOA)

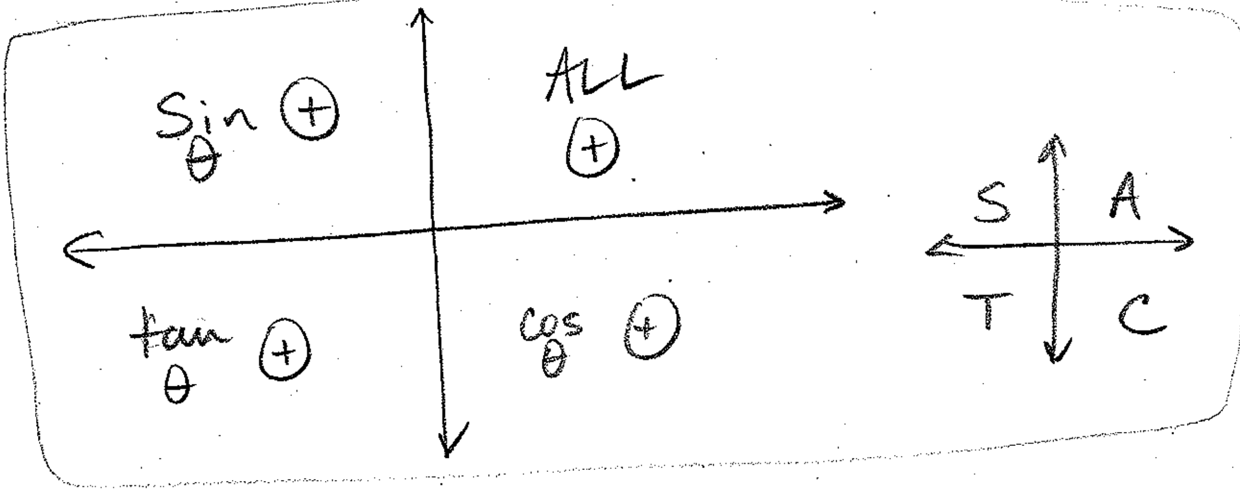
x is positive in quadrants I and IV

y is positive in quadrants I and II.

So... $\sin\theta$ $\oplus$ in Qs I & II

$\cos\theta$ $\oplus$ in Qs I & IV

$\tan\theta$ $\oplus$ in Qs I & III (when both x & y $\ominus$)



eg1: Identify the quadrant(s) for the angles satisfying the following conditions:

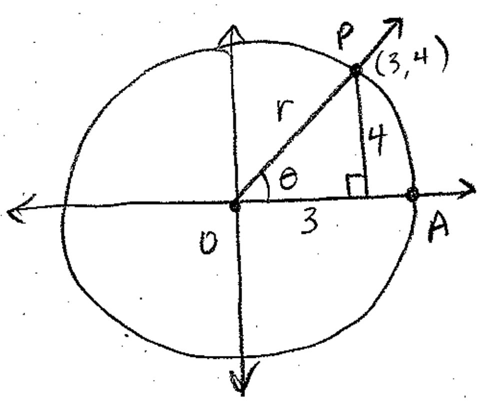
a) $\sin\theta < 0$, $\cos\theta > 0$

b) $\tan\beta < 0$, $\cos\beta < 0$

a) $\sin\theta$ neg. in III & IV, $\cos\theta$ pos. in I & IV $\Big\}$ $\boxed{\text{IV}}$

b) $\tan\beta$ neg. in II & IV, $\cos\beta$ neg. in II & III $\Big\}$ $\boxed{\text{II}}$

eg2: P is the point (3,4) that lies on the terminal arm of angle θ in standard position. Find $\sin\theta$, $\cos\theta$, and $\tan\theta$.



$3^2 + 4^2 = r^2$

$r = 5$

$\sin\theta = \frac{4}{5}$ $\tan\theta = \frac{4}{3}$

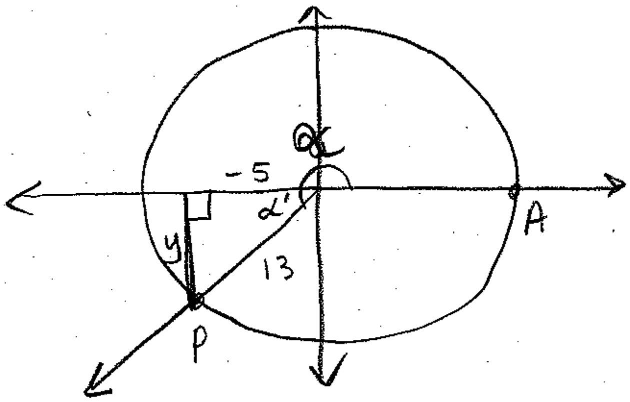
$\cos\theta = \frac{3}{5}$ (all ⊕) ⇒ QI.

If θ lies in quadrants II, III, or IV, we cannot use θ to construct a right Δ since $\theta > 90°$. We must use θ' (the ref. angle).

eg3: If $\cos\alpha = \frac{-5}{13}$ with α in QIII, find $\sin\alpha$ and $\tan\alpha$.

need ref $\angle$:

$$\cos\alpha' = \frac{-5}{13}$$



y will be negative!

$$(-5)^2 + y^2 = 13^2$$

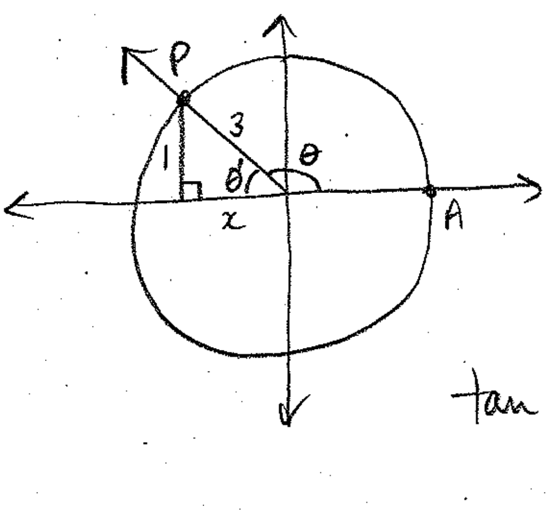
$$y^2 = 144$$

$$y = \pm 12 \Rightarrow \boxed{y = -12}$$

$$\sin\alpha' = \frac{-12}{13} \;;\quad \tan\alpha' = \frac{-12}{-5} = \frac{12}{5}$$

so $\boxed{\sin\alpha = \frac{-12}{13}}$ so $\boxed{\tan\alpha = \frac{12}{5}}$

eg4: Find $\tan\theta$ if $\sin\theta = \frac{1}{3}$ and θ is in QII.



x will be negative!

$$1^2 + x^2 = 3^2$$

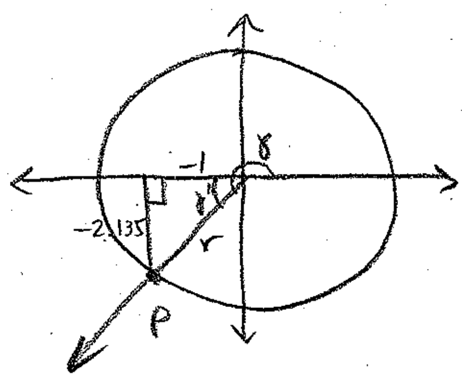
$$x^2 = 8$$

$$x = \pm 2\sqrt{2} \Rightarrow x = -2\sqrt{2}$$

$$\tan\theta' = \tan\theta = \frac{1}{-2\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{4}}$$

eg5: Find $\sin\gamma$ if $\tan\gamma = 2.135$ with γ in QIII.

$$\tan\gamma = \frac{\text{opp.}}{\text{adj.}} = \frac{2.135}{1}$$



both x & y negative in QIII.

$$\tan\gamma' = \frac{-2.135}{-1}$$

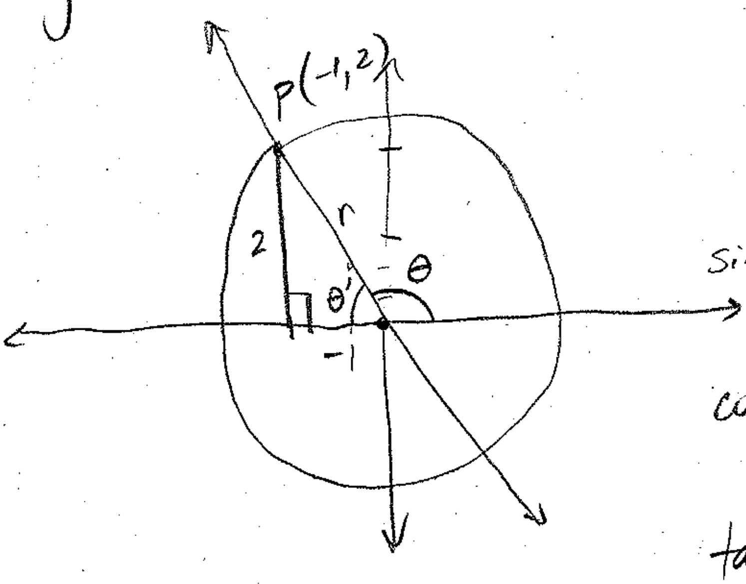
$$r^2 = (-1)^2 + (-2.135)^2$$

$$r = 2.358$$

$$\sin\gamma' = \sin\gamma = \frac{-2.135}{2.358} = \boxed{-0.906}$$

eg6: $y = -2x$ $(x \le 0)$ is the equation of the terminal side of an angle θ in standard position. Sketch the SMALLEST positive angle θ, and determine $\sin\theta$, $\cos\theta$, and $\tan\theta$.

$y = -2x$ is a line



$$(-1)^2 + 2^2 = r^2$$

$$r = \sqrt{5}$$

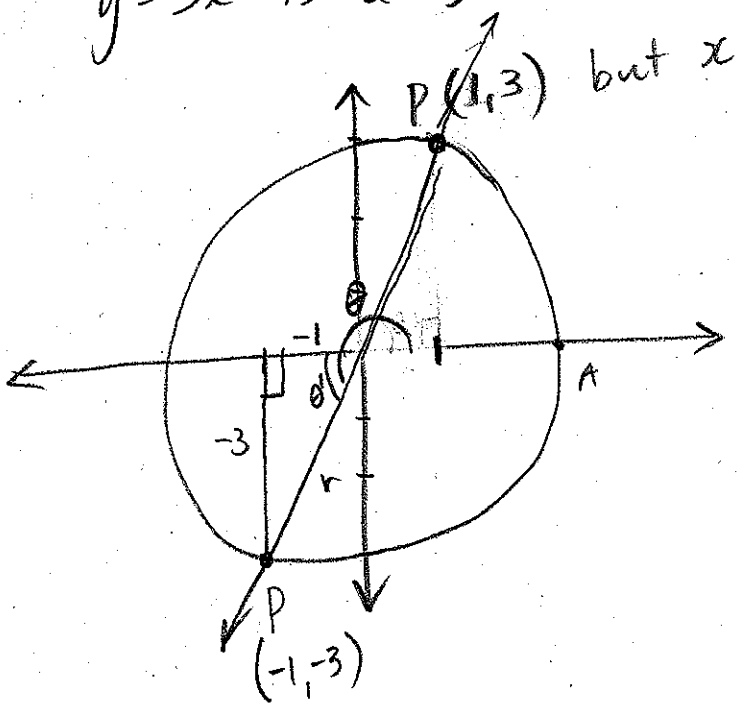
$$\sin\theta' = \sin\theta = \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$$

$$\cos\theta' = \cos\theta = \frac{-1}{\sqrt{5}} = \boxed{\frac{-\sqrt{5}}{5}}$$

$$\tan\theta' = \tan\theta = \frac{2}{-1} = \boxed{-2}$$

eg 7: $y = 3x \ (x \leq 0)$ is the equation of the terminal side of an angle θ in standard position. Sketch the smallest positive angle θ, and determine $\sin\theta$, $\cos\theta$, and $\tan\theta$.

$y = 3x$ is a line but $x \leq 0$



$$(-1)^2 + (-3)^2 = r^2$$

$$r = \sqrt{10}$$

$$\sin\theta' = \sin\theta = \frac{-3}{\sqrt{10}}$$

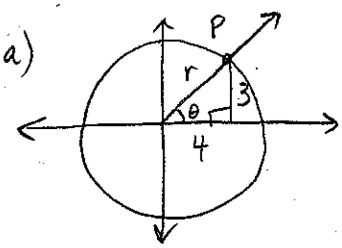
$$\cos\theta' = \cos\theta = \frac{-1}{\sqrt{10}}$$

$$\tan\theta' = \tan\theta = \frac{-3}{-1} = 3$$

p. 114-119 # 1-10

Ch. 3.2 Text Solutions

2a)



$3^2 + 4^2 = r^2$

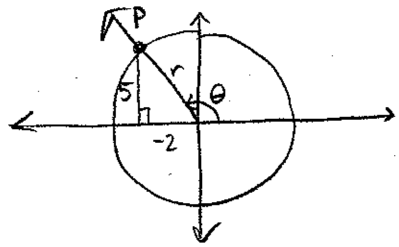
$r = 5$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

b)



$(-2)^2 + 5^2 = r^2$

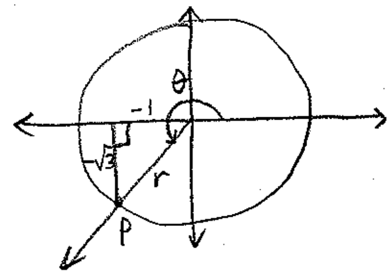
$r = \sqrt{29}$

$\sin\theta = \frac{5}{\sqrt{29}}$

$\cos\theta = \frac{-2}{\sqrt{29}}$

$\tan\theta = -\frac{5}{2}$

c)



$(-\sqrt{3})^2 + (-1)^2 = r^2$

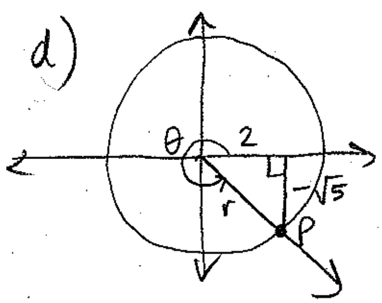
$r = 2$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{-1}{2}$

$\tan\theta = \sqrt{3}$

d)



$2^2 + (-\sqrt{5})^2 = r^2$

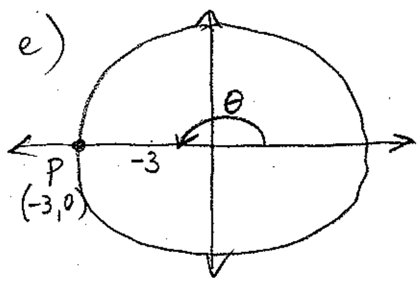
$r = 3$

$\sin\theta = -\frac{\sqrt{5}}{3}$

$\cos\theta = \frac{2}{3}$

$\tan\theta = -\frac{\sqrt{5}}{2}$

e)



$(-3)^2 + 0^2 = r^2$

$r = 3$

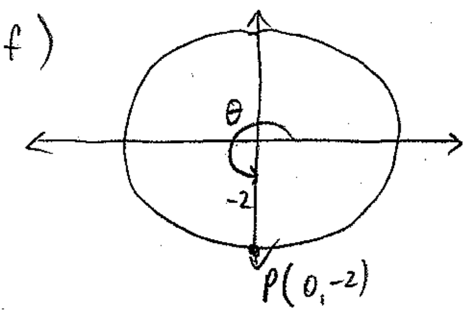
$\sin\theta = \frac{0}{3} = 0$

$\cos\theta = \frac{-3}{3} = -1$

$\tan\theta = \frac{0}{-3} = 0$

* more on this next section (3.3)

f)



$(-2)^2 + 0^2 = r^2$

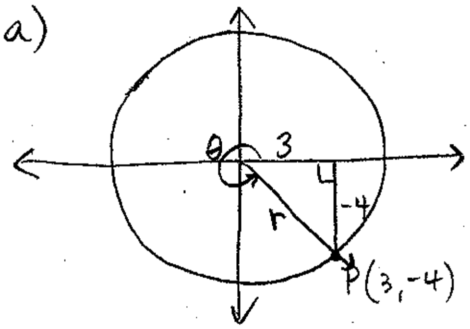
$r = 2$

$\sin\theta = \frac{-2}{2} = -1$

$\cos\theta = \frac{0}{2} = 0$

$\tan\theta = \frac{-2}{0}$ = undefined

3. a)



$3^2+(-4)^2=r^2$

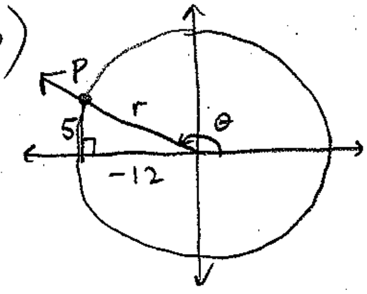
$r=5$

$\sin\theta=\frac{-4}{5}$

$\cos\theta=\frac{3}{5}$

$\tan\theta=\frac{-4}{3}$

b)



$(-12)^2+5^2=r^2$

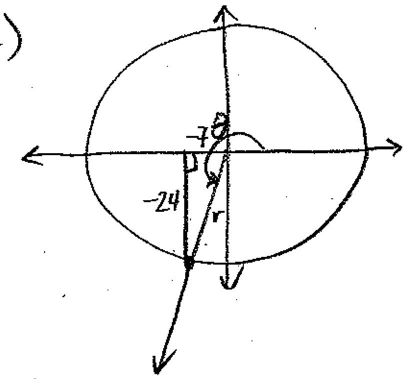
$r=13$

$\sin\theta=\frac{5}{13}$

$\cos\theta=\frac{-12}{13}$

$\tan\theta=\frac{-5}{12}$

c)



$(-7)^2+(-24)^2=r^2$

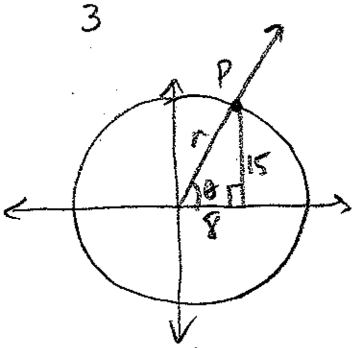
$r=25$

$\sin\theta=\frac{-24}{25}$

$\cos\theta=\frac{-7}{25}$

$\tan\theta=\frac{24}{7}$

d)



$8^2+15^2=r^2$

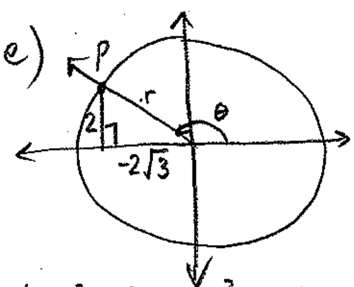
$r=17$

$\sin\theta=\frac{15}{17}$

$\cos\theta=\frac{8}{17}$

$\tan\theta=\frac{15}{8}$

e)



$(2)^2+(-2\sqrt{3})^2=r^2$

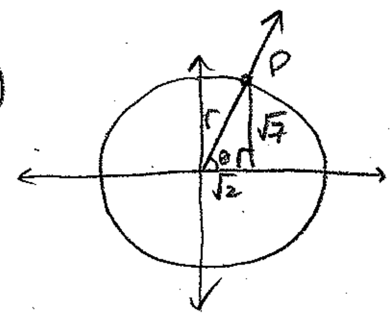
$r=4$

$\sin\theta=\frac{2}{4}=\frac{1}{2}$

$\cos\theta=\frac{-2\sqrt{3}}{4}=\frac{-\sqrt{3}}{2}$

$\tan\theta=\frac{2}{-2\sqrt{3}}=\frac{-1}{\sqrt{3}}$

f)



$(\sqrt{2})^2+(\sqrt{7})^2=r^2$

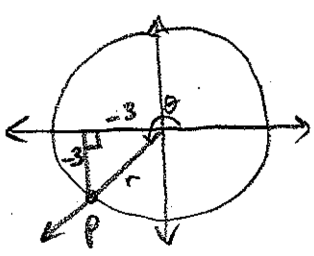
$r=3$

$\sin\theta=\frac{\sqrt{7}}{3}$

$\cos\theta=\frac{\sqrt{2}}{3}$

$\tan\theta=\frac{\sqrt{7}}{\sqrt{2}}=\frac{\sqrt{14}}{2}$

g)



$(-3)^2+(-3)^2=r^2$

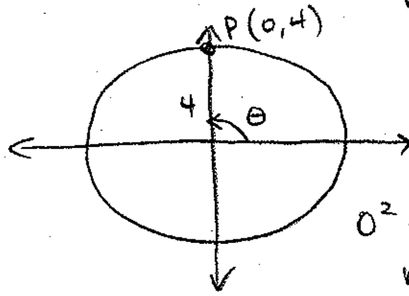
$r=3\sqrt{2}$

$\sin\theta=\frac{-3}{3\sqrt{2}}=\frac{-1}{\sqrt{2}}$

$\tan\theta=\frac{-3}{-3}=1$

$\cos\theta=\frac{-3}{3\sqrt{2}}=\frac{-1}{\sqrt{2}}$

h)



$0^2+4^2=r^2$

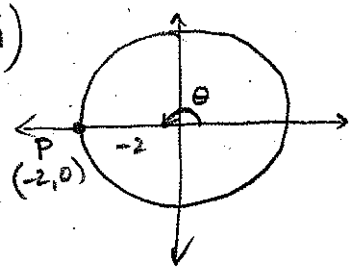
$r=4$

$\sin\theta=\frac{4}{4}=1$

$\cos\theta=\frac{0}{4}=0$

$\tan\theta=\frac{4}{4}=1$

i)



$(-2)^2 + 0^2 = r^2$

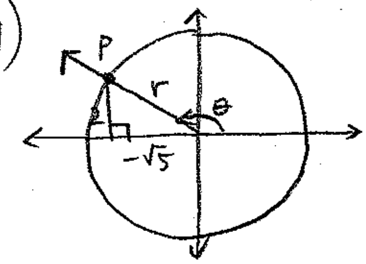
$r = 2$

$\sin\theta = \frac{0}{2} = 0$

$\cos\theta = \frac{-2}{2} = -1$

$\tan\theta = \frac{0}{-2} = 0$

j)



$2^2 + (-\sqrt{5})^2 = r^2$

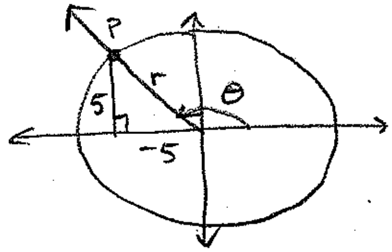
$r = 3$

$\sin\theta = \frac{2}{3}$

$\cos\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{-2}{\sqrt{5}}$

k)



$5^2 + (-5)^2 = r^2$

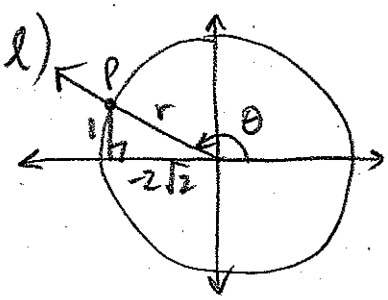
$r = 5\sqrt{2}$

$\sin\theta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{-5}{5\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\tan\theta = \frac{5}{-5} = -1$

l)



$1^2 + (-2\sqrt{2})^2 = r^2$

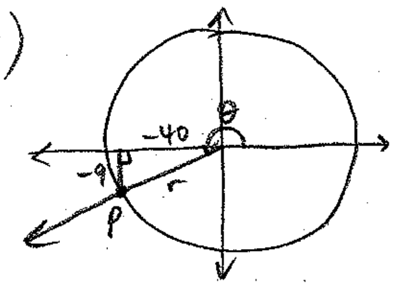
$r = 3$

$\sin\theta = \frac{1}{3}$

$\cos\theta = \frac{-2\sqrt{2}}{3}$

$\tan\theta = \frac{-1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$

m)



$(-40)^2 + (-9)^2 = r^2$

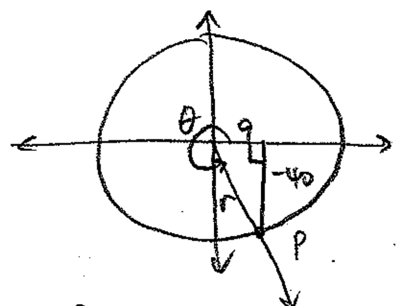
$r = 41$

$\sin\theta = \frac{-9}{41}$

$\cos\theta = \frac{-40}{41}$

$\tan\theta = \frac{-9}{-40} = \frac{9}{40}$

n)



$9^2 + (-40)^2 = r^2$

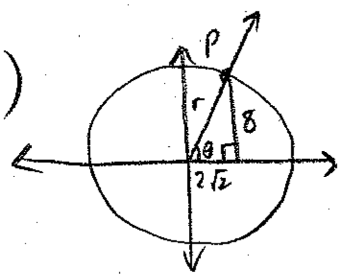
$r = 41$

$\sin\theta = \frac{-40}{41}$

$\cos\theta = \frac{9}{41}$

$\tan\theta = \frac{-40}{9}$

o)



$(2\sqrt{2})^2 + 8^2 = r^2$

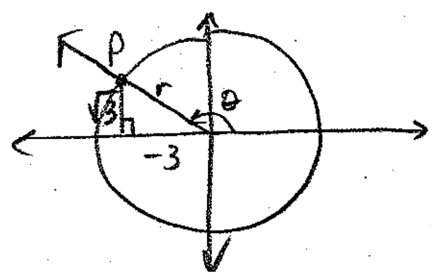
$r = 6\sqrt{2}$

$\sin\theta = \frac{8}{6\sqrt{2}} = \frac{4}{3\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

$\cos\theta = \frac{2\sqrt{2}}{6\sqrt{2}} = \frac{1}{3}$

$\tan\theta = \frac{8}{2\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

p)



$(\sqrt{3})^2 + (-3)^2 = r^2$

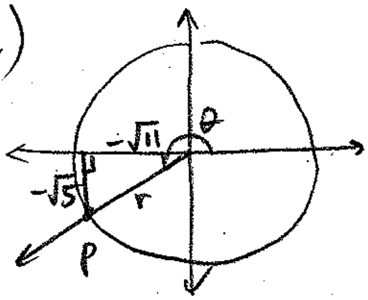
$r = 2\sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$

$\cos\theta = \frac{-3}{2\sqrt{3}} = \frac{-3\sqrt{3}}{6} = \frac{-\sqrt{3}}{2}$

$\tan\theta = -\frac{\sqrt{3}}{3}$

q)



$(-\sqrt{5})^2 + (-\sqrt{11})^2 = r^2$

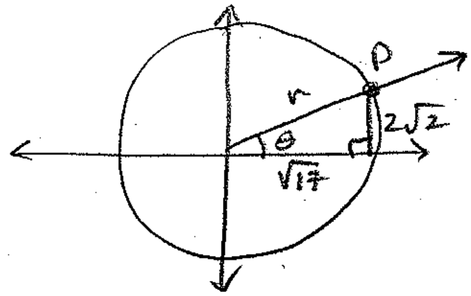
$r = 4$

$\sin\theta = -\frac{\sqrt{5}}{4}$

$\cos\theta = -\frac{\sqrt{11}}{4}$

$\tan\theta = \frac{-\sqrt{5}}{-\sqrt{11}} = \frac{\sqrt{55}}{11}$

r)



$(\sqrt{17})^2 + (2\sqrt{2})^2 = r^2$

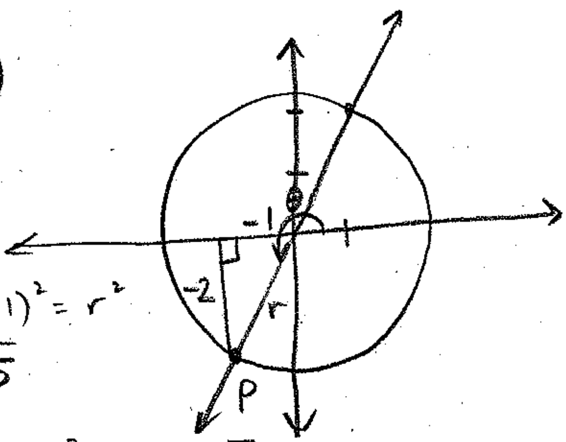
$r = 5$

$\sin\theta = \frac{2\sqrt{2}}{5}$

$\cos\theta = \frac{\sqrt{17}}{5}$

$\tan\theta = \frac{2\sqrt{2}}{\sqrt{17}} = \frac{2\sqrt{34}}{17}$

4a)



$(-2)^2 + (-1)^2 = r^2$

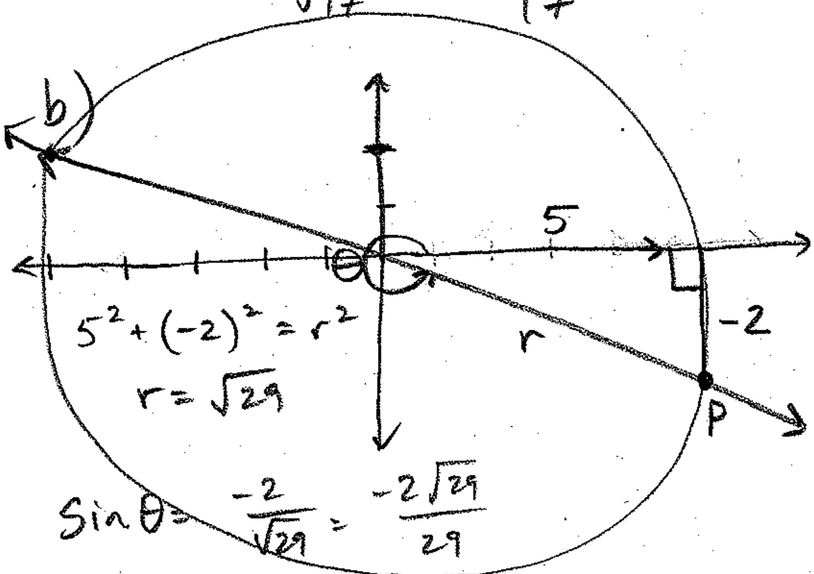
$r = \sqrt{5}$

$\sin\theta = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

$\cos\theta = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$

$\tan\theta = \frac{-2}{-1} = 2$

b)



$5^2 + (-2)^2 = r^2$

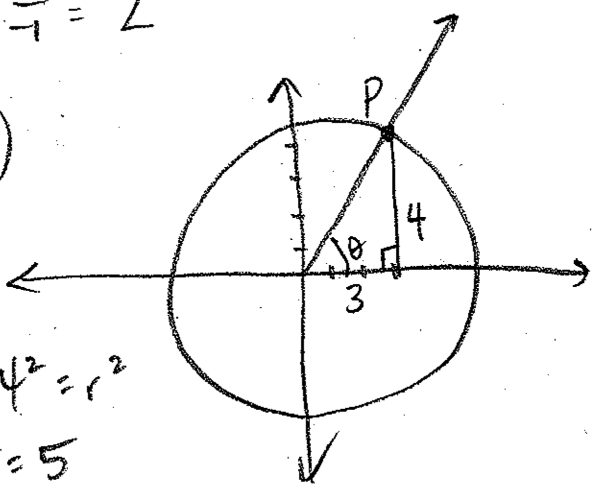
$r = \sqrt{29}$

$\sin\theta = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$

$\cos\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$

$\tan\theta = \frac{-2}{5}$

c)



$3^2 + 4^2 = r^2$

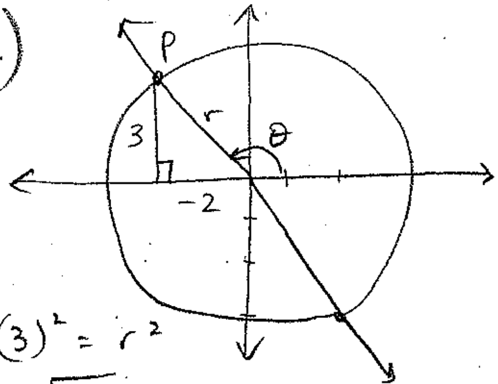
$r = 5$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

d)



$(-2)^2+(3)^2=r^2$

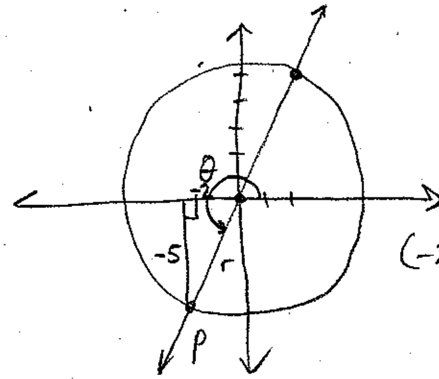
$r=\sqrt{13}$

$\sin\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\cos\theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

$\tan\theta = -\frac{3}{2}$

e)



$(-2)^2+(-5)^2=r^2$

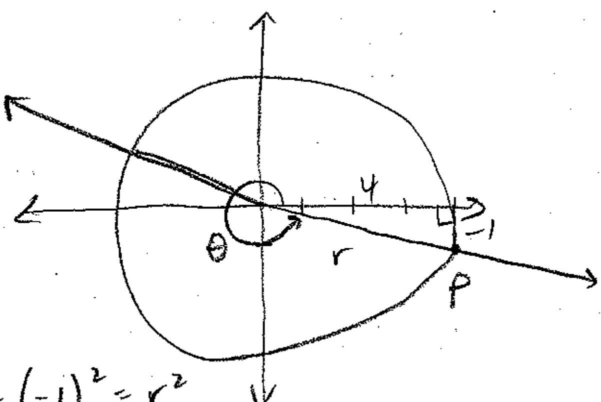
$r=\sqrt{29}$

$\sin\theta = \frac{-5}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$

$\tan\theta = \frac{-5}{-2} = \frac{5}{2}$

f)



$4^2+(-1)^2=r^2$

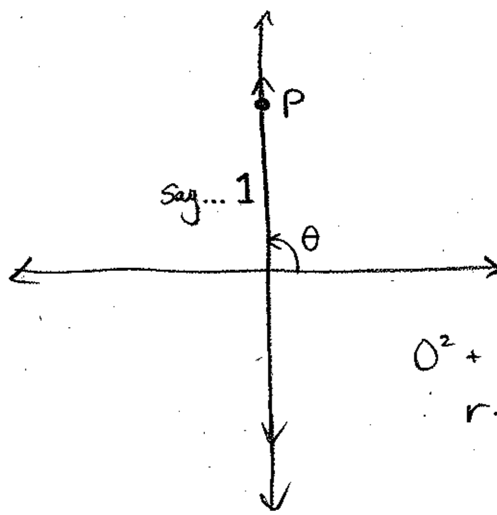
$r=\sqrt{17}$

$\sin\theta = \frac{-1}{\sqrt{17}} = \frac{-\sqrt{17}}{17}$

$\cos\theta = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$

$\tan\theta = \frac{-1}{4}$

g)



$0^2+1^2=r^2$

$r=1$

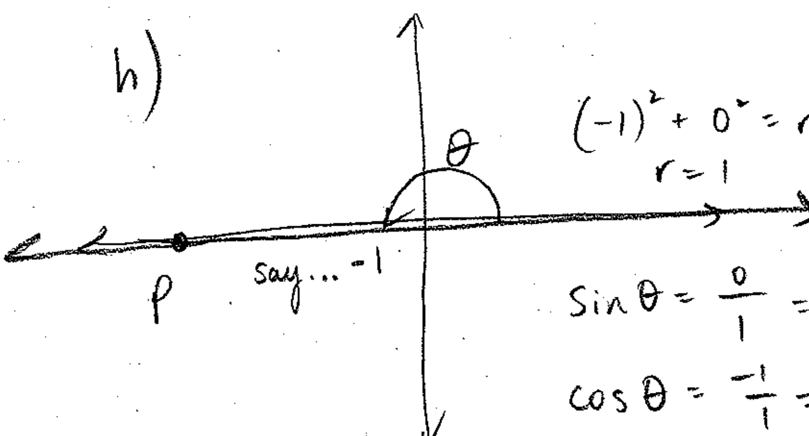
$\sin\theta = \frac{1}{1} = 1$

$\cos\theta = \frac{0}{1} = 0$

$\tan\theta = \frac{1}{1} = 1$

* more next section!

h)



$(-1)^2+0^2=r^2$

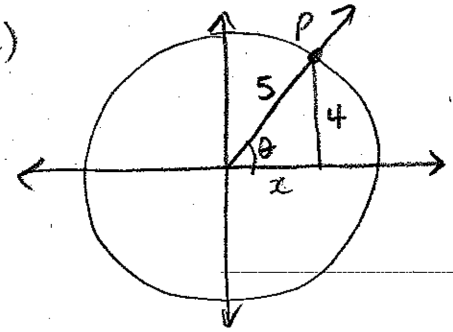
$r=1$

$\sin\theta = \frac{0}{1} = 0$

$\cos\theta = \frac{-1}{1} = -1$

$\tan\theta = \frac{0}{-1} = 0$

5a)

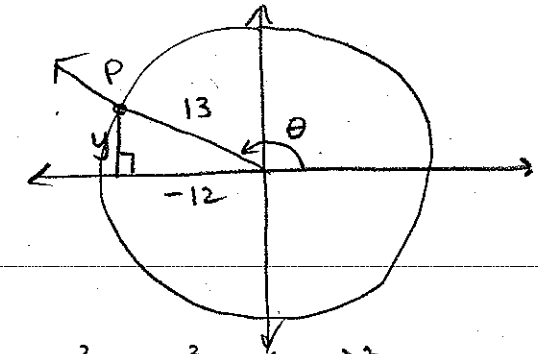


$x^2 = 5^2 - 4^2$

$x = 3$

$\cos\theta = \frac{3}{5}$; $\tan\theta = \frac{4}{3}$

b)



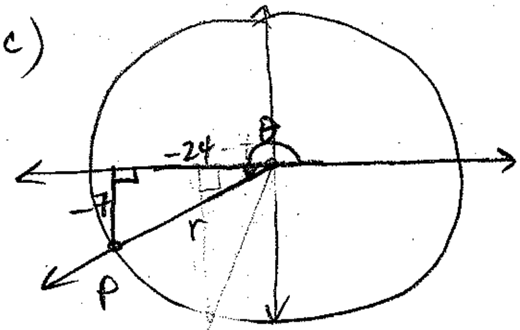
$y^2 = 13^2 - (-12)^2$

$y = 5$

$\sin\theta = \frac{5}{13}$

$\tan\theta = \frac{-5}{12}$

c)

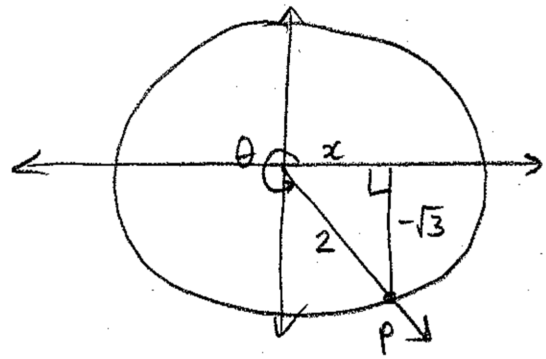


$(-7)^2 + (-24)^2 = r^2$

$r = 25$

$\sin\theta = \frac{-7}{25}$; $\cos\theta = \frac{-24}{25}$

d)



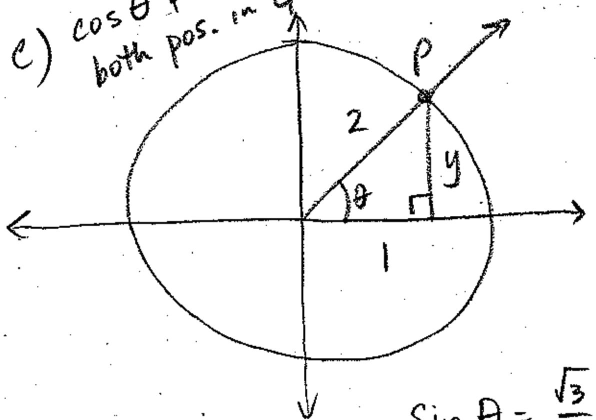
$x^2 = 2^2 - (-\sqrt{3})^2$

$x = 1$

$\cos\theta = \frac{1}{2}$

$\tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3}$

e) cos θ & tan θ both pos. in Q I



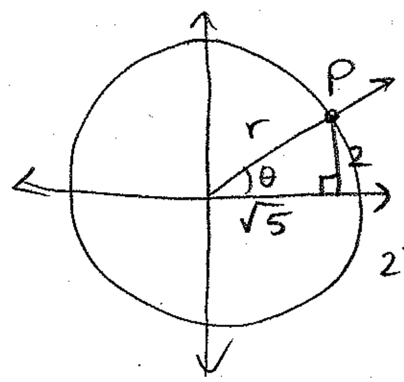
$y^2 = 2^2 - 1^2$

$y = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

f)



sin θ & tan θ both pos. in Q. I

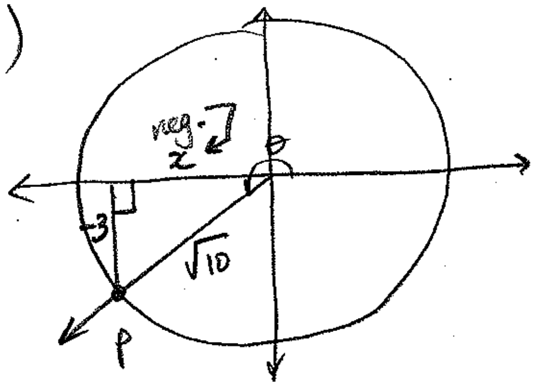
$2^2 + (\sqrt{5})^2 = r^2$

$r = 3$

$\sin\theta = \frac{2}{3}$

$\cos\theta = \frac{\sqrt{5}}{3}$

g)



Sin θ & cos θ both neg. in Q III

$$x^2 = (\sqrt{10})^2 - (-3)^2$$

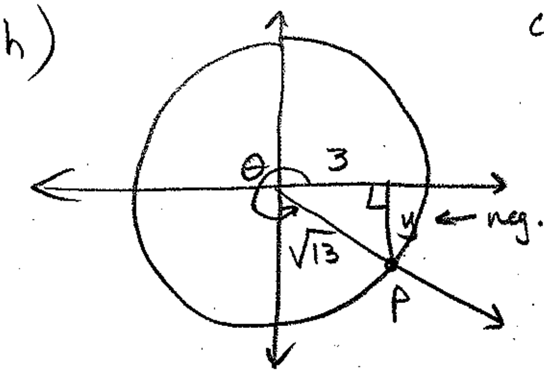
$$x = \pm\sqrt{1} = -\sqrt{1}$$

$$\cos\theta = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10}$$

$$\tan\theta = \frac{-3}{-1} = 3$$

h)

cos θ (+) and tan θ (−) in Q IV.



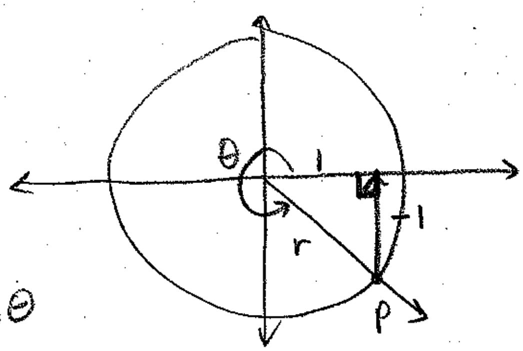
$$y^2 = (\sqrt{13})^2 - (3)^2$$

$$y = \pm 2 = -2$$

$$\sin\theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$$

$$\tan\theta = -\frac{2}{3}$$

i)



tan θ (−) and sin θ (−) in Q IV

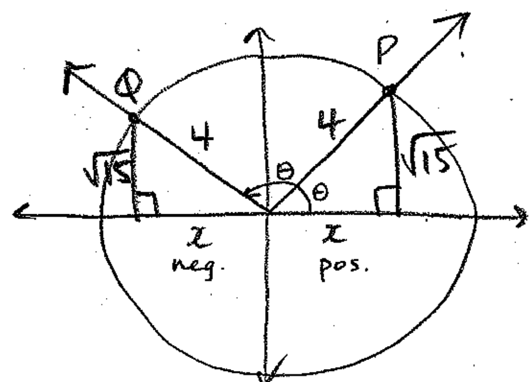
$$r^2 = 1^2 + (-1)^2$$

$$r = \sqrt{2}$$

$$\sin\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

j)

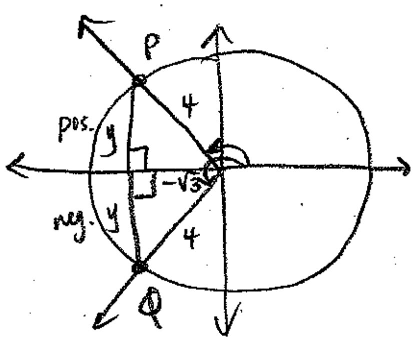


$$x^2 = 4^2 - (\sqrt{15})^2$$

$$x = \pm 1$$

$$\cos\theta = \frac{\pm 1}{4} \qquad \tan\theta = \frac{\pm\sqrt{15}}{1} = \pm\sqrt{15}$$

k)



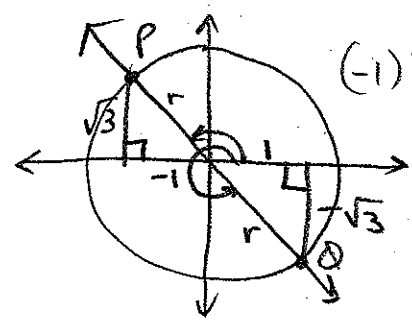
$$\sin\theta = \frac{\pm\sqrt{13}}{4}$$

$$\tan\theta = \frac{\pm\sqrt{13}}{\sqrt{3}} = \frac{\pm\sqrt{39}}{3}$$

$$y^2 = 4^2 - (-\sqrt{3})^2$$

$$y = \pm\sqrt{13}$$

l)



$$(-1)^2 + (\sqrt{3})^2 = r^2$$

$$r = 2$$

$$\sin\theta = \frac{\pm\sqrt{3}}{2}$$

$$\cos\theta = \pm\frac{1}{2}$$

#6) see back of text.

7)a) $y = -2x + 1$

$y = 3x + 6$

$-2x + 1 = 3x + 6$

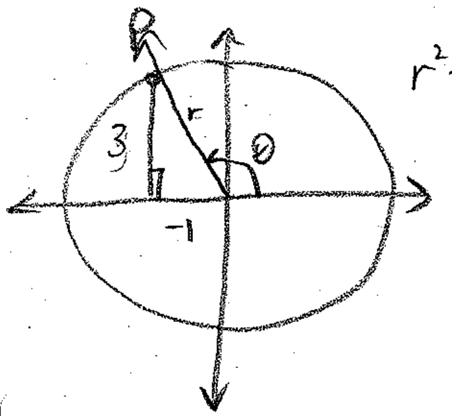
$-5 = 5x$

$x = -1$

$y = -2(-1) + 1$

$y = 3$

$P = (-1, 3)$



$r^2 = (-1)^2 + (3)^2$

$r = \sqrt{10}$

$\sin\theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\cos\theta = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

$\tan\theta = \frac{3}{-1} = -3$

b) $y = -3x + 10$

$4y = x - 12$

$y = \frac{1}{4}x - 3$

$-3x + 10 = \frac{1}{4}x - 3$

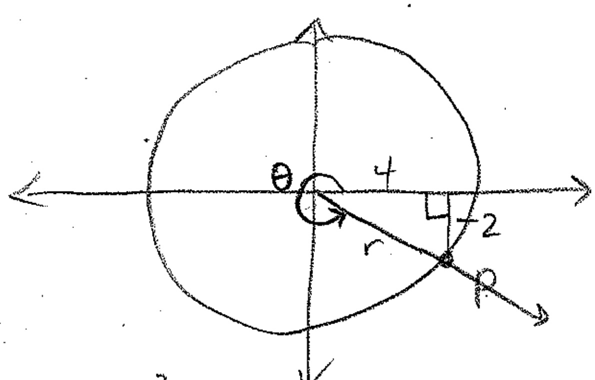
$13 = \frac{13}{4}x$

$x = 4$

$y = -3(4) + 10$

$y = -2$

$P = (4, -2)$



$r^2 = 4^2 + (-2)^2$

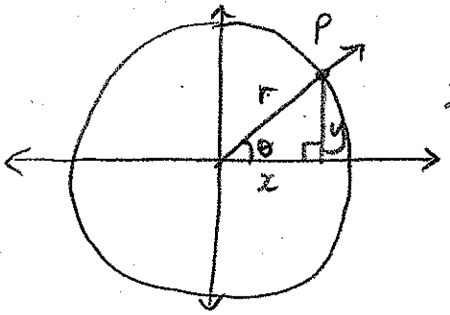
$r = 2\sqrt{5}$

$\sin\theta = \frac{-2}{2\sqrt{5}} = \frac{-\sqrt{5}}{5}$

$\cos\theta = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\tan\theta = -\frac{2}{4} = \frac{-1}{2}$

8. a)



$x = y$

let $x = 1$
$y = 1$ then $r = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

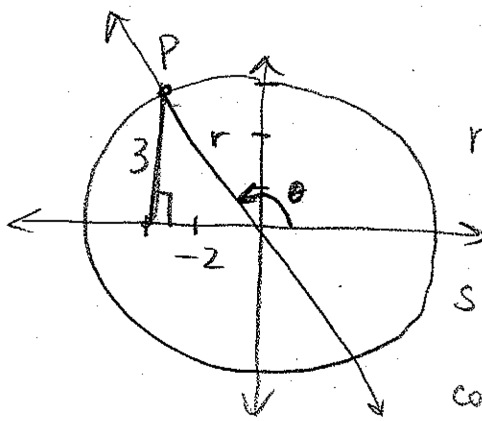
$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan\theta = 1$

b) $2y = -3x + 2$

$y = -\frac{3}{2}x + 1$

So ... our line $\Rightarrow$ $y = -\frac{3}{2}x$
(to go through ORIGIN)



$r^2 = 3^2 + (-2)^2$

$r = \sqrt{13}$

$\sin\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

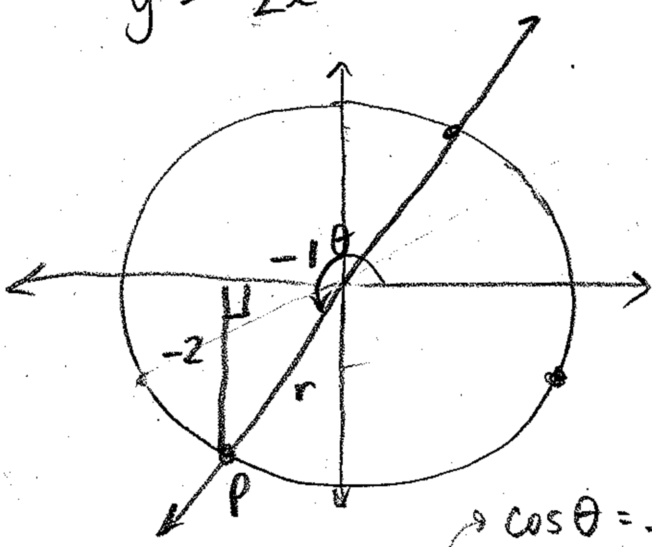
$\cos\theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

$\tan\theta = \frac{-3}{2}$

c) $A(1,4)$ $B(-3,-4)$

rise $= -8$
run $= -4$ slope $= +2$

$y = 2x$



$r^2 = (-2)^2 + (-1)^2$

$r = \sqrt{5}$

$\sin\theta = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

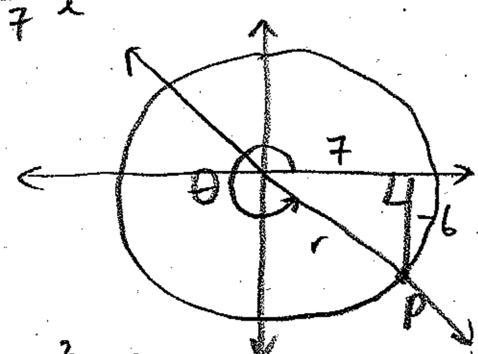
$\cos\theta = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$

$\tan\theta = 2$

d) $A(-2,4)$ $B(5,-2)$

rise $= -6$
run $= 7$ slope $= -\frac{6}{7}$

$y = -\frac{6}{7}x$



$r^2 = 7^2 + (-6)^2$

$r = \sqrt{85}$

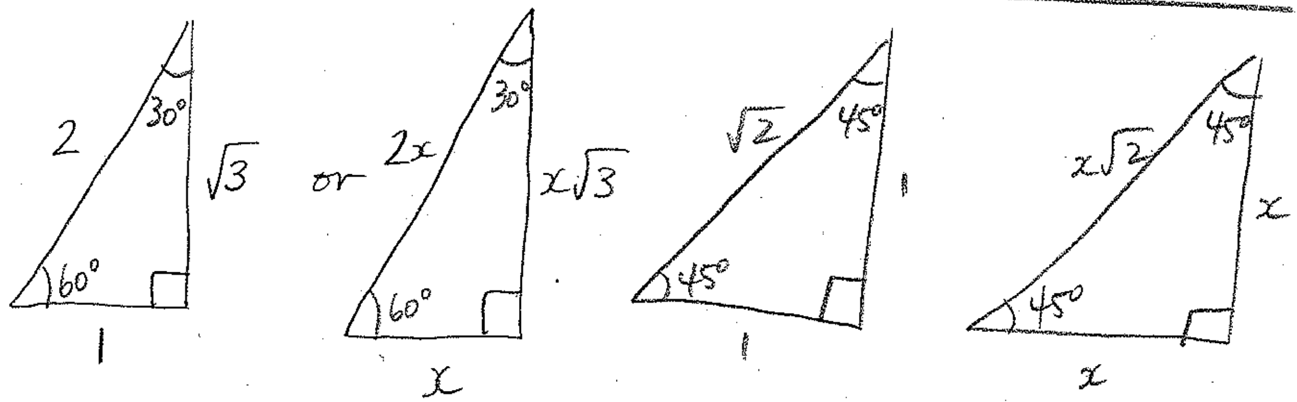
$\sin\theta = \frac{-6}{\sqrt{85}} = \frac{-6\sqrt{85}}{85}$

$\cos\theta = \frac{7}{\sqrt{85}} = \frac{7\sqrt{85}}{85}$

$\tan\theta = -\frac{6}{7}$

Ch. 3.3 - Special Angles (Triangles)

30°-60°-90° Δ 45°-45°-90° Δ



- all are proven with the PYTHAGOREAN THEOREM.

- special Δs can be used to find the EXACT VALUE (no decimal = no rounding error) of the three trig. ratios of certain angles (30°, 60°, 45° as angles or reference angles).

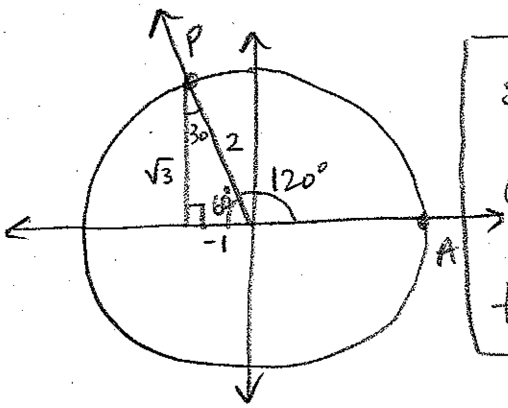
* remember, the hypotenuse (radius) is always POSITIVE.

TRIG. RATIOS OF SPECIAL ANGLES

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\left(\frac{\sqrt{3}}{3}\right)$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\left(\frac{\sqrt{2}}{2}\right)$	$\frac{1}{\sqrt{2}}$ or $\left(\frac{\sqrt{2}}{2}\right)$	1

eg1: Find the exact value of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for each of the following: (DIAGRAM REQUIRED)

a) $\theta = 120°$, so $\underset{(\text{ref})}{\theta'} = 60°$

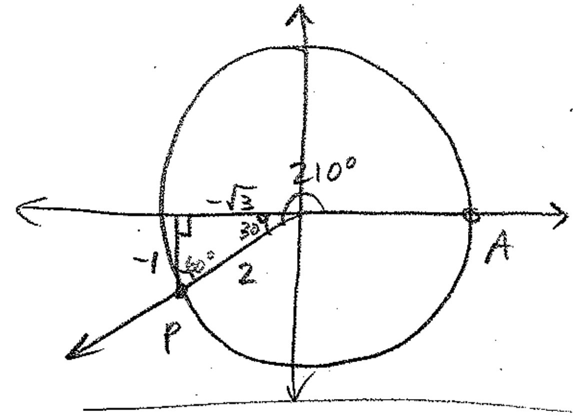


$\sin\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = -\frac{1}{2}$

$\tan\theta = -\sqrt{3}$

b) $\theta = 210°$, so $\theta' = 30°$

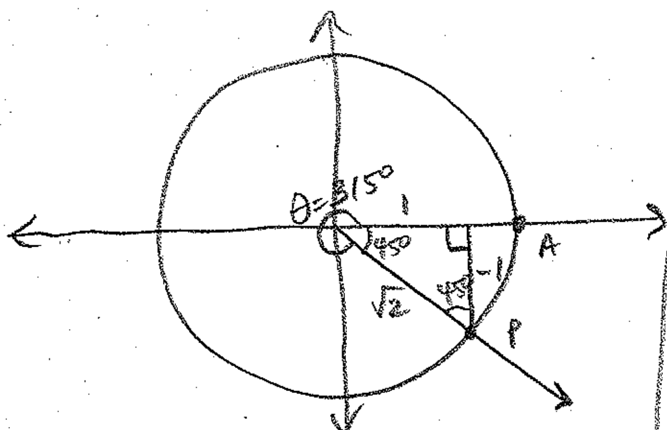


$\sin\theta = -\frac{1}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

c) $\theta = 315°$, so $\theta' = 45°$

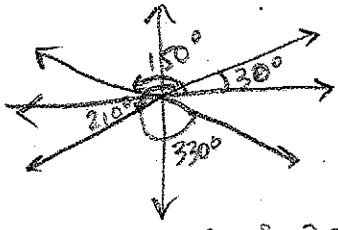


$\sin\theta = \frac{-1}{\sqrt{2}}$ or $\frac{-\sqrt{2}}{2}$

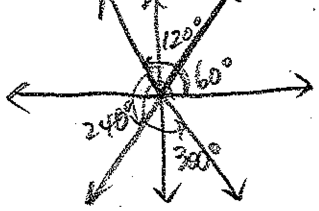
$\cos\theta = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$\tan\theta = -1$

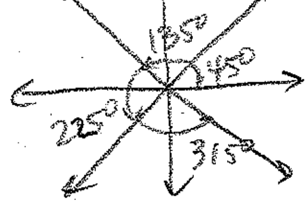
* 'RELATED' angles: (all special)



30°, 150°, 210°, 330°



60°, 120°, 240°, 300°

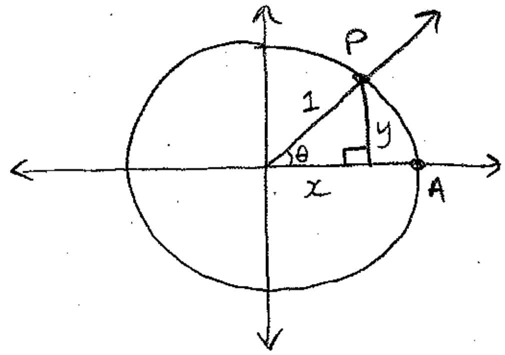


45°, 135°, 225°, 315°

Quadrantal Angles $(0°, 90°, 180°, 270°)$

- requires use of a UNIT CIRCLE (radius = 1)

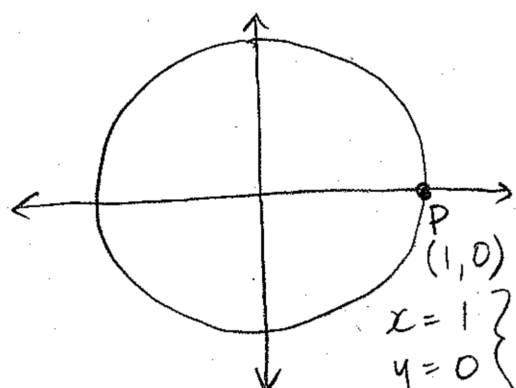
$$x^2 + y^2 = 1^2$$
$$x^2 + y^2 = 1$$



$$\boxed{\sin\theta = \frac{y}{1} = y \qquad \cos\theta = \frac{x}{1} = x \qquad \tan\theta = \frac{y}{x}}$$

eg 2: Find the EXACT VALUE of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for each of the following (Diagram req'd):
$(0° \leq \theta < 360°)$

a) $\theta = 0°$
(coterminal to $360°$)

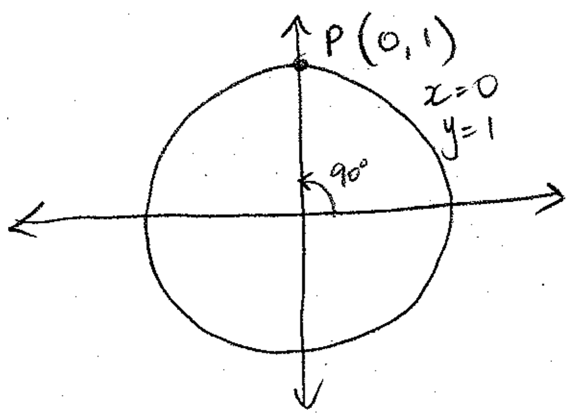


$\left.\begin{matrix} x = 1 \\ y = 0 \end{matrix}\right\}$

* $0°$ is a QUADRANTAL angle, so use the UNIT CIRCLE

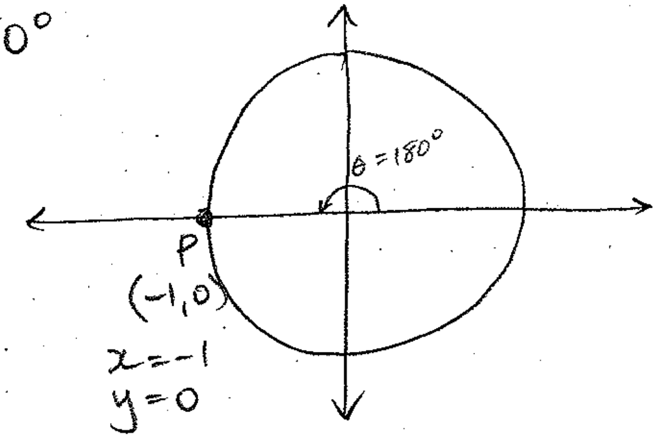
$\sin 0° = 0$
$\cos 0° = 1$
$\tan 0° = \frac{0}{1} = 0$

b) $\theta = 90°$



$\sin 90° = 1$
$\cos 90° = 0$
$\tan 90° = \frac{1}{0} = \text{undefined}$

c) $\theta = 180°$

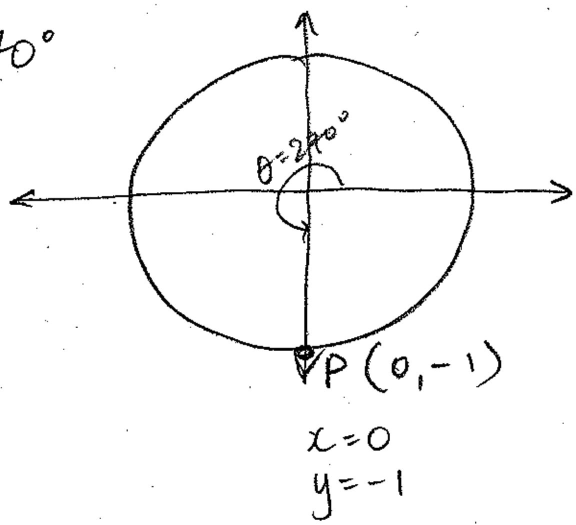


$\sin 180° = 0$

$\cos 180° = -1$

$\tan 180° = \frac{0}{-1} = 0$

d) $\theta = 270°$



$\sin 270° = -1$

$\cos 270° = 0$

$\tan 270° = \frac{-1}{0} = \text{undef.}$

FOR NOW... p. 126 - 127

\# 1-3 (diagrams!)

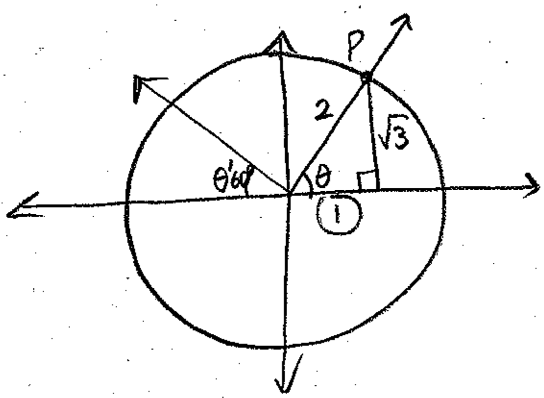
The Reverse Process ... Given the EXACT ratio, solve for θ

If $0° \leq \theta < 360°$, determine all values of θ such that: (diagram required)

a) $\sin\theta = \dfrac{\sqrt{3}}{2}$

HINTS:

* Where do we see $\sqrt{3}$ and 2? $\Rightarrow$ a 30-60-90 Δ!
* where is sin positive? $\Rightarrow$ Quadrants I & II.



QI	QII
$\boxed{\theta = 60°,}$	$\boxed{120°}$

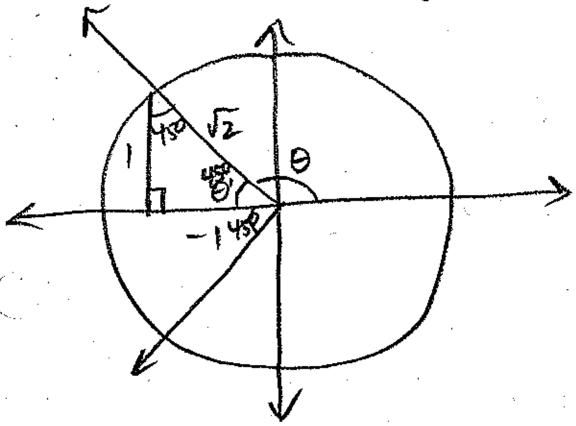
$(180° - 60°)$

Another HINT:

Quadrant I:	$\theta = \theta'$
Q II:	$\theta = 180° - \theta'$
Q III:	$\theta = 180° + \theta'$
Q IV:	$\theta = 360° - \theta'$

b) $\cos\theta = -\dfrac{1}{\sqrt{2}}$ (could be stated as $\cos\theta = -\dfrac{\sqrt{2}}{2}$, but not on test)

* 45-45-90 Δ
* cos negative in Qs II and III

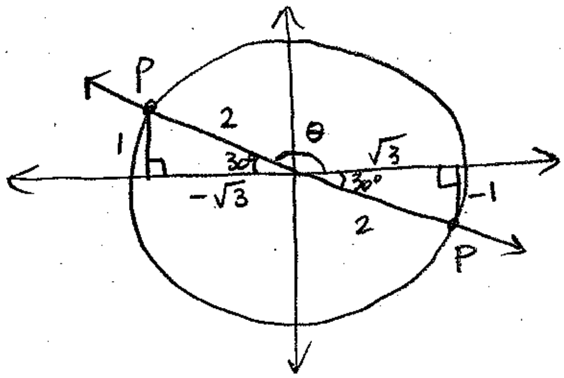


Q II	Q III
$\theta = 180° - 45°$	$\theta = 180° + 45°$
$\boxed{\theta = 135°}$	$\boxed{\theta = 225°}$

c) $\tan\theta = \frac{-1}{\sqrt{3}}$

* 30-60-90 Δ

* tan negative in Qs II and IV



Q II

$\theta = 180° - 30°$

$\boxed{\theta = 150°}$

Q IV

$\theta = 360° - 30°$

$\boxed{\theta = 330°}$

Given decimal ratios, find θ $(0° \leq \theta < 360°)$ for the following: (nearest tenth)

a) $\cos\theta = 0.632$

cos positive in Qs I & IV

$\theta = \cos^{-1} 0.632$

$\boxed{\theta = 50.8° \ (Q\,I)}$

$\theta = \theta'$

Q IV

$\theta = 360° - 50.8°$

$\boxed{\theta = 309.2°}$

b) $\sin\theta = -0.711$

sin negative in Qs III & IV

Solve $\sin\theta = 0.711$ to get θ', the ref. angle.

$\theta = \sin^{-1} 0.711 = 45.3° = \theta'$

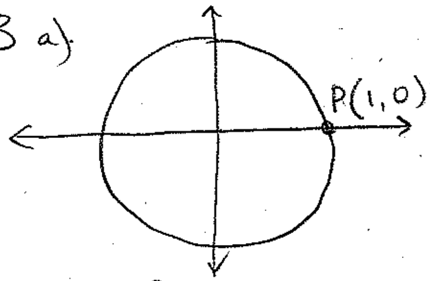
Q III: $\theta = 180 + 45.3° = \boxed{225.3°}$

Q IV: $\theta = 360° - 45.3° = \boxed{314.7°}$

Hwk: p. 127-128 # 4-6. diagrams!

Ch. 3.3 Solutions #3 & 4 only

3 a)



$\sin\theta = y$

$\boxed{\sin 0^\circ = 0}$

b) see ⓐ for diagram

$\cos\theta = x$

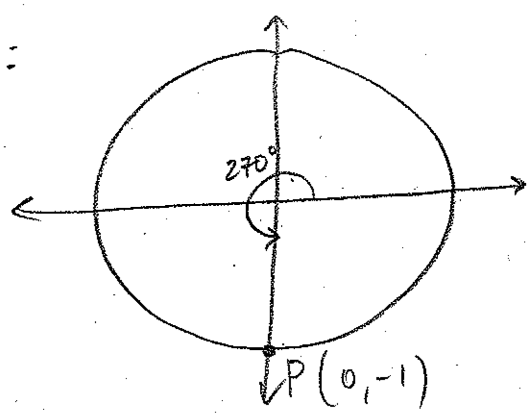
$\boxed{\cos 0^\circ = 1}$

c) see ⓐ for diagram

$\tan\theta = \frac{y}{x}$

$\tan 0^\circ = \frac{0}{1} = \boxed{0}$

d), e), f) :



$\sin\theta = y$

$\boxed{\sin 270^\circ = -1}$

$\cos\theta = x$

$\boxed{\cos 270^\circ = 0}$

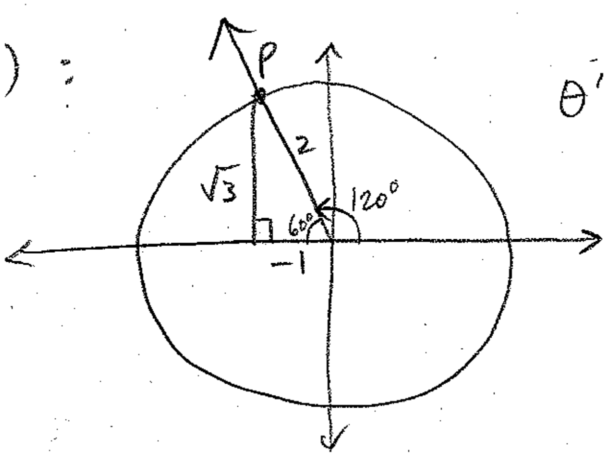
$\tan\theta = \frac{y}{x}$

$\tan 270^\circ = \frac{-1}{0} = \boxed{\text{undefined}}$

or

$\boxed{\infty}$

g), h), i) :



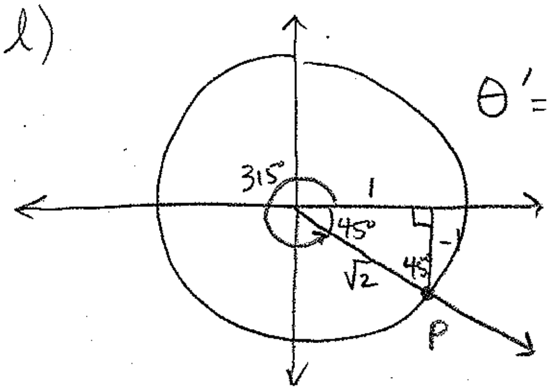
$\theta' = 60^\circ$

$\boxed{\sin 120^\circ = \frac{\sqrt{3}}{2}}$

$\boxed{\cos 120^\circ = -\frac{1}{2}}$

$\tan 120^\circ = \frac{\sqrt{3}}{-1} = \boxed{-\sqrt{3}}$

j), k), l)



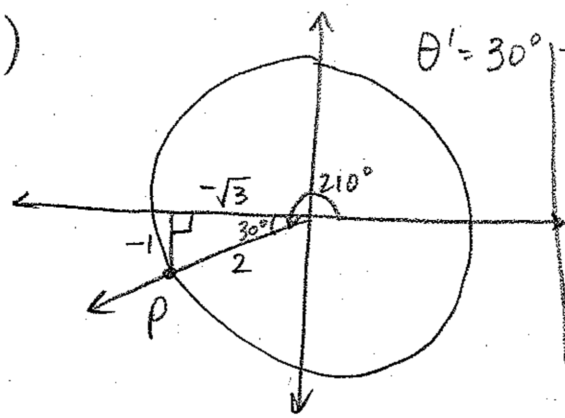
$\theta' = 45°$

$\sin 315° = \frac{-1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

$\cos 315° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$\tan 315° = \frac{-1}{1} = -1$

m), n), o)



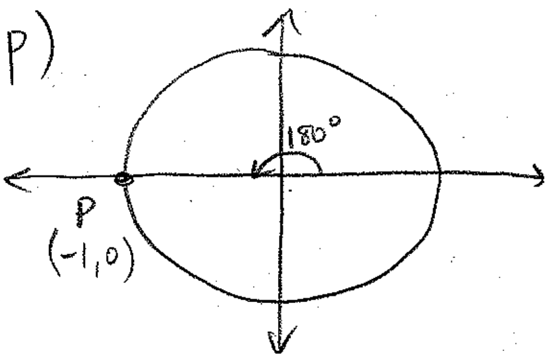
$\theta' = 30°$

$\sin 210° = \frac{-1}{2}$

$\cos 210° = \frac{-\sqrt{3}}{2}$

$\tan 210° = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

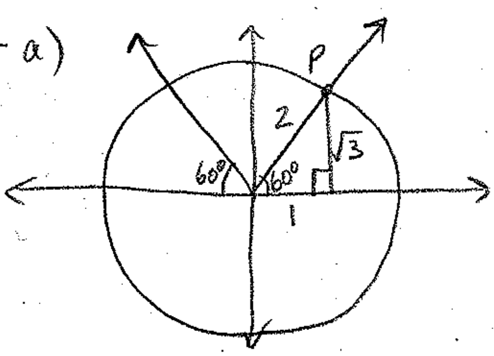
p)



$\tan\theta = \frac{y}{x}$

$\tan 180° = \frac{0}{-1} = \boxed{0}$

4 a)

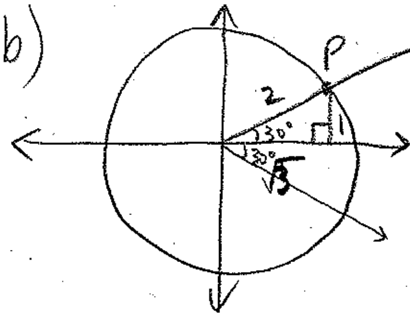


QI QII

$\boxed{\theta = 60°,\ 120°}$

$(180° - 60°)$

b)

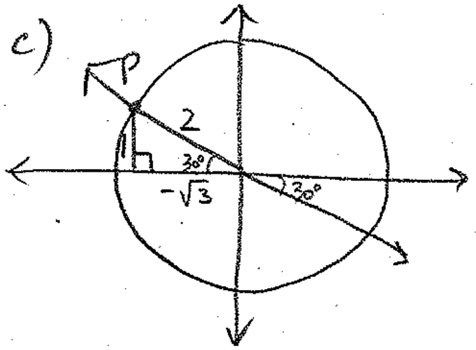


QI QIV

$\boxed{\theta = 30°,\ 330°}$

$360° - 30°$

c)

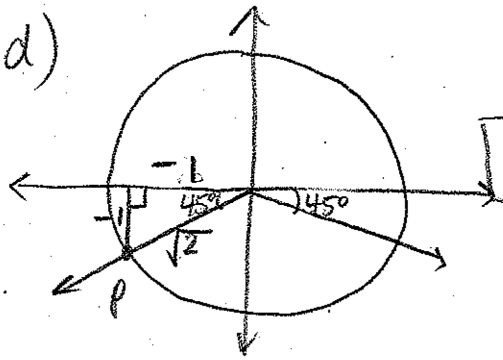


Q II Q IV

$\boxed{\theta = 150°, \quad 330°}$

$(180° - 30°)$ $(360° - 30°)$

d)

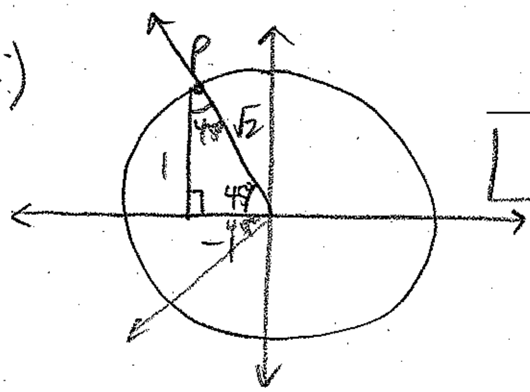


Q III Q IV

$\boxed{\theta = 225°, \quad 315°}$

$(180° + 45°)$ $(360° - 45°)$

e)

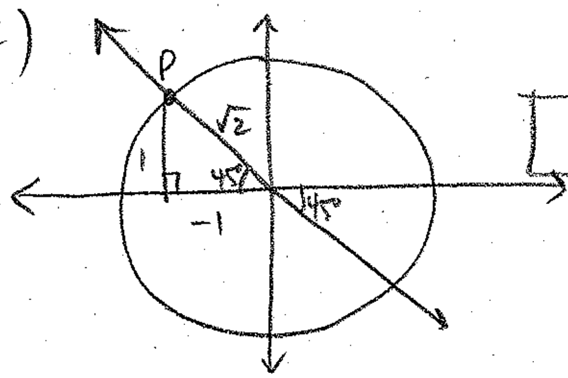


Q II Q III

$\boxed{\theta = 135°, \quad \theta = 225°}$

$(180° - 45°)$ $(180° + 45°)$

f)

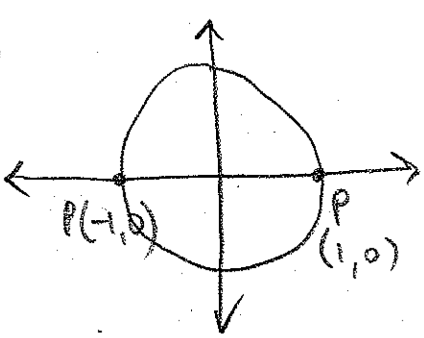


Q II Q IV

$\boxed{\theta = 135°, \quad 315°}$

$(180 - 45°)$ $(360° - 45°)$

g)

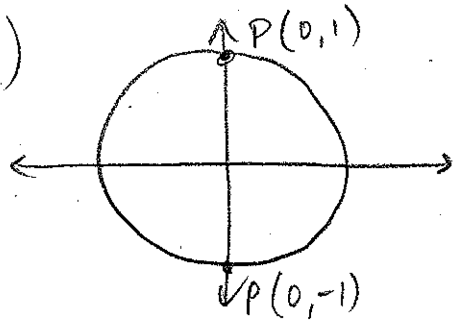


$\sin\theta = y$

when is $y = 0$?

$\boxed{\theta = 0°, \ 180°}$

h)

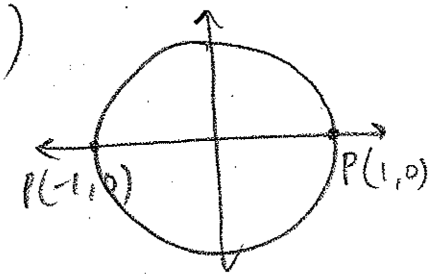


$\cos\theta = x$

when is $x = 0$?

$\boxed{\theta = 90°, 270°}$

i)

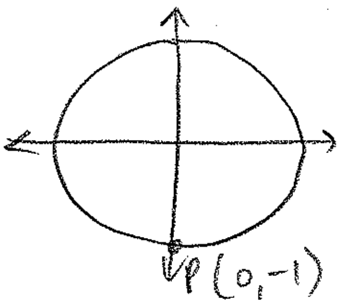


$\tan\theta = \frac{y}{x}$

when is $y = 0$?

$\boxed{\theta = 0°, 180°}$

j)

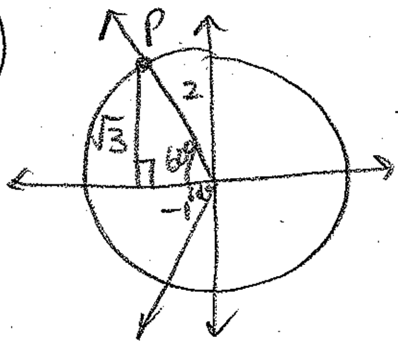


$\sin\theta = y$

when is $y = -1$?

$\boxed{\theta = 270°}$

k)

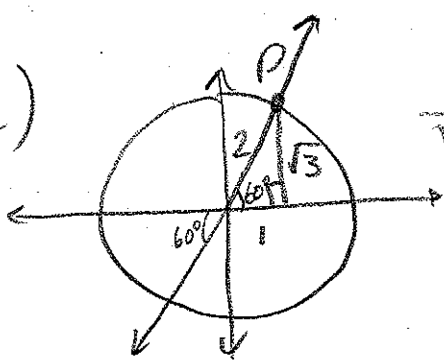


Q II Q III

$\boxed{\theta = 120° \quad , \quad 240°}$

$(180° - 60°)$ $(180° + 60°)$

l)



Q I Q III

$\boxed{\theta = 60° \quad , \quad \theta = 240°}$

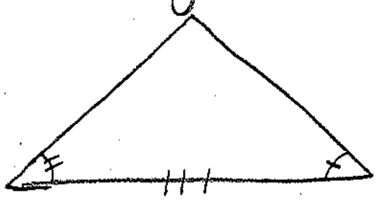
$(180° + 60°)$

Ch. 3.4 - Oblique Triangles

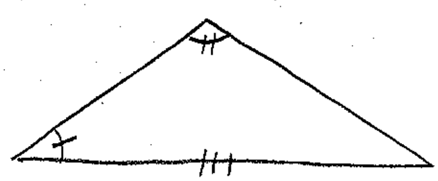
Oblique Δ - a Δ that does not contain a right angle.

- to solve an oblique Δ (ie. find all side lengths and all angle measures), three pieces of information are necessary. This info can be categorized in four ways:

① Two angles and one side of a triangle



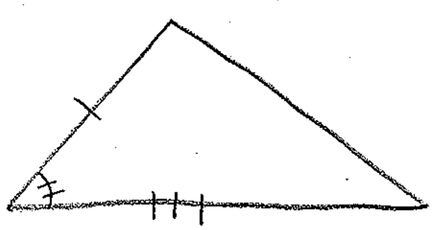
ASA (Angle - Side - Angle)



AAS (Angle - Angle - Side)

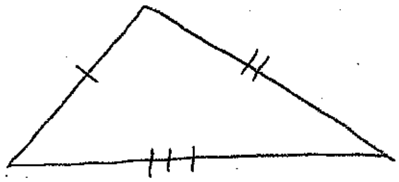
Note: If you know two angles in a Δ, then you know the third as all interior angles of a Δ add to 180°.

② Two sides and their contained (included) angle.



SAS (Side - Angle - Side)

③ Three sides



SSS (Side - Side - Side)

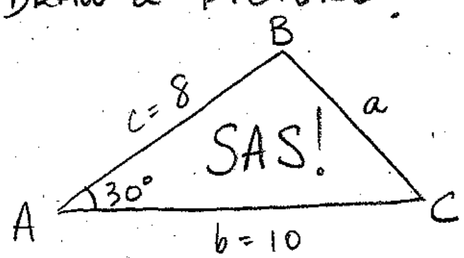
④ Two sides and an angle opposite one of the sides.

ASS (Angle - Side - Side)

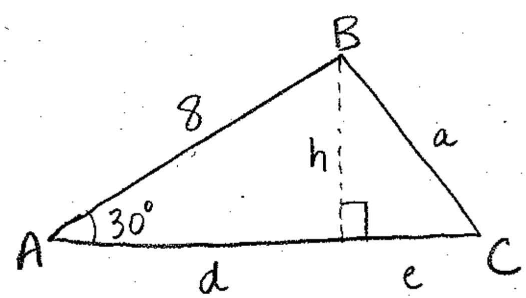
Note: ASS is referred to as the Donkey Theorem because the triangle is not always unique. In this section, however, the ASS Δs will be unique.

eg1: Given ΔABC, with $\angle A = 30°$, $b = 10$, $c = 8$, find: a) a b) $\angle C$ c) $\angle B$ (nearest tenth)

DRAW a PICTURE!



a) To find a: ⓘ - find h
ⓘⓘ - find e (1st find d)
ⓘⓘⓘ - use Pythagoras



$d + e = 10$

ⓘ $\sin 30° = \frac{h}{8}$

$h = 8 \sin 30°$

$h = 4$

ⓘⓘ $d^2 + h^2 = 8^2$

$d^2 + 4^2 = 8^2$

$d^2 = 48$

$d = 6.93$

$e = 10 - d$

$e = 10 - 6.93$

$e = 3.07$

ⓘⓘⓘ. $a^2 = h^2 + e^2$

$a^2 = 4^2 + (3.07)^2$

$\boxed{a = 5.0}$

b) $\tan C = \frac{h}{e}$

$\tan C = \frac{4}{3.07}$

$\tan C = 1.303$

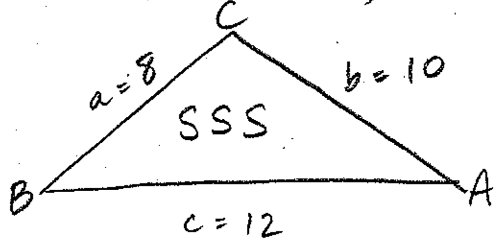
$C = \tan^{-1}(1.303)$

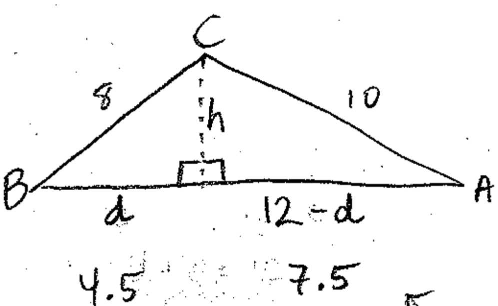
$\boxed{C = 52.5°}$

c) $\angle B = 180° - 30° - 52.5°$

$\boxed{\angle B = 97.5°}$

eg2: Given ΔABC, with $a = 8$, $b = 10$, and $c = 12$, find: a) $\angle A$ b) $\angle B$ c) $\angle C$ (nearest tenth)





a) Two Pythagorean relations for h:

$h^2 = 8^2 - d^2$

$h^2 = 10^2 - (12-d)^2$

So... $8^2 - d^2 = 10^2 - (12-d)^2$

$64 - d^2 = 100 - 144 + 24d - d^2$

$108 = 24d$

$d = 4.5$

$12 - d = 7.5$

$\cos A = \frac{7.5}{10}$

$A = \cos^{-1}(0.75)$

$\boxed{A = 41.4°}$

b) $\cos B = \frac{4.5}{8}$

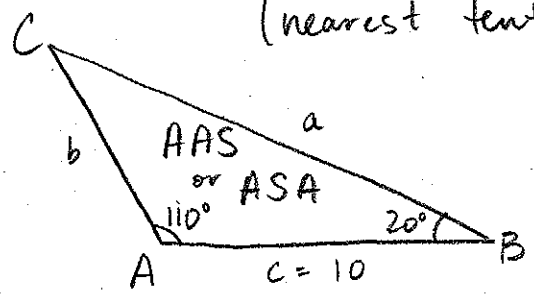
$B = \cos^{-1}(0.5625)$

$\boxed{B = 55.8°}$

c) $\angle C = 180° - 41.4° - 55.8°$

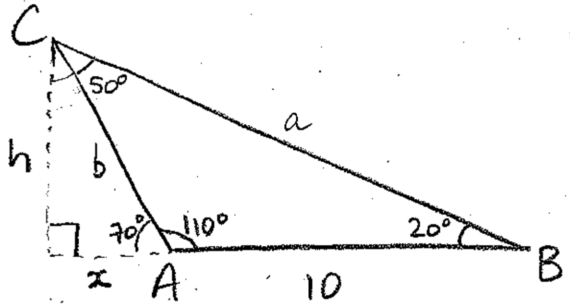
$\boxed{\angle C = 82.8°}$

eg3: Given ΔABC, with $\angle A = 110°$, $\angle B = 20°$, and $c = 10$, find: a) $\angle C$ b) b c) a
(nearest tenth)



a) $\angle C = 180° - 110° - 20°$

$\boxed{\angle C = 50°}$



1.53

b) $\tan 70° = \frac{h}{x}$; $\tan 20° = \frac{h}{x+10}$

$h = x \tan 70°$; $h = (x+10) \tan 20°$

$x \tan 70° = (x+10) \tan 20°$

$x \tan 70° = x \tan 20° + 10 \tan 20°$

$x \tan 70° - x \tan 20° = 10 \tan 20°$

$x(\tan 70° - \tan 20°) = 10 \tan 20°$

$x = \frac{10 \tan 20°}{(\tan 70° - \tan 20°)}$

$x = 1.53$

$\cos 70° = \frac{1.53}{b}$

$b = \frac{1.53}{\cos 70°}$

$\boxed{b = 4.5}$

c) need $\underline{\underline{h}}$!

$\tan 70° = \frac{h}{x}$

$\tan 70° = \frac{h}{1.53}$

$h = 1.53 \tan 70°$

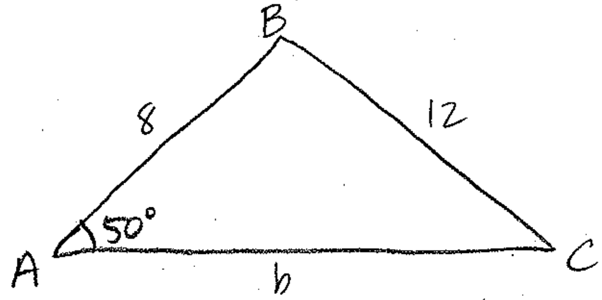
$h = 4.204$

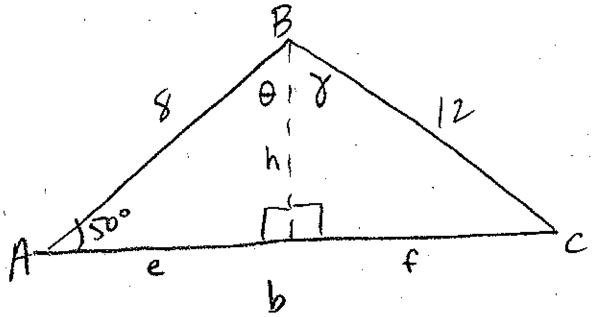
$\sin 20° = \frac{h}{a}$

$\sin 20° = \frac{4.204}{a}$

$a = \frac{4.204}{\sin 20°} = \boxed{12.3 = a}$

eg 4: Given ΔABC, with $\angle A = 50°$, $a = 12$, and $c = 8$, find: a) $\angle B$ b) $\angle C$ c) b (nearest tenth)





a) $\angle B = \theta + \gamma$

$\sin 50° = \frac{h}{8}$

$h = 8 \sin 50°$

$h = 6.13$

So... $\cos \theta = \frac{6.13}{8}$

$\theta = \cos^{-1}(0.76625)$

$\theta = 39.98°$

$\cos \gamma = \frac{6.13}{12}$

$\gamma = \cos^{-1}(0.511)$

$\gamma = 59.27°$

$\angle B = 39.98° + 59.27°$

$\boxed{\angle B = 99.3°}$

b) $\angle C = 180° - 50° - 99.3°$

$\boxed{\angle C = 30.7°}$

c) $b = e + f$

$\cos 50° = \frac{e}{8}$

$e = 8 \cos 50°$

$e = 5.14$

$\cos 30.7° = \frac{f}{12}$

$f = 12 \cos 30.7°$

$f = 10.32$

$b = 5.14 + 10.32 = \boxed{15.5 = b}$

Hwk:

p. 135–137

\# 1–12

$\boxed{\text{Area}(\Delta) = \frac{1}{2} b h}$

Ch. 3.5 - The Law of Sines

- allows ASA, AAS, and ASS triangle problems to be solved much easier than by the right-triangle method shown in the last section.

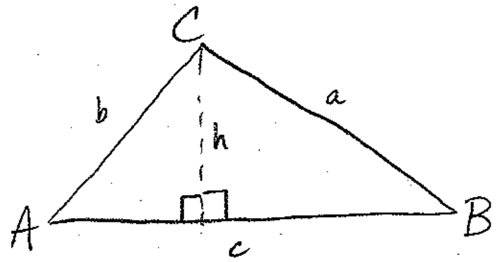
The Sine Law

If ΔABC is a triangle (acute or obtuse) with sides a, b, and c, then:

$$\boxed{① \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{OR} \quad ② \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

* if solving for an angle, use ①; if solving for a side, use ②

DERIVATION: (using an acute Δ)



let h = height of Δ

$$\sin A = \frac{h}{b} \Rightarrow h = b\sin A$$

$$\sin B = \frac{h}{a} \qquad h = a\sin B$$

So... $b\sin A = a\sin B$

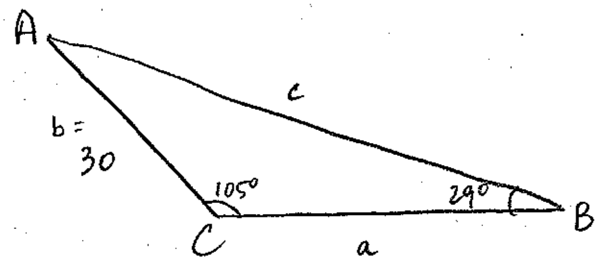
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

In a similar manner, by constructing a height (h) from B to AC, it can be shown that

$$\boxed{\frac{\sin A}{a} = \frac{\sin C}{c}}$$

eg1: Solve $\triangle ABC$, given $\angle B = 29°$, $\angle C = 105°$, and $b = 30$. (nearest tenth)

DRAW A DIAGRAM



Need to find: $\angle A$, a, c

$\angle A = 180° - 105° - 29°$

$$\boxed{\angle A = 46°}$$

AAS example

(could be ASA too)

easy to find 1st!

a:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 46°} = \frac{30}{\sin 29°}$$

$$a = \frac{30 \sin 46°}{\sin 29°}$$

$$\boxed{a = 44.5}$$

c:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

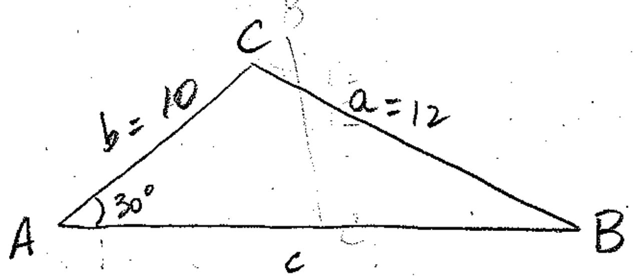
$$c = \frac{b \sin C}{\sin B}$$

$$c = \frac{30 \sin 105°}{\sin 29°}$$

$$\boxed{c = 59.8}$$

eg2: Given $\triangle ABC$, with $\angle A = 30°$, $a = 12$, and $b = 10$, solve the triangle. (nearest tenth)

GIVEN DIAGRAM:



Need to find: $\angle B$, $\angle C$, c

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{b \sin A}{a}$$

$$B = \sin^{-1}\left(\frac{b \sin A}{a}\right)$$

$$B = \sin^{-1}\left(\frac{10 \sin 30°}{12}\right)$$

$$\boxed{\angle B = 24.6°}$$

ASS example

(always unique in Math 11)

$$\angle C = 180° - 30° - 24.6° = \boxed{125.4°}$$

c: $$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{12 \sin 125.4°}{\sin 30°}$$

$$\boxed{c = 19.6}$$

Hwk: p.142 - 146

#1, 6 (don't solve for 2nd $\triangle$ in k, l, q, r)

7-10, 12

Ch. 3.6 - Law of Cosines

- must be utilized when the Law of Sines cannot be. (SAS and SSS cases)
  - * in fact, always use the Sine Law when you can; only use the Cosine Law when you must!

The Law of Cosines

For any ΔABC, and corresponding sides a, b, and c:

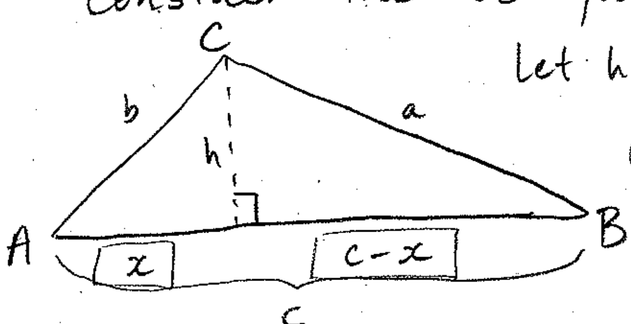
$$a^2 = b^2 + c^2 - 2bc(\cos A) \; ; \; \cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac(\cos B) \; ; \; \cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab(\cos C) \; ; \; \cos C = \frac{a^2+b^2-c^2}{2ab}$$

Derivation

Consider the oblique ΔABC:



Length c divided into two parts

Let h = height

$\cos A = \frac{x}{b} \Rightarrow x = b\cos A$

Using Pythag: ① $x^2 + h^2 = b^2$

and ② $(c-x)^2 + h^2 = a^2$

① $h^2 = b^2 - x^2$ ② $h^2 = a^2 - (c-x)^2$

$b^2 - x^2 = a^2 - (c^2 - 2cx + x^2)$

$b^2 - \cancel{x^2} = a^2 - c^2 + 2cx - \cancel{x^2}$

$a^2 = b^2 + c^2 - 2cx$

$\boxed{a^2 = b^2 + c^2 - 2cb(\cos A)}$

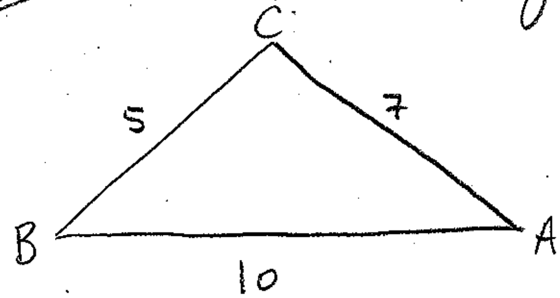
* can draw h in two other ways! ⟶

SSS Example

* Always find the largest angle first in a SSS problem (guarantees that the other two are acute).

- largest angle is opposite the longest side.

eg1: Solve ΔABC, given $a = 5$, $b = 7$, $c = 10$ (nearest tenth)



largest $\angle$ is C.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{5^2 + 7^2 - 10^2}{2(5)(7)}$$

$$\cos C = \frac{-26}{70}$$

$$C = \cos^{-1}\left(\frac{-26}{70}\right)$$

$$\boxed{\angle C = 111.8^\circ}$$

now use Sine Law to find $\angle A$ or $\angle B$:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

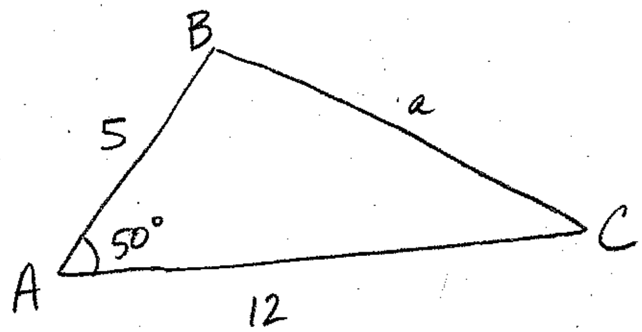
$$\sin A = \frac{5 \sin 111.8^\circ}{10}$$

$$\boxed{\angle A = 27.7^\circ}$$

$$\angle B = 180^\circ - 111.8^\circ - 27.7^\circ = \boxed{40.5^\circ}$$

eg2: SAS Example

Solve ΔABC, given $\angle A = 50°$, $b = 12$, and $c = 5$. (nearest tenth)



Find a w/ Cosine law:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$a^2 = 12^2 + 5^2 - 2(12)(5)(\cos 50°)$$

$$a^2 = 169 - 120\cos 50°$$

$$a^2 = 91.865$$

$$\boxed{a = 9.6}$$

IMPORTANT!!! Now use Sine law to find SMALLEST remaining angle!

* in this case, $\angle C$:

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \frac{5\sin 50°}{9.6}$$

$$\boxed{\angle C = 23.5°}$$

$$\angle B = 180° - 50° - 23.5°$$

$$\boxed{\angle B = 73.6°}$$

Summary of Law of Sines and Law of Cosines

Given	Method of Solving
ASA or AAS	1. Find the remaining angle using $\Delta = 180^0$ 2. Find the remaining sides using the Law of *Sines*
ASS (will ALWAYS be a unique Δ -- no ambiguous case)	1. Find an angle using the Law of *Sines* 2. Find the remaining angle using $\Delta = 180^0$ 3. Find the remaining side using the Law of *Sines*
SAS	1. Find the remaining side using the Law of *Cosines* 2. Find the **smaller** of the two remaining angles using the Law of *Sines* 3. Find the remaining angle using $\Delta = 180^0$
SSS	1. Find the **largest** angle using the Law of *Cosines* 2. Find either of the remaining angles using the Law of *Sines* 3. Find the final angle using $\Delta = 180^0$

Homework: p.150-154 #1-14 (omit 8f)

Chapter Review p.155-157 #1-11 (omit 9g)